Ministry of Education and Science of Ukraine
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of National Aviation University

O. Kuz`menko, N. Sadoviy

PHYSICS

FOR TECHNICAL SPECIALITIES

MECHANICS. MOLECULAR PHYSICS AND THERMODYNAMICS. ELECTRICITY AND ELECTROMAGNETISM. OSCILLATIONS AND WAVE OPTICS. QUANTUM AND ATOMIC PHYSICS.

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R e v i e w e r s:

O.V. Volchanskiy - Ph.D., Associate Professor of Physics and Methods of Teaching Department the Kirovohrad Volodymyr Vynnychenko State Pedagogical University.

O.N. Gavrylenko - Ph.D., Associate Professor, Associate Professor of Foreign Languages Department of the Central Ukrainian National Technical University.

I.A. Slipukhina - Doctor of Pedagogical Sciences, Professor, Department of Physics of National Aviation University.

N.G. Cherednychenko, Senior Lecturer in English Kirovohrad Flight Academy of National Aviation University.

Пропонований посібник містить основний навчальний матеріал з розділів «Механіка», «Молекулярна фізика та термодинаміка», «Електромагнетизм», «Коливання та хвильова оптика», «Квантова та атомна фізика».

Навчальний посібник призначений для іноземних студентів вищих навчальних закладів технічного напрямку навчання, а також для викладачів вузів.

O. Kuz`menko, …

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The proposed guide provides basic educational material sections «Mechanics», «Molecular Physics and Thermodynamics», «Electromagnetism», «Oscillations and wave optics», «Quantum and atomic physics».

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Recommended by the Academic Council of Kirovohrad Flight Academy of National Aviation University as a textbook for foreign students of technical specialties and teachers (Protocol № 5 from 29.06.2017)

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CONTENTS

Useful Constants and Units

Physical constants

speed of light $c = 2.997\ 924\ 58 \times 10^{8}$ m /s
gravitational constant $G = 6.673 \times 10^{-11}$ N m^2 kg^{-2}
Planck's constant/$2\pi = 1.054\ 57 \times 10^{-34}$ J s
mass of hydrogen atom $m_H = 1.673\ 52 \times 10^{-27}$ kg
mass of electron $m_e = 9.109\ 38 \times 10^{-31}$ kg
charge of proton $e = 1.602\ 18 \times 10^{-19}$ C
permittivity of vacuum $\varepsilon_0 = 8.854\ 19 \times 10^{-12}$ F m^{-1}
permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

Defined standard values

standard gravitational acceleration $g_n = 9.806\ 65$ m s^{-2}
normal atmospheric pressure 1 atm = 1.013 25 × 105 Pa = 1.013 25 bar

Properties of Earth

mass	$M = 5.974 \times 10^{24}$ kg
	$G_M = 3.9860 \times 1014$ m^3 s^{-2}
radius (polar)	Rp = 6356.8 km
(equatorial)	$R_e = 6378.1$ km
(mean)[= (R^2 eR_p)1/3]	R = 6371.0 km
semi-major axis of orbit	$a = 1.495\ 98 \times 10^{8}$ km
eccentricity of orbit	e = 0.016 722
orbital period (sidereal year)	$\tau = 3.155\ 75 \times 10^{7}$ s
mean orbital velocity	v = 29.785 km s^{-1}
surface escape velocity (mean)	$v_e = 11.18$ km s^{-1}
rotational angular velocity	$\omega = 7.2921 \times 10^{-5}$ s^{-1}

Properties of Sun and Moon

mass of Sun $M_S = 1.989 \times 10^{30}$ kg $= 3.329\ 5 \times 10^{5}$ M
GMS = $1.327\ 12 \times 10^{20}$ m^3 s^{-2}
mass of Moon $M_M = 7.348 \times 10^{22}$ kg = 0.012 300 M
semi-major axis of lunar orbit $a_M = 3.8440 \times 10^{5}$ km
lunar orbital period (sidereal month) $\tau_M = 2.3606 \times 10^{6}$ s

SI units

kilogramme (kg): mass of the international standard kilogramme kept at S`evres in France.
second (s): 9 192 631 770 oscillation periods of the hyperfine transition between the levels F = 4, $m_F = 0$ and F = 3, $m_F = 0$ in the ground state of ^{133}Cs.

metre (m): distance travelled by light in vacuum in (1/299 792 458) s.
ampere (A): defined so that the force per unit length between two infinitely long parallel wires of negligible cross section 1 m apart in vacuum, each carrying a current of 1 A is 2×10^{-7} N m^{-1} (or, equivalently, so that the constant μ_0 has the precise value $4\pi \times 10^{-7}$ N A^{-2}).

Subsidiary units

newton	1 N = 1 kg m s^{-2}	pascal	1 Pa = 1 N m^{-2}
joule	1J = 1 Nm	ton(ne)	1 t = 10^3 kg
watt	1W = 1 Js^{-1}	bar	1 bar = 10^5 Pa
coulomb	1C = 1 As	hertz	1 Hz = 1 s^{-1}

British and American units

foot 1 ft = 0.3048 m

pound 1 lb = 0.452 592 37 kg

Prefixes denoting multiples and submultiples

10^3 kilo (k)	10^{-3} milli (m)
10^6 mega (M)	10^{-6} micro (μ)
10^9 giga (G)	10^{-9} nano (n)
10^{12} tera (T)	10^{-12} pico (p)
10^{15} peta (P)	10^{-15} femto (f)
10^{18} exa (E)	10^{-18} atto (a)

List of Symbols

Table 1

$\vec{a}$	acceleration	прискорення
a	absorptivity (absorption factor)	коефіцієнт поглинання
a_p	coefficient peltie	коефіцієнт Пельтьє
a_v	spectral absorption	спектральне поглинання
A	work, amplitude	робота
A	number of nucleons	кількість нуклонів
$\vec{B}$	induction of the magnetic field	індукція магнітного поля
B	charge of baryons	число баріонів
B_r	remanence	коерцитивної сила
C, C_V, C_p	heat capacities	теплоємності
C	electric capacity	електрична ємність
d	diameter, constant of diffraction grating, lattice spacing	діаметр, постійна дифракційної решітки, крок решітки
d_m	diameter of molecule	діаметр молекули
D	coefficient of diffusion, barrier transparency	коефіцієнт дифузії, прозорість бар'єру
e	charge of electron	заряд електрона
ε_k	kinetic energy of molecule	кінетична енергія молекул
ε	electromotive Force (EMF), dielectric permittivity, deformation	електрорушійна сила (ЕРС), діелектрична проникність, деформація
ε_{si}	back electromotive force (self-induced EMF)	електрорушійна сила (ЕРС самоіндукції)
ε_m	magnetomotive Force (MMF)	Магніторушійна сила (МДС)
E, W	energy, field strength	енергія, напруженість поля
E	elastic moduls (Young`s modulus)	модуль пружності (модуль Юнга)

Continuation of Table 1

E	radiant emittance	випромінювання
E_k	kinetic energy	кінетична енергія
$\vec{F}$	force	сила
F	free energy	вільна енергія
$\vec{g}$	free fall acceleration	прискорення вільного падіння
G, P	weight of a body	вага тіла
G	shear modulus	модуль зсуву
h	height	висота
i	degrees of freedom of molecule	кількість ступенів руху
I, J	enthalpy	ентальпія
j	denisity of current	щільність струму
I	electrical carent, indensity	електричний струм, інтенсивність
I	momentum of inertia	момент інерції
Δl	length	довжина відрізка
$\vec{L}$	angular momentum	момент імпульсу
k	coefficient absorption, coefficient of elasticity	коефіцієнт поглинання, коефіцієнт еластичності
L	inductance	індуктивність
l, d	length	довжина
m, M	mass, spectrum order	маса
$\vec{M}$	moment of force (torque)	момент сили
n	concentration, refraction index	концентрація, індекс заломлення
n	frequency of rotation	обертання
N , n_0	amount of moleculas	кількість молекул
N	power	потужність
N	number of neutrons	кількість нейтронів
$\vec{p}$	momentum	імпульс

Continuation of Table 1

P	loss of power	втрата потужності
$P,\ p$	pressure	тиск
$\vec{P}$	total momentum	повний імпульс
Q	amount of heat	кількість теплоти
Q	quality factor, charge electric	коефіцієнт в\`язкості, електричний заряд
$\vec{r}$	position vector	радіус-вектор
R, r	radius	радіус
$R\ ,\ R_x$	Resistance ----wave	опір, ----хвилі
s	distance	шлях
S	entropy	ентропія
S	area	площа
T	absolute temperature, coefficient of transmission	абсолютна температура, коефіцієнт передачі
$T_{\deg}$	temperature of degeneration	температура виродження
T	period of oscillations	період коливань
$T_{1/2}$	half-life period	період напіврозпаду
t	time, temperature	час, температура
U	potential energy	потенціальна енергія
U	voltage	напруга
$U_{R,}U_{C,}U_{L}$	voltage drope	падіння напруги
υ	speed	швидкість
$\vec{\upsilon}$	velocity	швидкість
V	volume	об\`єм
$\vec{x}\ ,\ \vec{y}\ ,\ \vec{z}$	cartesian coordinates	декартові координати
$X\ ,\ X_C,\ X_L$	reactance	реактивний опір
$\vec{\varepsilon}$	angular acceleration	кутове прискорення
η	efficiency	коефіцієнт корисної дії
λ_c	length of Compton	довжина Комптона

Continuation of Table 1

λ	free path of molecule, wavelength, constant of radioactivity decay	довжина вільного пробігу, довжина хвилі, постійна радіоактивного розпаду
μ	molar mass, magnetic permittivity, chemical potential	молярна маса, магнітна проникність, хімічний потенціал
ν	frequency	частота
ρ	density	густина
ρ	coefficient of photons	коефіцієнт фотонів
Z	Impedance, number of protons	опір, число протонів
Z_e	Charge of nucleus	число нуклонів
σ	cross sectional area	площа поперечного перерізу
σ	surface tension	коефіцієнт поверхневого натягу
τ	time of free path	час вільного пробігу
φ	angle of rotation	кут повороту
$\vec{\varphi}$	angle displacement vector	вектор кутового переміщення
χ	coefficient of heat conduction	коефіцієнт теплопровідності
χ	dielectric susceptibility	діелектрична сприйнятливість
ϖ, P	probability of the state	ймовірність стану
ϖ	energy density	густина енергії
ω	cyclic frequency	циклічна частота
$\vec{\omega}$	angular velocity	кутова швидкість
α_T	coefficient thermo-electromotive force	коефіцієнт термо-ЕРС
α	rotation number	число обертів
β	damping factor	коефіцієнт затухання
β_T	coefficient Thomson	коефіцієнт Томсона
γ	damping constant, -- logarithmic	коефіцієнт згасання, логарифмічний коефіцієнт
δ	small variation	мала величина
δ	Feigenbaum number	номер Фейгенбаума

Continuation of Table 1

θ	polar angle, Euler angle	полярний кут, кут Ейлера
Θ, θ, θ^*	scattering angle	кут розсіювання
λ, λ_i	Lyapunov exponent	експонента Ляпунова
λ_i	Eigen values	власні значення
μ	reduced mass	зведена маса
μ_0	permeability of free space	проникність вільного простору
ξ, ξ,η	linearized phase-space co-ordinates	координати в фазовому просторі
π	circumference/diameter ratio of circle	число π
ρ	cylindrical polar co-ordinate	циліндричні полярні координати
$\mathrm{\rho}$	mass or charge density	щільність маси або заряду
σ, $d\sigma$	cross-section	поперечний переріз
σ	surface charge density	поверхнева щільність заряду
σ_v	volume charge denisity	об`ємна щільність заряду
σ_l	liner charge denisity	лінійна щільність заряду
τ	Period, time of life	період, час життя
Φ	gravitational potential, chemical potential, Magnetic flux	гравітаційний потенціал, хімічний потенціал, магнітний потік
φ	electrostatic potential	електростатичний потенціал
ϕ	azimuth angle, Euler angle, phase of oscilations	кут азимуту, кут Ейлера, фаза коливань
φ_i	angle variables	кутові змінні
ψ	Euler angle, wave function	кут Ейлера, хвильова функція
$d\Omega$	solid angle	тілесний кут

Continuation of Table 1

η	efficiency	ефективність
æ	magnetic susceptibilyti	магнітна проникність
ω	angular frequency	кутова частота
ω_i	natural angular frequencies	природні кутові частоти
∇	vector differential operator	векторний оператор
Δ	optical path difference	різниця оптичного шляху
a b	scalar product of *a* and *b*	скалярний добуток
a ∧ b	vector product of *a* and *b*	векторний добуток

INTRODUCTION

Physics (from Ancient Greek: φυσική (ἐπιστήμη) *phusikḗ (epistḗmē)* "knowledge of nature", from φύσις *phúsis*"nature"[1; 2; 3]) is the natural science that involves the study of matter [4] and its motion and behavior throughspace and time, along with related concepts such as energy and force [5]. As one of the most fundamental scientific disciplines, the main goal of physics is to understand how the universe behaves [6; 7; 8].

Physics is one of the oldest academic disciplines, perhaps the oldest through its inclusion of astronomy [9]. Over the last two millennia, physics was a part of natural philosophy along with chemistry, biology, and certain branches of mathematics, but during the scientific revolution in the 17-th century, the natural sciences emerged as unique research programs in their own right. Physics intersects with many interdisciplinary areas of research, such as biophysics and quantum chemistry, and the boundaries of physics are not rigidly defined. New ideas in physics often explain the fundamental mechanisms of other sciences [6] while opening new avenues of research in areas such as mathematics and philosophy.

Physics also makes significant contributions through advances in new technologies that arise from theoretical breakthroughs. For example, advances in the understanding of electromagnetism or nuclear physics led directly to the development of new products that have dramatically transformed modern-day society, such as television, computers, domestic appliances, and nuclear weapons;[6] advances in thermodynamics led to the development of industrialization, and advances in mechanics inspired the development of calculus.

Physics became a separate science when early modern Europeans used experimental and quantitative methods to discover what are now considered to be the laws of physics [18].

Major developments in this period include the replacement of the geocentric model of the solar system with the heliocentric Copernican model, the laws governing the motion of planetary bodies determined by Johannes Kepler between 1609 and 1619, pioneering work on telescopes and observational astronomy by Galileo Galilei in the 16-th and 17-th Centuries, and Isaac Newton's discovery and unification of the laws of motion and universal gravitation that would come to bear his name [19] Newton also developed calculus, the mathematical study of change, which provided new mathematical methods for solving physical problems [20].

The discovery of new laws in thermodynamics, chemistry, and electromagnetics resulted from greater research efforts during the Industrial Revolution as energy needs increased [21]. The laws comprising classical physics remain very widely used for objects on everyday scales travelling at non-relativistic speeds, since they provide a very close approximation in such situations, and theories such as quantum mechanics and the theory of relativity simplify to their classical equivalents at such scales. However, inaccuracies in classical mechanics for very small objects and very high velocities led to the development of modern physics in the 20-th century.

Classical mechanics is one of the most familiar scientific theories. Its basic concepts - mass, acceleration, force, and others become very much a part of our everyday modes of thought. So we may easily regard their physical meaning as more obvious than it really is. For this reason, a large part of this introductory chapter will be devoted to a critical examination of the fundamental concepts and principles of mechanics.

Every scientific theory starts from a set of hypotheses, which are suggested by our observations, but represent an idealization of them. The theory is then tested by checking the predictions deduced from these hypotheses against experiment. When persistent discrepancies are found, we try to modify the hypotheses to restore the agreement with observation. If many of such tests are made and no serious disagreements emerge, then the hypotheses gradually acquire the status of ‘laws of nature’. When results that apparently contradict well-established laws appear, as they often do, we tend to look for other possible explanations - for simplifying assumptions we have made that may be wrong, or neglected effects that may be significant.

The laws of classical mechanics are no exception. Since they were first formulated by Galileo and by Newton in his Principia, their range of known validity has been enormously extended, but in two directions they have been found to be inadequate. For the description of the small-scale phenomena of atomic and nuclear physics, classical mechanics has been superseded by quantum mechanics, and for phenomena involving speeds approaching that of light, by relativity. This is not to say that classical mechanics has lost its value. Indeed both quantum mechanics and the special and general theories of relativity are extensions of classical mechanics in the sense that they reproduce its results in appropriate limiting cases. Thus the fact that these theories have been confirmed, actually reinforces our belief in the correctness of classical mechanics within its own vast range of validity. Indeed, it is a remarkably successful theory, which provides a coherent and satisfying account of phenomena as diverse as the planetary orbits, the tides and the motion of a gyroscope. Moreover, even outside this range, many of the results of classical mechanics still apply. In particular, the conservation laws of energy, momentum and angular momentum are, so far as we yet know, of universal validity.

Modern physics began in the early 20-th century with the work of Max Planck in quantum theory and Albert Einstein's theory of relativity. Both of these theories came about due to inaccuracies in classical mechanics in certain situations. Classical mechanicspredicted a varying speed of light, which could not be resolved with the constant speed predicted byMaxwell's equations of electromagnetism; this discrepancy was corrected by Einstein's theory ofspecial relativity, which replaced classical mechanics for fast-moving bodies and allowed for a constant speed of light [22].

Black body radiation provided another problem for classical physics, which was corrected when Planck proposed that the excitation of material oscillators is possible only in discrete steps proportional to their frequency; this, along with the photoelectric effect and a complete theory predicting discrete energy levels of electron orbitals, led to the theory of quantum mechanics taking over from classical physics at very small scales [23].

Quantum mechanics would come to be pioneered by Werner Heisenberg, Erwin Schrödinger and Paul Dirac [23]. From this early work, and work in related sphere, the Standard Model of particle physics was derived [24]. Following the discovery of a particle with properties consistent with the Higgs boson at CERN in 2012 [25], all fundamental particles predicted by the standard model, and no others, appear to exist; however, physics beyond the Standard Model, with theories such as super symmetry, is an active area of research [26]. Areas of mathematics in general are important to this field, such as the study of probabilities and groups.

The proposed guide provides basic educational material sections «Mechanics», «Molecular Physics and Thermodynamics», «Electromagnetism», «Oscillations and wave optics», «Quantum and atomic physics».

The manual is intended for foreign students of technical training universities as well as for professors and for teachers.

Chapter 1 **MECHANICS**

1.1 KINEMATICS

1.1.1 Kinematics of material point

The elementary view of a motion in the nature is the mechanical motion, consisting in change of a relative positioning of bodies or their parts in space eventually. The section of physics which is engaged in studying of laws of a mechanical motion is termed **as a mechanics**.

Distinguish the **classical mechanics**, when velocities of macroscopical bodies of essentially less light speed. The classical mechanics is grounded on Newton's laws, therefore it often term as a Newtonian mechanics. Motions of bodies with velocities close to light speed it is studied in a **relativistic mechanics**, and laws of a motion of microparticles in a **quantum mechanics**.

The classical mechanics consists of three basic sections – a statics, kinematics and dynamics. The **statics** – studies laws of a composition of forces and a requirement of balance of bodies. **The Kinematics** (motion) – gives the mathematical description of a motion of bodies without reason causing this motion. **Dynamics** – studies a motion of bodies taking into account **forces operating on them.**

The **motion** in the mechanic terms change of a relative positioning of bodies. For the description of a motion of bodies it is necessary to choose prestressly a **reference system**, i.e. to choose one or several bodies which conventionally are accepted to immobile, and to them to relate any coordinate system and hours.

Perfectly rigid body terms a body which strain in the conditions of the given problem can be neglected. The distance between any two points perfectly rigid body does not change at any interactions.

The body, in relation to which the motion of other bodies is considered, is termed **as a body frame**.

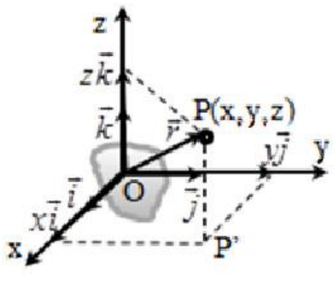


Figure 1.1

The rectangular, Cartesian frame formed by three crossly perpendicular axes *X*, *Y*, *Z* is most often used.

Unit vector along these axes are termed orts - $\vec{i}$, $\vec{j}$, $\vec{k}$. They lay out the origin of *O*. Position of a point *P* is characterized by the radius vector $\vec{r}$,connecting the origin *O* with a point of *P* (Figure 1.1).

X, *Y*, *Z* – Cartesian coordinates of the point *P* or projections of the radius vector

$\vec{r}$ on the respective axes of coordinates. Character of a motion of a body in space will be set, if we know, how change in time of co-ordinate or its radius vector, i.e. dependences $x=x(t)$; $y = y\ (t)$; $z = z(t)$ will be determined.

Solving a physical problem some factors which in the given problem not essential, neglect, for example, it is often possible to neglect the sizes of the body which motions it is studied.

The body which sizes in the conditions of the given problem can be neglected, is termed **as the material point**.

Line, described by a material point in its motion in space, called the trajectory. The distance between two point position, measured along a path called the path traveled by the body (A path – trajectory length.)

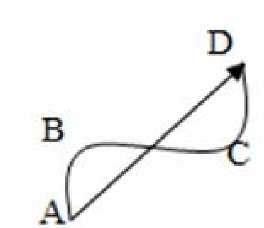


Figure 1.2

Vector between the initial position of the body and the end position, called the **displacement vector** (Figure 1.2).

ABCD – a trajectory; $\overrightarrow{AD}$ - displacement vector.

Depending on the trajectory shape distinguish a rectilinear and curvilinear motion of a point.

If the body trajectory represents a straight line, a motion – rectilinear, a curve – curvilinear.

Besides distinguish translational and a rotary motion (Figure 1.3).

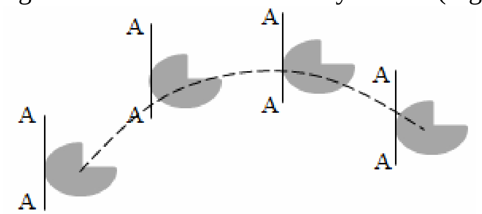


Figure 1.3

The body motion is termed translational if any straight line spent in a body, remains at a motion of this body parallel to itself (at this motion of a trajectory of all points of a body identical) (Figure 1.4).

Speed and Velocity

Average speed on any part of the trajectory is the ratio of the increment of the radius vector of a point in the time interval t +Δt to its duration Δt.

$$\overrightarrow{\upsilon_{av}} = \frac{\Delta\vec{r}}{\Delta t}, \qquad (1.1)$$

(The average velocity of the body in any part of the trajectory is the ratio of the length *S* of the site at the time *t*, during which the body passed this site).

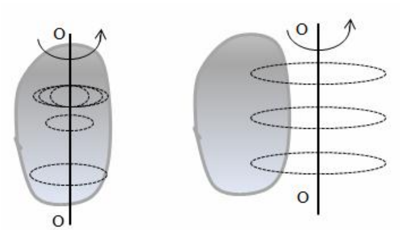


Figure 1.4

At a rotary motion all points of the body move in circles whose centers lie on the same straight line, called the axis of rotation, the axis of rotation can be outside of the body.

If for the sites of any length taken in various places of a trajectory, this relation is identical, velocity of a body along a trajectory is constant also such motion is termed as the uniform (Figure 1.5).

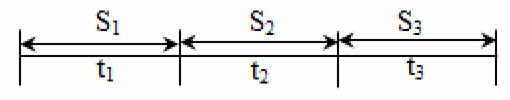


Figure 1.5

$$\overrightarrow{\upsilon_1} = \frac{S_1}{t_1},\ \overrightarrow{\upsilon_2} = \frac{S_2}{t_2},\ \overrightarrow{\upsilon_3} = \frac{S_3}{t_3}\ , \tag{1.2}$$

$$S = \upsilon \cdot t\ , \tag{1.3}$$

$$\vec{\upsilon} = \frac{S}{t} = \frac{S_1 + S_2 + S_3}{t_1 + t_2 + t_3}, \tag{1.4}$$

Speed υ point is called a vector quantity $\vec{\upsilon}$, equal to the first time derivative of the radius vector $\vec{r}$ of the viewed point.

$$\vec{\upsilon} = \frac{d\vec{r}}{dt}\ , \tag{1.5}$$

$$[\upsilon] = \frac{m}{s}, \tag{1.6}$$

(Velocity of a point at time t is equal to the limit of the average speed v_{av} at $\Delta t \to 0$).

$$\vec{\upsilon} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \lim_{\Delta t \to 0} \upsilon_{med}\ . \tag{1.7}$$

In general, the path S is different from the module move $|\Delta r|$ (Figure 1.6). Equally, if we consider the way dS, passable point for a small period of time dt, then $dS = |dr|$. Therefore the modulus of the velocity vector is the first derivative of the path length of the time.

Average ground speed of uneven movement of the point on the section of its trajectory is called a scalar quantity equal to the ratio of the length υ_{av} of this section, the trajectory to the duration Δt of its passage point.

Average path velocity of a non-uniform motion of a point on the section of its trajectory is called scalar value υ_{av} equal to the relation of length of this section a trajectory, to duration Δt passages by its point.

It is possible to present a velocity vector in a view

$$\upsilon = \left|\vec{\upsilon}\right| = \frac{dS}{dt}\ , \tag{1.8}$$

It is possible to present a velocity vector in a view

$$\vec{\upsilon} = \upsilon_x \vec{l_x} + \upsilon_y \vec{l_y} + \upsilon_z \vec{l_z}\ , \tag{1.9}$$

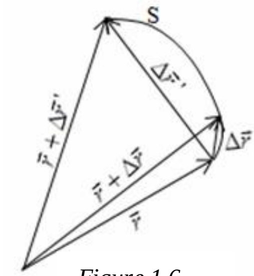


Figure 1.6

$\upsilon_x, \upsilon_y, \upsilon_z$ - projection of the vector $\vec{\upsilon}$ on the axes

$$\upsilon_x = \dot{X} = \frac{dx}{dt}, \upsilon_y = \dot{Y} = \frac{dy}{dt}, \upsilon_z = \dot{Z} = \frac{dz}{dt}, \tag{1.10}$$

$$\upsilon = \sqrt{\upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\ . \tag{1.11}$$

The vector of velocity $\vec{\upsilon}$ of a point is guided on a tangent to a trajectory towards a motion as well a vector $dr = \upsilon\ dt$ of small displacement of a point for all time interval *d*t (a vector that is tangent to be from the physical meaning of the first derivative - is tangent to the graph of the function indicates the velocity of motion at time *t*).

Acceleration

Tangential and normal components of the acceleration

Acceleration - is a vector quantity that characterizes the rate of change of velocity of the moving body in magnitude and direction.

Point **average acceleration** in the time interval Δ*t* is the vector a_{av} increment equal to the ratio of the velocity vector Δ*v* to the time interval Δ*t*

$$\overrightarrow{a_{av}} = \frac{\Delta\vec{\upsilon}}{dt}\ , \tag{1.12}$$

Acceleration (instantaneous acceleration) point is called a vector quantity $\vec{a}$, which is equal to the first derivative of the velocity *v* in time (or the second derivative of the radius vector $\vec{r}$ of the time *t*)

$$\vec{a} = \frac{d\vec{\upsilon}}{dt} = \frac{d^2\vec{r}}{dt^2}, \tag{1.13}$$

$$[a] = \frac{m}{s^2}, \quad (1.14)$$

Acceleration of a point at time t is equal to the limit of the average acceleration of $\overrightarrow{a_{av}}$ when $\Delta t \to 0$

$$a = \lim_{\Delta t \to 0} \frac{\Delta \vec{\upsilon}}{\Delta t} = \lim_{\Delta t \to 0} \vec{a}_{av}, \quad (1.15)$$

In a Cartesian coordinate system vector $\vec{a}$ can be written in terms of its coordinates

$$\vec{a} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k}, \quad (1.16)$$

where

$$a_x = \frac{d\upsilon_x}{dt} = \frac{d^2 x}{dt^2}, a_y \frac{d\upsilon_y}{dt} = \frac{d^2 y}{dt^2}, a_z = \frac{d\upsilon_z}{dt} = \frac{d^2 z}{dt^2}, \quad (1.17)$$

Module of the acceleration vector

$$a = |\vec{a}| = \sqrt{\left(\frac{d\upsilon_x}{dt}\right)^2 + \left(\frac{d\upsilon_y}{dt}\right)^2 + \left(\frac{d\upsilon_z}{dt}\right)^2} = \sqrt{\left(\frac{d^2 x}{dt^2}\right)^2 + \left(\frac{d^2 y}{dt^2}\right)^2 + \left(\frac{d^2 z}{dt^2}\right)^2}, \quad (1.18)$$

Vector $\vec{a}$ can be expressed as the sum of two components:

$\vec{a}_\tau$ - tangential component of acceleration is tangential to the trajectory of the point and is equal to

$$\vec{a}_\tau = \frac{d\upsilon}{dt}\vec{\tau}, \; a_\tau = \frac{d\upsilon}{dt}. \quad (1.19)$$

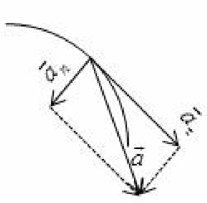


Figure 1.7

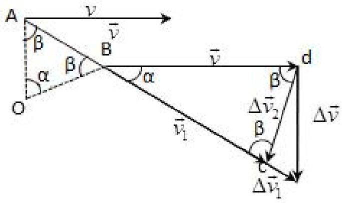


Figure 1.8

where the vector $\vec{\tau} = \frac{\vec{\upsilon}}{\upsilon}$ - the unit vector of the tangent drawn at the point of the trajectory and direction of the velocity $\vec{\upsilon}$.

Vectors $\vec{a}_\tau$ and $\vec{\upsilon}$ collinear with uniformly accelerated motion; $\vec{a}_\tau \uparrow\downarrow \vec{\upsilon}$ at $a_\tau < 0$ *i*.e. at uniformly retarded motion.

Tangential acceleration $\vec{a}_\tau$ - characterizes the quickness of change of velocity vector modulus of (measures change in velocity magnitude).

For uniform motion

$$\overrightarrow{a_\tau} = const \neq 0 \quad \vec{\upsilon} = \overrightarrow{\upsilon_0} + \overrightarrow{a_\tau} t \quad S = S_0 + \upsilon_0 t + a_\tau \frac{t^2}{2}, \tag{1.20}$$

$\vec{a}_n$ - normal component of acceleration (normal acceleration) along the normal to the trajectory and the given point in the direction of the center of curvature of the trajectory. Curved trajectory can be represented as a set of elementary sections, each of which can be seen as a circular arc of radius R (called the radius of curvature of a circle of a given point of the trajectory).

$$\Delta\vec{\upsilon} = \Delta\overrightarrow{\upsilon_1} + \Delta\overrightarrow{\upsilon_2} \quad \Delta AOB \sim \Delta dbc \Rightarrow \frac{AB}{R} = \frac{\Delta\upsilon_2}{\upsilon}, \; \upsilon = \frac{AB}{\Delta t}, \tag{1.21}$$

$$|Bd| = |BC| \quad a_n = \lim_{\Delta t \to 0} \frac{\Delta\upsilon_2}{\Delta t} = \lim_{\Delta t \to 0} \frac{AB}{t} \frac{\upsilon}{R} = \frac{\upsilon^2}{R}, \tag{1.22}$$

$$a_n = \frac{\upsilon^2}{R}, \tag{1.23}$$

Normal acceleration characterizes the speed change of direction of the velocity vector (characterizes the change in the direction of the velocity).

Full acceleration module:

$$a = \sqrt{a_\tau^2 + a_n^2} = \sqrt{\left(\frac{dv}{dt}\right)^2 + \frac{\upsilon^2}{R}}, \tag{1.24}$$

The classification depends on the movements of the tangential and normal components:

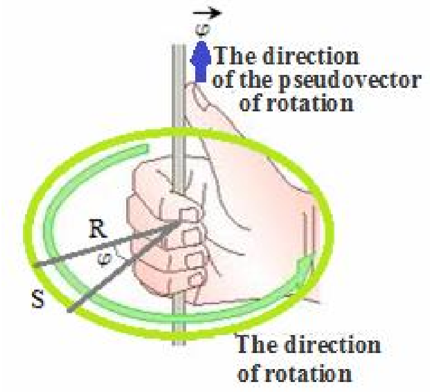
1. $a_\tau = 0; a_n = 0; \; \upsilon = const; \; S = \upsilon \cdot t$ - constant motion;
2. $a_\tau = const > 0; a_n = 0; \upsilon_2 > \upsilon_1; \; S = S_0 + \upsilon_0 t + \frac{at^2}{2}$ - uniformly accelerated motion;
3. $a_\tau = const < 0; a_n = 0; \upsilon_2 < \upsilon_1; \; S = S_0 + \upsilon_0 t - \frac{at^2}{2}$ - uniformly retarded motion;
4. $a_\tau = f(t); a_n = 0$ - linear motion with variable acceleration;
5. $a_\tau = 0; a_n = const; a_n = \frac{\upsilon^2}{R}$ - uniform circular motion;
6. $a_\tau = 0; a_n = f(t)$ - uniform curvilinear motion;
7. $a_\tau = const > 0; a_n \neq 0; \upsilon_2 > \upsilon_1$ - curved uniformly accelerated motion;
8. $a_\tau = const < 0; a_n \neq 0; \upsilon_2 < \upsilon_1$ - curvilinea uniformly retarded motion;
9. $a_\tau = f(t); a_n \neq 0$ - curvilinear motion with variable acceleration.

1.1.2 The kinematics of rotational motion

Rotation of the body at a certain angle φ can be described by a vector of length φ, and the direction coincides with the axis of rotation is determined by the rule of the right screw (corkscrew, right hand):

Four fingers of the right hand – on the direction of rotation, bent thumb indicates the direction of the vector $\vec{\varphi}$.

The direction of the rotation vector φ, associated with the direction of **rotation right-hand rule** (Figure 1.9). Such vectors are referred to as the axial or pseudo-to distinguish them from ordinary (sometimes called field) vectors. Called the angular velocity vector $\vec{\omega}$ which is numerically equal to the first derivative of the angle of rotation $\vec{\varphi}$ on time *t* and is directed along a fixed axis on the right hand rule.



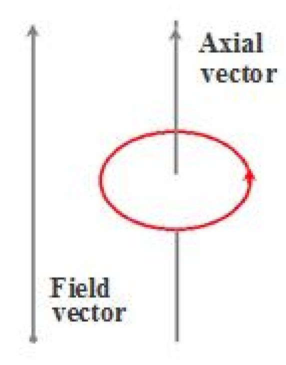


Figure 1.9 *Figure 1.10*

$$\vec{\omega} = \frac{d\vec{\varphi}}{dt} \quad \left|\vec{\omega}\right| = \frac{d\,|\vec{\varphi}|}{dt} \quad [\omega] = \frac{rad}{c}$$

Angular velocity $\vec{\omega}$, as $\vec{\varphi}$ is an axial vector. Axial vectors do not have certain points of application, they can be deposited from any point on the axis of rotation (Figure 1.10). Often they are put off by a stationary point of the rotation axis, taken simultaneously as the origin of the reference framet. Rotation of the body is called uniform if $\varphi = \omega t$.

$\Delta S = \Delta\varphi \cdot R$ divide by Δt

$$\frac{\Delta S}{\Delta t} = \frac{\Delta\varphi}{\Delta t} \cdot R \quad \upsilon = \omega \cdot R. \tag{1.25}$$

Speed $\vec{\upsilon}$ points as opposed to the angular velocity $\vec{\omega}$ of a body, called the linear speed. It is perpendicular to both the axis of rotation (i.e., the vector $\overrightarrow{\omega}$), and the radius – vector *R*, drawing to a point *P* from the center of the circle and about equal to the vector product:

$$\overrightarrow{\upsilon} = [\overrightarrow{\omega}\overrightarrow{R}] = [\overrightarrow{\omega}\overrightarrow{R}] \;\; \upsilon = \omega \cdot \rho \quad \overrightarrow{r} = \overrightarrow{OO`} + \overrightarrow{R}. \tag{1.26}$$

Uniform rotation can be a characterization of the rotation period *T*, which are defined as the time in which the body makes one revolution, i.e. rotated by an angle $\varphi = 2\pi$. Then

$$\omega = \frac{\varphi}{t} = \frac{2\pi}{T}, \tag{1.27}$$

- relationship of the angular velocity with the circulation period.

$$T = \frac{2\pi}{\omega}, \tag{1.28}$$

- rotating speed - number of revolutions per unit of time.

$$\nu = n = \frac{1}{T} = \frac{\omega}{2\pi}; \; [\nu] = \frac{1}{s}; \; [n] = \frac{1}{s}. \tag{1.29}$$

In the case of variable rotational motion angular velocity $\overrightarrow{\omega}$ of material point changes in both magnitude and direction. To characterize the rate of change of the angular velocity $\overrightarrow{\omega}$ in irregular rotation around a fixed axis vector $\overrightarrow{\varepsilon}$ is introduced - angular acceleration of the body is equal to the first derivative of its angular velocity $\overrightarrow{\omega}$ on time

$$\overrightarrow{\varepsilon} = \frac{d\overrightarrow{\omega}}{dt} = \frac{d^2\overrightarrow{\varphi}}{dt^2}, \tag{1.30}$$

$$[\varepsilon] = \frac{rad}{s^2}. \tag{1.31}$$

Vector $\overrightarrow{\varepsilon}$ is also an axial (or pseudovector). Vectors $\overrightarrow{\varepsilon}$ and $\overrightarrow{\omega}$ same direction for accelerated rotation

$$\frac{d\omega}{dt} > 0, \text{ i.e. } \varepsilon > 0 \tag{1.32}$$

and $\vec{\varepsilon} \uparrow\downarrow \vec{\omega}$ opposite directions during decelerated rotation

$$\frac{d\omega}{dt} < 0, \text{ i.e. } \varepsilon < 0. \tag{1.33}$$

Acceleration $\overrightarrow{a}$ arbitrary point *P* of the body in contrast to the angular acceleration $\overrightarrow{\varepsilon}$ of the body is called linear acceleration.

$$\overrightarrow{a} = \frac{d\overrightarrow{\upsilon}}{dt} = \frac{d^2\overrightarrow{r}}{dt^2} = \overrightarrow{a_\tau} + \overrightarrow{a_n}, \tag{1.34}$$

$$\overrightarrow{a_\tau} = [\vec{\varepsilon} \times \overrightarrow{R}]; \; \overrightarrow{a_n} = -\omega^2 \overrightarrow{R}. \tag{1.35}$$

For uniformly accelerated rotational motion can be written:

$$\varepsilon = const \neq 0;\ \omega = \omega_0 + \varepsilon t;\ \varphi = \varphi_0 + \omega_0 t + \frac{\varepsilon t^2}{2}. \qquad (1.36)$$

Table 2

Relationship between linear and angular values

Liner		Angular		Relationship	The time dependence of
Path (displacement)	$S(\vec{r})$	Rotation vector	$\vec{\varphi}$	$\vec{r} = [\vec{\varphi} \times \vec{R}]$ $\varphi = \frac{S}{R}$	$S = S_0 + \upsilon_0 t + \frac{at^2}{2}$ $\varphi = \varphi_0 + \omega_0 t + \frac{\varepsilon t^2}{2}$
Liner speed	$\vec{\upsilon} = \frac{d\vec{S}}{dt}$	Angular velocity	$\vec{\omega} = \frac{d\vec{\varphi}}{dt}$	$\vec{\upsilon} = [\vec{\omega} \times \vec{R}]$	$\upsilon = \upsilon_0 + at$ $\omega = \omega_0 + \varepsilon t$
Liner acceleration	$\vec{a} = \frac{d\vec{\upsilon}}{dt}$	Angular acceleration	$\vec{\varepsilon} = \frac{d\vec{\omega}}{dt}$	$\vec{a_\tau} = [\vec{\varepsilon} \times \vec{R}]$ $\vec{a_n} = -\omega^2 \vec{R}$ $\vec{a} = \vec{a_n} + \vec{a_\tau} =$ $= [\vec{\varepsilon} \times \vec{R}] - \omega^2 \vec{R}$	$a = \frac{\upsilon_2 - \upsilon_1}{t_2 - t_1}$ $\varepsilon = \frac{\omega_2 - \omega_1}{t_2 - t_1}$

1.2 DYNAMICS OF TRANSLATIONAL MOTION

1.2.1 Newton's First law. Inertial reference system

Newton's First law: Every body is at rest or in uniform motion as long as the effects of other bodies do not bring it out of this state

$\vec{a} = 0$, $\vec{\upsilon} = const(or = 0)$, if $\vec{F} = 0$ - **Newton's First law.**

This law is called the law of inertia. **Inertia -** the body's ability to maintain speed. **Inertial motion -** motion with constant velocity.

Newton's 1 law holds not in all frames. Reference frame in which the 1-Newton's law is valid, called inertial. Any system, moving relative to an inertial system uniformly, will also inertial.

An example of an inertial reference frame can serve as the heliocentric reference frame, i.e., the reference frame of the Sun (Figure 1.11).

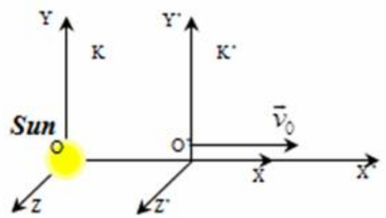


Figure 1.11

Any system, moving relative to the heliocentric uniform rectilinear will be inertial.

The laboratory frame, the axes of which are rigidly connected to the Earth, is not inertial, because of non-inertial rotation of the earth. However, the Earth's rotation is very slow, with a = 0.034 m/s^2, and so in most applications laboratory frame of reference can be considered as approximately inertial.

The content of the First Newton's law reduced to two statements:

1) all have the property of inertia of the body;

2) there are inertial frames of reference.

Inertial reference systems play a special role not only in mechanics, but also in other areas of physics, because the principle of relativity Einstein's mathematical notation of any physical law must have the same form in all inertial reference frames.

1.2.2 Mass, momentum of the body. Newton`s Second Law

The same effect in different ways alters the movement of various bodies. When exposed to any body changes its velocity at once, but gradually. The body's ability to maintain its speed is called inertia. The measure of inertia is mass. Body **mass -** a positive scalar value, a measure of inertia of a body, that is characterized by the body's ability to maintain its speed.

Under the action of the body changes its velocity is not instantaneous, but gradually, i.e., acquires a finite acceleration, which is smaller, the greater the mass, that is, when exposed to the same forces

$$\frac{m_2}{m_1} = \frac{a_1}{a_2}, \tag{1.37}$$

$$[m] = kg\,. \tag{1.38}$$

Density is the ratio of body mass dm small volume dV to the value of this volume

$$\rho = \frac{dm}{dV}, \tag{1.39}$$

if the body is homogeneous, then ρ = const and

$$\rho = \frac{m}{V}, \tag{1.40}$$

$$[\rho] = \frac{kg}{m^3}, \tag{1.41}$$

Center of mass, or center of mass system of particles is a point with a radius vector $\vec{r}_c$, which is equal to

$$\vec{r}_c = \frac{\sum_{i=1}^{n} m_i r_i}{\sum_{i=1}^{n} m_i} = \frac{m_1 r_1 + m_2 r_2 + \ldots + m_n r_n}{m_1 + m_2 + \ldots + m_n}\,. \tag{1.42}$$

Vector quantity $\vec{p}_i$ equal to the mass m of a point on its velocity is called **momentum** (or linear momentum) of the material point

$$\vec{p}_i = m\vec{\upsilon}_i\,,\ \vec{p}_i \uparrow\downarrow \vec{\upsilon}_i\,, \tag{1.43}$$

$$[p] = \frac{kg \cdot m}{s}. \tag{1.44}$$

Momentum of the system is the vector of point $\overrightarrow{p_i}$ equal to the geometric sum (i.e, the sum of the vectors) of all material points

$$\overrightarrow{p_i} = \sum_{i=1}^{n} \overrightarrow{p_i}. \tag{1.45}$$

Velocity of the center of inertia:

$$\overrightarrow{\upsilon_c} = \frac{d\overrightarrow{r_0}}{dt} = \frac{\sum_{i=1}^{n} m_1 \overrightarrow{\upsilon_i}}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} \overrightarrow{p_i}}{m} = \frac{\overrightarrow{p}}{m}. \tag{1.46}$$

that is, the momentum of the system is equal to the mass of the entire system on the speed of its center of inertia.

Newton`s Second Law: the rate of change of momentum of the body is acting on the body force *F*

$$\frac{d\overrightarrow{p}}{dx} = \overrightarrow{F}. \tag{1.47}$$

If the body has several forces, under the force *F* in Newton's second law is necessary to understand the resultant force (net force) - geometrical sum of all forces acting on the body**.**

From Newton's second law, it follows that

$$d\overrightarrow{p} = \overrightarrow{F} \cdot dt. \tag{1.48}$$

Vector quantity *F*d*t* called the elementary impulse of forcer.

Impulse, for a finite time interval t_2 -t_1 is

$$\int_{t_1}^{t_2} \overrightarrow{F} dt, \tag{1.49}$$

where

$$\overrightarrow{F} = \overrightarrow{F}(t), \tag{1.50}$$

$$dp = Fdt \Rightarrow \Delta p = p_2 - p_1 = \int_{t_1}^{t_2} \overrightarrow{F} dt, \tag{1.51}$$

$$\frac{d\overrightarrow{p}}{dt} = \frac{d(m\overrightarrow{\upsilon})}{dt} = \frac{md\overrightarrow{\upsilon}}{dt} = m\overrightarrow{a} = \overrightarrow{F}, \tag{1.52}$$

$$\overrightarrow{F} = m\overrightarrow{a}. \tag{1.53}$$

The basic equation of the dynamics of the translational motion of a rigid body. The force acting on a body is equal to the mass of the body to its acceleration (Figure 1.12).

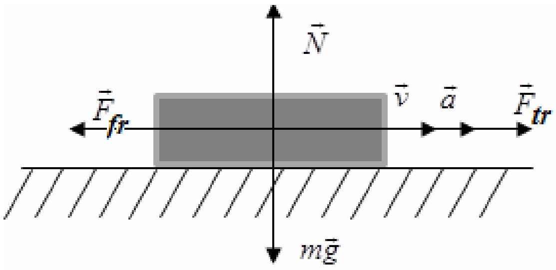


Figure 1.12

$$[F] = \frac{kg \cdot m}{c^2} = N, \tag{1.54}$$

$$\vec{F} = m\vec{g} + \vec{N} + \vec{F}_{jriction} + \vec{F}_{tyaction}, \tag{1.55}$$

$$m\vec{g} + \vec{N} + \vec{F}_{jr} + \vec{F}_{ty} = m\vec{a}, \tag{1.56}$$

$$x: F_{jr} - F_{ty} = ma, \tag{1.57}$$

$$y: -mg + N = 0. \tag{1.58}$$

Tangent and normal acceleration determined by the appropriate component of the force *F*

$$\vec{a} = \vec{a_\tau} + \vec{a_n}, \tag{1.59}$$

$$\vec{F} = \vec{F_\tau} + \vec{F_n}, \tag{1.60}$$

$$\vec{F} = m\vec{a}, \tag{1.61}$$

$$\vec{F_\tau} + \vec{F_n} = m\vec{a_\tau} + m\vec{a_n}, \tag{1.62}$$

$$\vec{F_\tau} = m\vec{a_\tau}, \ |F_\tau| = m \cdot \varepsilon R, \tag{1.63}$$

$$\vec{F_n} = m\vec{a_n}, \ |F_n| = m \cdot \frac{\upsilon^2}{R}. \tag{1.64}$$

Force $\vec{F_n}$, which imparts at normal acceleration is directed towards the center of curvature of the trajectory and is therefore called the **centripetal force**.

1.2.3 Newton's Third Law

Every action of the body to each other is in the nature of interaction: if the body 1 acts on the body 2 with a force F_{21}, then the body 2 acts on the body with the force of one F_{12} (Figure 1.13).

Newton's Third law: the forces that act on each other interacting bodies are equal in magnitude and opposite in direction. (Action force equal to the force of the reaction).

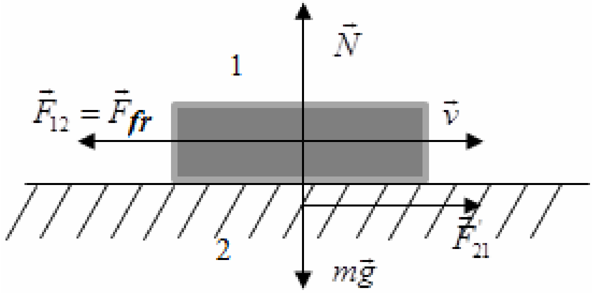


Figure 1.13

$$\vec{F}_{12} = -\vec{F}_{21} \quad . \tag{1.65}$$

Newton's III law is not always valid. It is strictly valid in the case of contact interactions, as well as the interaction of some distance from each other stationary bodies.

III of Newton's law that in any mechanical system geometric sum of all internal forces exactly equal to 0.

$$\sum_{i=1}^{n}\sum_{k=1}^{n}\varphi_{ik}^{\text{int}ema} = 0 \equiv \varphi_{12}+\varphi_{13}+\varphi_{14}+\ldots+\varphi_{in}-\varphi_{21}-\varphi_{31}-\varphi_{41}-\varphi_{in} = 0 \cdot \tag{1.66}$$

The law of gravity: Two point bodies attract each other through space with a force directly proportional to the two masses and inversely proportional to the square of the distance between them (Figure 1.14).

$$\vec{F}_{12} = \gamma\frac{m_1 m_2}{r_{12}^3}\cdot\overrightarrow{r_{12}}, \tag{1.67}$$

$$\vec{F}_{12} = \gamma\frac{m_1 m_2}{r_{12}^2}, \tag{1.68}$$

$$\gamma = 6{,}672\cdot 10^{-11}\frac{N\cdot m^2}{kg^2}, \tag{1.69}$$

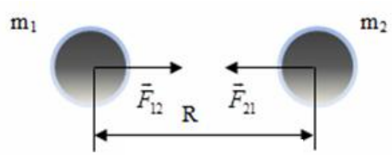


Figure 1.14

γ - the gravitational constant (numerically equal to the force of mutual attraction 2 material points of unit mass at a distance of 1 *m*).

1.2.4 The law of conservation of momentum

Consider a system consisting of *n* material points interacting (Figure 1.15).

$$\frac{m_1}{\upsilon_1};\frac{m_2}{\upsilon_2};\ldots;\frac{m_i}{\upsilon_i};\ldots;\frac{m_n}{\upsilon_n}, \tag{1.70}$$

$$\frac{\upsilon_1}{p_1};\frac{\upsilon_2}{p_2};\ldots;\frac{\upsilon_i}{p_i};\ldots;\frac{\upsilon_n}{p_n}. \tag{1.71}$$

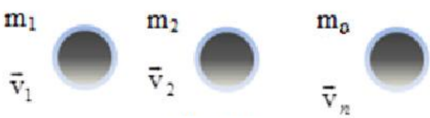


Figure 1.15

The forces of interaction between the bodies that make up the system, let f_k. Interaction of external forces from the body not in the file system on the *I* body system denoted F_i.

We write Newton's Second law applied to all bodies that form the system:

$$\frac{d\overrightarrow{p_1}}{dt}=\overrightarrow{f_{12}}+\overrightarrow{f_{13}}+\overrightarrow{f_{ji}}+\overrightarrow{f_{in}}+\overrightarrow{F_1}\ , \qquad (1.71)$$

$$\frac{d\overrightarrow{p_2}}{dt}=\overrightarrow{f_{21}}+\overrightarrow{f_{23}}+...+\overrightarrow{f_{21}}+\overrightarrow{f_{24}}+\overrightarrow{F_2}\ , \qquad (1.72)$$

$$\frac{d\overrightarrow{p_n}}{dt}=\overrightarrow{f_{n1}}+\overrightarrow{f_{n2}}+...+\overrightarrow{f_{ni}}+\overrightarrow{f_{n_{n-1}}}+\overrightarrow{F_n}\ , \qquad (1.73)$$

On Newton's third law:

$$\frac{d}{dt}(\overrightarrow{p_1}+\overrightarrow{p_2}+...+\overrightarrow{p_n})=\frac{d}{dt}\left(\sum_{i=1}^{n}\overrightarrow{p_i}\right)=\frac{d\overrightarrow{p}}{dt}=\sum_{i=1}^{n}F_i=\overrightarrow{F_{bxt}}\ , \qquad (1.74)$$

$$\vec{f}_{12}=-\vec{f}_{21}\ , \qquad (1.75)$$

$$\frac{d\overrightarrow{p_1}}{dt}+\frac{d\overrightarrow{p_2}}{dt}+...+\frac{d\overrightarrow{p_i}}{dt}+...+\frac{d\overrightarrow{p_n}}{dt}=F_1+F_2+...+F_i+...+F_n\,\cdot \qquad (1.76)$$

Vector sum of the momenta of all bodies forming this system is called the resultant momentum of the system.

$$\frac{d\overrightarrow{p_p}}{dt}=\sum_{i=1}^{n}F_i\,\cdot \qquad (1.77)$$

If external forces no effect on the body system (no interaction between bodies within the system and external bodies), or the action of external forces is compensated, then the system is called a **closed** or **isolated**.

In this case,

$$\frac{d\overrightarrow{p_p}}{dt}=0\Rightarrow\overrightarrow{p_p}=const;\ \vec{F}_{bxt}=0\,\cdot \qquad (1.78)$$

The law of conservation of momentum:
geometric (vector) sum of the momentum of a closed system remains constant over time in all interactions within the system:

$$\text{LCM: } \overrightarrow{p_1}+\overrightarrow{p_2}+...+\overrightarrow{p_n}=\vec{p}\,'_1+\vec{p}\,'_2+...+\vec{p}\,'_n=const\,, \qquad (1.79)$$

$$\vec{F}_{bxtmal}=0\,. \qquad (1.80)$$

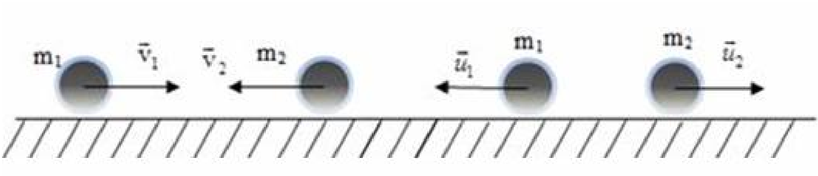


Figure 1.16

that is, the interaction between the bodies of the pulses of individual bodies can vary in magnitude and direction, but within a framework that the vector sum of the momenta of all the bodies that make this system remains constant.

1.3 WORK AND KINETIC ENERGY

"Work" is done whenever a force is applied to move an object from one place to another. If the force is *F*, and the displacement of the object is *S*, then the work done *W* is given by

$$A = \vec{F} \cdot \vec{S} \ , \qquad (1.81)$$

Note that both $\vec{F}$ and $\vec{S}$ are vectors, while *A* is a scalar. Recall the scalar product of two vectors (Figure 1.17)

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}|\cos\theta = AB\cos\theta \ , \qquad (1.82)$$

So we also have

$$A = F \cdot S\cos\theta \ , \qquad (1.83)$$

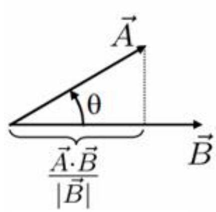


Figure 1.17

Units:

Force, F : Newtons [N]

Displacement, S : metres [m]

Work, A : Joules [J]

1 Joule = 1 Newton metre

Example: Consider a force of 5 *N* pushing an object 2 *m* in the same direction as the direction of the force (Figure 1.18). 1 *J* = 1 *N m*.

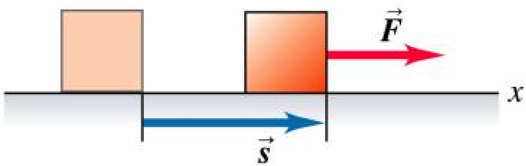


Figure 1.18

The work done is

$$A = \vec{F} \cdot \vec{S} = F \cdot S = 5N \cdot 2m = 10N \cdot m = 10J$$

If the motion is in a different direction from the force, we have to take the direction of the vectors into account.

Example: What is the work done by gravity when a 5*kg* object moves 2*m* down a frictionless 30° slope (Figure 1.19)?

Force downwards is $F = mg$.

Displacement down the slope is $S = 2m$.

Angle between force and displacement is $\theta = 60^{o}$.

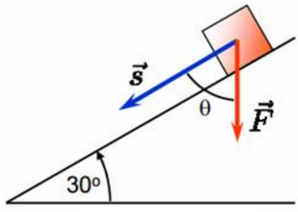


Figure 1.19

Work done is

$$A = \vec{F}\cdot\vec{S} = F\cdot S\cdot\cos\theta = mgs\cos\theta = 5kg\cdot 9.8\, m/_{s^2}\cdot\cos 60^0 = 49kgm^2/s^2 = 49J$$

$1N = 1kgm/s^2$

Work can also be negative (Figure 1.20).

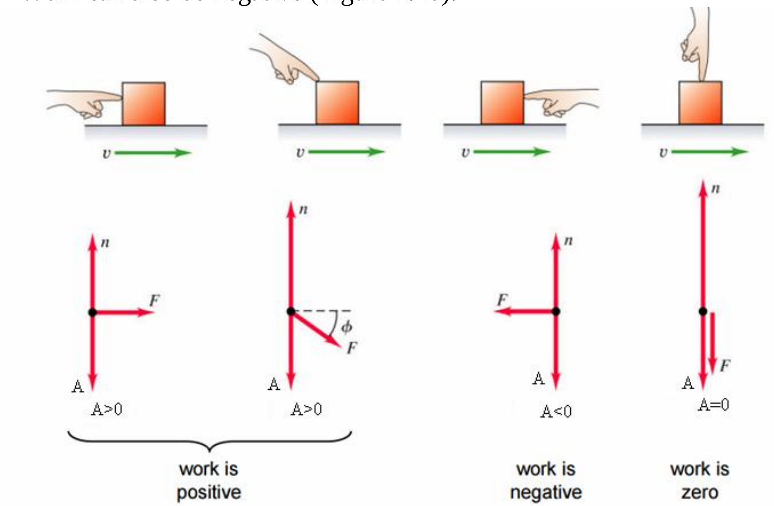


Figure 1.20

1.3.1 Work done by a varying force

We can divide the displacement $\vec{S}$ into lots of little (infinitesimal) displacements $\Delta\vec{S}_i$ such that (Figure 1.21)

$$\vec{S} = \Delta\vec{S_1} + \Delta\vec{S_2} + ... + \Delta\vec{S_n} = \sum_{i=1}^{n}\Delta\vec{S}_i\,. \qquad (1.84)$$

Let the force on the object for a displacement $\Delta\vec{S}_i$ be $\vec{F_i}$

Then the work done on segment *i* is

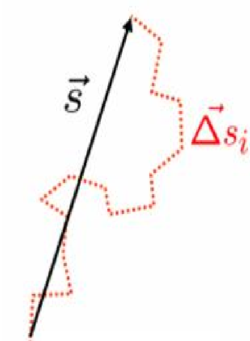


Figure 1.21

$$\Delta A_i = \vec{F}_i \cdot \Delta \vec{S}_i \ . \tag{1.85}$$

The total work done is

$$A = \sum_{i=1}^{n} \Delta A_i = \sum_{i=1}^{n} \vec{F}_i \cdot \Delta \vec{S}_i \cdot \tag{1.86}$$

Taking the limit $\Delta \vec{S}_i \to 0$ this turns into an integral:

$$A = \int \vec{F} \cdot d\vec{S} \ , \tag{1.87}$$

Imagine a simpler case, where the motion is along only the x direction, starting at x_1 and ending at x_2.

Now $d\vec{S} = dx\hat{i}$ and $\vec{F} \cdot d\vec{S} = F_x dx \Rightarrow A = \int_{x_1}^{x_2} F_x dx$, (1.88)

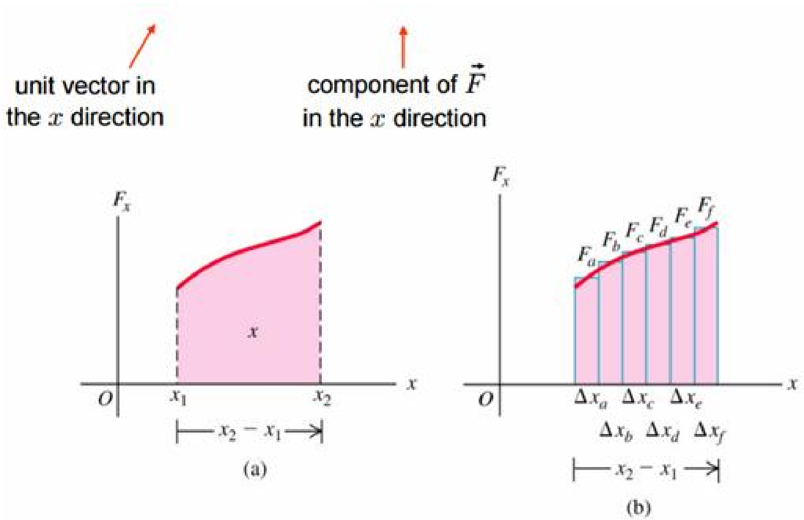


Figure 1.22

The two shaded areas above are equal when $\Delta x \to 0$.

Notice that when F_x is independent of the position (Figure 1.22), then we return to our old formula:

$$A = \int_{x_1}^{x_2} F_x dx = F_x \int_{x_1}^{x_2} dx = F_x (x_2 - x_1) = F_x S \ , \tag{1.89}$$

Alternatively

$$A = \int \vec{F} \cdot d\vec{S} = \vec{F} \cdot \int d\vec{S} = \vec{F} \cdot \vec{S} \ . \tag{1.90}$$

Example: An example of a force which varies with displacement is the force exerted by a spring when you pull it.

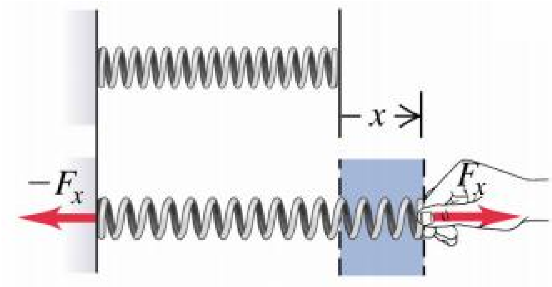


Figure 1.23

The force applied by the spring is given by Hooke's Law (Figure 1.23)

$$F_x = -kx, \tag{1.91}$$

k is known as the spring constant.

Hooke's Law tells us that the force required to stretch a spring increases linearly with the distance that you pull it.

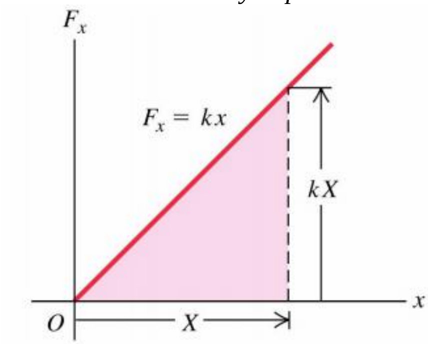


Figure 1.24

The work done in stretching the spring by an amount *X*, is the area under the curve to the left (Figure 1.24).

$$A = \frac{1}{2} X \cdot kX = \frac{1}{2} kX^2 . \tag{1.92}$$

Alternatively:

$$A = \int_0^x F_x dx = \int_0^x kxdx = \left[\frac{1}{2} kx^2\right]_0^X = \frac{1}{2} kX^2 . \tag{1.93}$$

Example: Work done pushing a swing

Let's say we push a child in a swing. The child has a weight *w*, and the length of the chain is *R*. Assuming that we push him very slowly, what is the work done by each of the forces acting on the child?

"slowly" ⇒ If T is the tension on the rope and F is the force with which we push him, then Y&F: Ex. First we need to know how the forces vary with θ. x-direction: y-direction: all the forces are (approximately) in equilibrium (i.e. sum to zero).

First we need to know how the forces vary with θ.

If T is the tension on the rope and F is the force with which we push him, then (Figure 1.25)

$$\left.\begin{aligned} x-direction: &\quad F = T\sin\theta \\ y-direction: &\quad \omega = T\cos\theta \end{aligned}\right\} \Rightarrow F = \omega \cdot tg\theta, \tag{1.94}$$

Now we can calculate the work done by each force moving from A (θ=0) to B ($\theta = \theta_0$).

Remember that it is only the force in the direction $d\vec{S}$ which counts!

For the force $\vec{F}$:

$$A_F = \int_A^B \vec{F} \cdot d\vec{S} = \int_A^B F\cos\theta\, dS = \int_0^{\theta_0} \omega \cdot tg\,\theta \cdot \cos\theta \cdot R \cdot d\theta = \tag{1.95}$$

$$= R\omega \int_0^{\theta_0} \sin\theta\, d\theta = R\omega(1-\cos\theta_0).$$

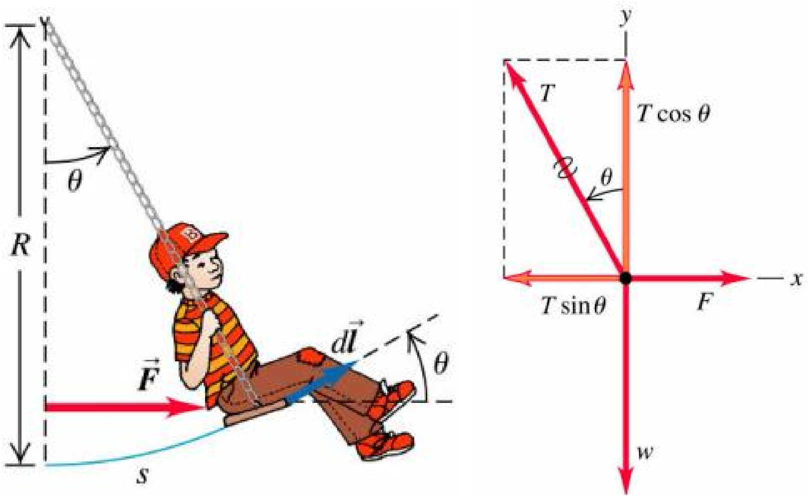


Figure 1.25

For the force $\vec{\omega}$ (gravity):

$$A_\omega = \int_A^B \vec{\omega}\cdot d\vec{S} = \int_A^B \omega\cos(90^0+\theta)dS = -\int_0^{\theta_0} \omega\cdot\sin\theta\cdot Rd\theta = -R\omega(1-\cos\theta_0)\cdot \quad (1.96)$$

For the force $\vec{T}$ (tension): the angle between $d\vec{S}$ and $\vec{T}$ is always 90^o, so $\vec{T}\cdot d\vec{S}=0$ and the work done by $\vec{T}$ is zero (*Figure 1.26*).

Notice that $A_F + A_\omega + A_T = 0$ since the total net force is zero, so can do no work!

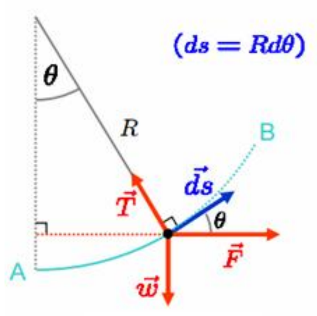


Figure 1.26

1.3.2 Kinetic energy

Consider an object of mass m, under a constant force $\vec{F}$ as it moves from x_1 to x_2. Since $\vec{F}=m\vec{a}$, the object is accelerated. Its velocity changes from υ_1 to υ_2 (Figure 1.27).

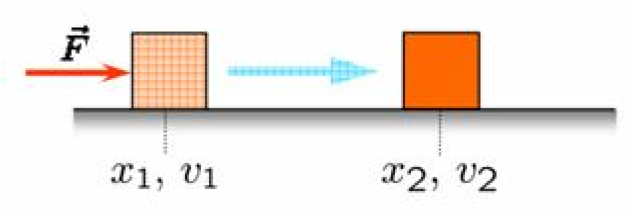


Figure 1.27

Displacement: $s = x_2 - x_1$

Velocities: $\upsilon_2^2 = \upsilon_1^2 + 2aS \Rightarrow \frac{1}{2}(\upsilon_2^2 - \upsilon_1^2)$

But Newton's second Law tells us that $F = ma = \frac{1}{2}m(\upsilon_2^2 - \upsilon_1^2)\cdot$

The work done by the force is $A = F\cdot S = \frac{1}{2}m\upsilon_2^2 - \frac{1}{2}m\upsilon_1^2\cdot$

Definition: The Kinetic Energy of an object of mass *m* moving with velocity υ is given by

$$E_K = \frac{1}{2}m\upsilon^2\cdot \quad (1.97)$$

It's units are *kg* $(ms^{-1})^2$ = *N m* = *J.*

Work-energy theorem: The work done on an object by the total net force is given by the change of the object's kinetic energy.

$$A_{tot} = E_{K_2} - E_{K_1} = \Delta E_K\cdot \quad (1.98)$$

N.B. Notice that this is the work done by the total net force. In our swing example, $A_{tot} = 0$ so the change in kinetic energy was zero. We could not have used this to calculate the work done by the individual forces.

The work energy theorem is still true even if the force changes during the motion.

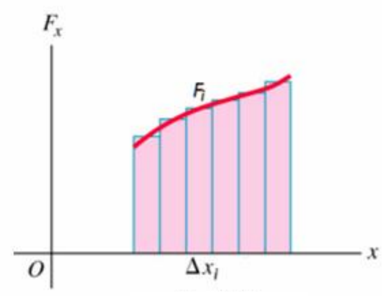


Figure 1.28

If we go back to our small steps Δx_i, then for step i the work done is (Figure 1.28)

$$A_i = F_i \Delta x_i = \Delta E_i \cdot \tag{1.99}$$

The total work done is the sum over all the steps:

$$A_{tot} = \sum_{i=1}^{n} A_i = \sum_{i=1}^{n} \Delta E_i = \Delta E_{tot} \cdot \tag{1.100}$$

So the total work done is still equal to the change in kinetic energy.

1.4 POWER

It is often useful to know how the work done by a force changes over time. We define Power as the rate of change of work done with time:

$$P = \frac{dA}{dt} \cdot \tag{1.101}$$

Unit: *Watt* [W] 1 W = 1 Js^{-1}.

The work done between time 0 and time T is given by $A = \int_0^T P \cdot dt \cdot$

If the power is constant then total work done is $A = P \times T$.

We can relate Power to Force and Velocity, just as we related Work to Force and Displacement:

Recall that the work done for a constant force is $A = \vec{F} \cdot \vec{S}$.

So the power is

$$P = \frac{dA}{dt} = \vec{F} \cdot \frac{d\vec{S}}{dt} = \vec{F} \cdot \vec{\upsilon} . \tag{1.102}$$

1.5 POTENTIAL ENERGY

Imagine an object of mass m, falling freely under gravity. Let’s say it falls from a height y_1 to a height y_2.

The work done by the force of gravity is

$$A = F \cdot S = mg(y_1 - y_2), \tag{1.103}$$

and this causes an increase in the kinetic energy of the object.

The higher the object is at the start, the more potential it has to gain energy when it falls.

We say that an object of mass *m*, at a height *h*, has gravitational potential energy, given by (Figure 1.29)

$$U = mgh \tag{1.104}$$

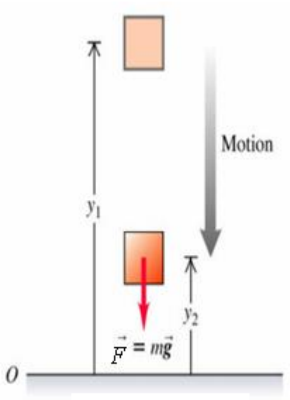


Figure 1.29

Notice that it doesn't really matter where the point h = 0 is, since only changes in potential energy matter.

In the example just shown, the object loses potential energy, but objects can also gain potential energy by moving against the gravitational field.

If we apply an upwards force *F* to the object of mass *m*, moving it from h_1 to h_2, we will be doing work

$$A_F = F(h_2 - h_1)\,. \tag{1.105}$$

If we choose $F = mg$ then the object moves with zero acceleration. Its velocity doesn't change, so it has the same kinetic energy as before, but it gains potential energy

$$U = mg(h_2 - h_1)\,. \tag{1.106}$$

The work done is stored as potential energy.

If *F* is larger than *mg*, then there will be a net upwards force and the object will accelerate upwards. The extra work done by the force will contribute to the kinetic energy of the object (Figure 1.30).

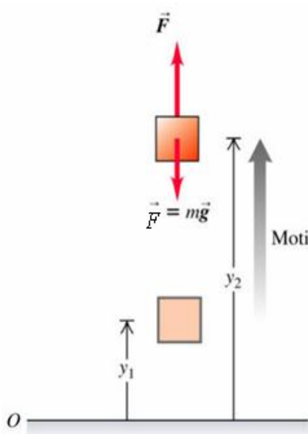


Figure 1.30

Let's return to our free-falling object *i.e.* $\vec{F} = 0$.

Change in potential energy

$$U = -mg(h_1 - h_2)\,. \tag{1.107}$$

Work done by gravitational force

$$A = mg(h_1 - h_2)\,. \tag{1.108}$$

Work-energy theorem $\Rightarrow \Delta E_K = mg(h_1 - h_2)$

$$\Delta U + \Delta E_K = 0\,. \tag{1.109}$$

We define the total energy to be the sum of the kinetic and potential energies:

$$W = U + E_K\,. \tag{1.110}$$

The total energy is conserved. It does not change.

When a force conserves the total energy, we say it is a conservative force.

Example: A projectile is fired from point 1 with an initial speed v_0 at an initial angle α_0. What is the maximum height it reaches (Figure 1.31)?

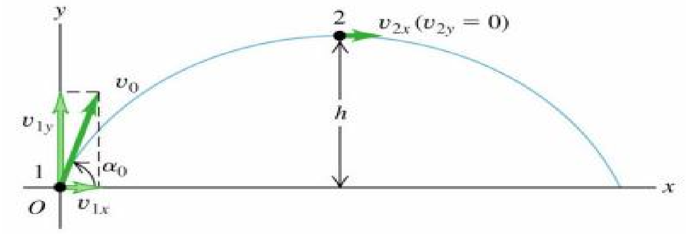


Figure 1.31

At point 1: $E_{K_1} = \frac{1}{2} m\upsilon_0^2 \;\; U_1 = 0$.

At point 2: $E_{K_2} = \frac{1}{2} m\upsilon_2^2 \;\; U_2 = mgh$.

Conservation of energy

$$\Rightarrow E_{K_1} + U_1 = E_{K_2} + U_2 \Rightarrow \frac{1}{2} m\upsilon_0^2 = \frac{1}{2} m\upsilon_2^2 + mgh.$$

Therefore the maximum height is $h = \dfrac{\upsilon_0^2 - \upsilon_2^2}{2g}$.

Also, since there is no force in the *x*-direction, we know that the velocity in the *x*-direction cannot change $\Rightarrow \upsilon_2 = \upsilon_{1x} = \upsilon_0 \cos\alpha_0$ and we have

$$h = \frac{\upsilon_0^2 - \upsilon_0^2 \cos^2\alpha_0}{2g} = \frac{\upsilon_0^2 \sin^2\alpha_0}{2g}.$$

There are many other forms of potential energy too. For example, a spring can contain **elastic potential energy.**

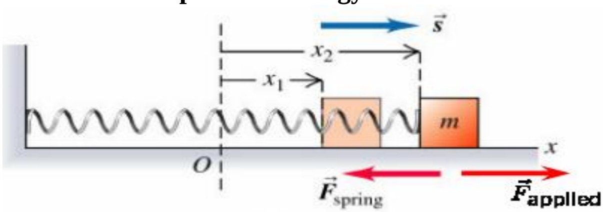


Figure 1.32

$$\vec{F}_{spring} = -kx. \tag{1.111}$$

So when we extend the spring from x_1 to x_2 we do work (*Figure 1.32*)

$$A = \int_{x_1}^{x_2} F_{applied} dx = \int_{x_1}^{x_2} kx dx = \left[\frac{1}{2}kx^2\right]_{x_1}^{x_2} = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2 \cdot \qquad (1.112)$$

The *elastic potential energy* stored in a spring is therefore given by

$$U = \frac{1}{2}kx^2 \cdot \qquad (1.113)$$

What happens if we take away our hand and let the spring pull the mass back?

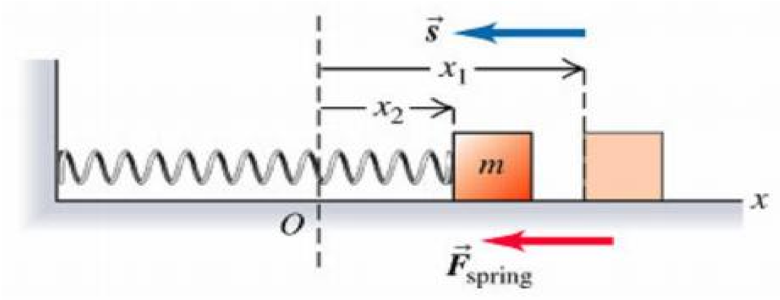


Figure 1.33

Work done by the spring is

$$A_{spring} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = U_1 - U_2 \qquad (1.114)$$

Work-energy theorem tells us that the work done is equal to the change in kinetic energy

$$A_{spring} = E_{K2} - E_{K1}, \qquad (1.115)$$

So

$$U_1 + K_1 = U_2 + K_2 \cdot \qquad (1.116)$$

Again, total energy (potential + kinetic) is conserved. Potential energy is the area under the curve of F.

We can work in the opposite direction, and derive the *force from the potential* (Figure 1.33).

In 1 dimension,

$$F = -\frac{dU}{dx} \cdot \qquad (1.117)$$

For our two examples:

$$F_{spring} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx, \qquad (1.118)$$

$$F_{gravity} = -\frac{d}{dy}(mgh) = -mg, \qquad (1.119)$$

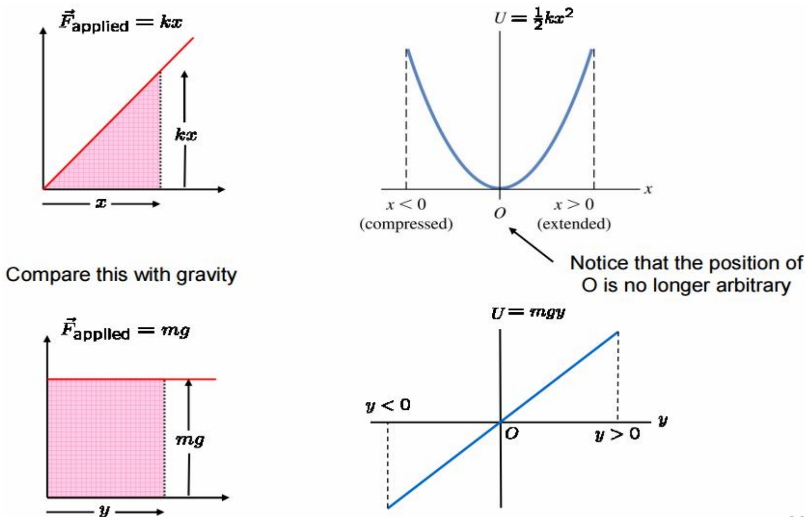


Figure 1.34

More generally, in 3 dimensions,

$$\vec{F} = -\vec{\nabla} \cdot U = \left(-\frac{dU}{dx}, -\frac{dU}{dy}, -\frac{dU}{dz} \right), \quad (1.120)$$

The gravitational force exerted on a mass m by another mass M at a distance r (Figure 1.35), is

$$F = -\frac{GmM}{r^2}, \quad (1.121)$$

G is Newton's constant $= 6.67 \cdot 10^{-11} m^3 kg^{-1} s^{-2}$.

The potential energy is $U = \int F dr = -GmM \int \frac{1}{r^2} dr = -\frac{GmM}{r} + const \cdot$

Choose at $U \to 0$ at $r \to \infty$ $\Rightarrow U = -\frac{GmM}{r}$.

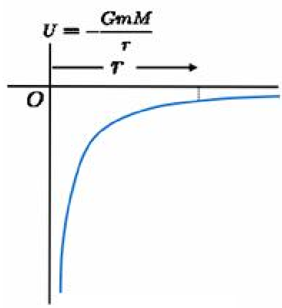


Figure 1.35

The potential we used before was an approximation which is valid when the change in height is small

If $r = R + y$ with y small

height above the Earth's surface

radius of the Earth

"small" means $y \ll R$

Then

$$U = -\frac{GmM}{R+y} = -\frac{GmM}{R}\frac{1}{\left(1+\frac{y}{R}\right)} \approx -\frac{GmM}{R}\left(1-\frac{y}{R}\right) = -\frac{GmM}{R} + mgy$$

with $g = \frac{GM}{R^2} = \frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{(6.38 \times 10^6)^2}\,\mathrm{m\,s^{-2}} = 9.8\,\mathrm{m\,s^{-2}}$

(Radius of the Earth = 6,380,000 m
Mass of Earth = 5.97×10^{24} kg)

Example: Escape Velocity of a satellite.

A satellite of mass *m* is launched from the Earth's surface with velocity υ. What v do we need, in order that the satellite escapes the Earth's pull (Figure 1.36)?

At launch $U_1 = -\frac{GmM}{R}$ and $E_{k1} = -\frac{1}{2}m\upsilon^2$.

"Escape" means that the satellite reaches $r \to \infty$ with $E_k = 0$.

So $U_2 = -\frac{GmM}{\infty} = 0$.

Conservation of energy $\Rightarrow -\frac{GmM}{R} + \frac{1}{2}m\upsilon^2 = 0$

$$\upsilon = \sqrt{\frac{2GM}{R}} = \sqrt{2gR} \approx 11.2\,{}^{km}/_{s}.$$

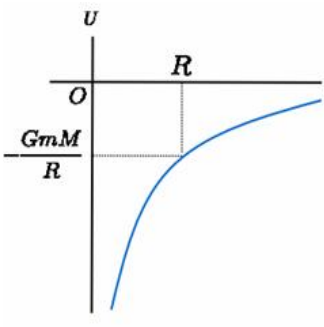


Figure 1.36

1.6 CONSERVATIVE AND NON-CONSERVATIVE FORCES

We have already seen two examples of a *conservative force*: gravity and a spring.

If the force is conservative, then we can convert potential energy into kinetic energy and vice versa, with no energy lost (Figure 1.37).

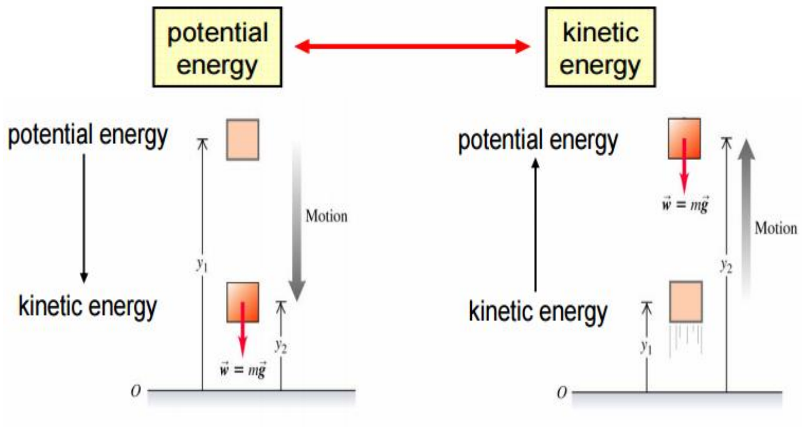


Figure 1.37

Total mechanical energy ($K + U$) is conserved.

Imagine a conservative force $\vec{F}$, which depends on position, acting on an object. What is the work done by the force when an object moves from point *A* to point *B* (Figure 1.38)?

Work done taking path 1 is

$$A_1 = \int_{A\to B:path} \vec{F} \cdot d\vec{S}, \qquad (1.122)$$

Work done taking path 2 is

$$A_2 = \int_{A\to B:path\,2} \vec{F} \cdot d\vec{S}. \qquad (1.123)$$

For a conservative *force* we know the potential energy at both *A* and *B*, so the work done must simply be

$A_1 = A_2 = U_B - U_A$. (1.124)

The work done depends only on the endpoints, not on the path taken.

This is really the *definition* of a conservative force. It is this definition which allows us to define potential energy in the first place.

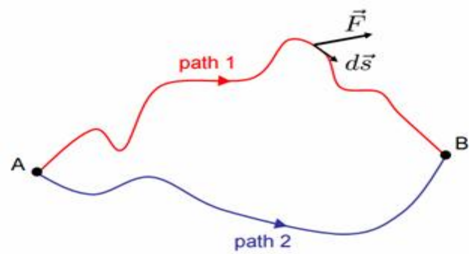


Figure 1.38

Imagine we now regard $A \to B \to A$ as the complete path.

Work done by $\vec{F}$ is $A_1 - A_2 = 0$ and is independent of the path.

This means that the potential energy at point *A* (or *B*) is well defined and does not change depending on the past history of the object.

A *non-conservative* force is a force where the work done by a force moving an object from point A to point B depends on the path taken. This is sometimes called a *dissipative force* (Figure 1.39).

A good example of a *non-conservative* force is *friction.*

The frictional force always opposes the motion so $\overrightarrow{F} \cdot d\overrightarrow{S} < 0$ always, so the work $A\,B$ done moving $A \to B \to A$ is

$$A = \oint \overrightarrow{F} \cdot d\overrightarrow{S} < 0 \cdot \qquad (1.125)$$

The work done depends on the path taken $\Rightarrow$ we cannot define potential energy.

The work done is usually dissipated as heat.

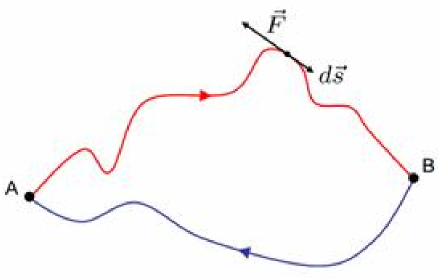


Figure 1.39

1.7 DYNAMICS OF MECHANICAL SYSTEMS

1.7.1 The centre-of-mass

Conservation of momentum is true for more complicated systems too. If there are no external forces on the system, then momentum is conserved.

$$\sum_i \overrightarrow{p_i} = const \cdot \qquad (1.126)$$

the sum here is over all the bodies in the system

Definition: The centre-of-mass of a collection of *n* objects of mass m_i at positions $\overrightarrow{r_i}$ is (Figure 1.40)

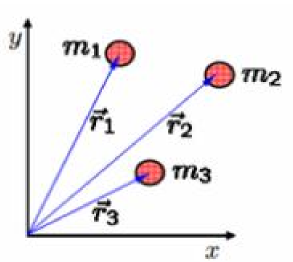


Figure 1.40

$$\overrightarrow{r_{cm}} = \frac{m_1\vec{r_1} + m_1\vec{r_2} + ... + m_1\vec{r_n}}{m_1 + m_2 + ... + m_n}. \qquad (1.127)$$

The centre-of mass will move with a velocity

$$\overrightarrow{\upsilon_{cm}} = \frac{m_1\overrightarrow{\upsilon_1} + m_1\overrightarrow{\upsilon_2} + ... + m_1\overrightarrow{\upsilon_n}}{m_1 + m_2 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i\overrightarrow{\upsilon_i}}{\sum_{i=1}^{n} m_i}, \qquad (1.128)$$

Remember momentum

$$\overrightarrow{p_i} = m\overrightarrow{\upsilon_i}, \text{ so } \sum \overrightarrow{p_i} = \sum_i m\overrightarrow{\upsilon_i} = const \cdot \qquad (1.129)$$

So as long as the masses don't change,

$$\overrightarrow{\upsilon_{cm}} = \frac{\sum_{i=1}^{n} m_i \overrightarrow{\upsilon_i}}{\sum_{i=1}^{n} m_i} = const \cdot \tag{1.130}$$

1.7.2 The velocity of the centre-of-mass is the same after the collision as it was before

The centre-of-mass is also useful if there is a net force on the system and momentum is not conserved.

The acceleration of the centre-of-mass is

$$\overrightarrow{a_{cm}} = \frac{\sum_{i=1}^{n} m_i \overrightarrow{a_i}}{\sum_{i=1}^{n} m_i} = \frac{\sum_{i=1}^{n} \overrightarrow{F_i}}{\sum_{i=1}^{n} m_i}. \tag{1.131}$$

$$\sum_{i=1}^{n} \overrightarrow{F_i} = \left(\sum_{i=1}^{n} m_i\right)\overrightarrow{a_{cm}} \cdot \tag{1.132}$$

The centre-of-mass moves like an object of mass $\sum_{i=1}^{n} m_i$ **under a force** $\sum_{i=1}^{n} \overrightarrow{F_i}$ **.**e.g. Consider an object breaking up in flight.

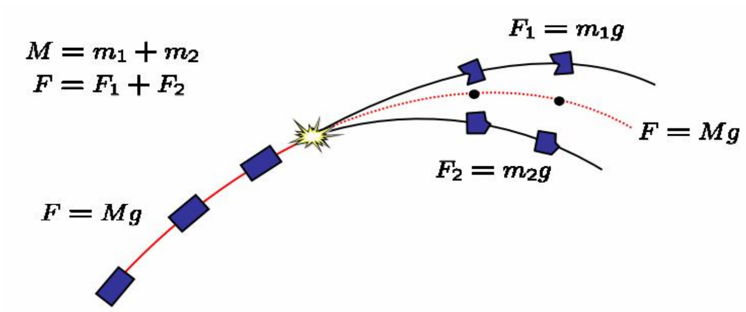


Figure 1.41

The motion of the centre-of-mass after the collision is the same is if the object had stayed in one piece.

1.8 DYNAMICS OF ROTATIONAL MOTION

1.8.1 The moment of inertia. Steiner's theorem

The moment of inertia of a point is

$$I = mr^2. \tag{1.133}$$

The moment of inertia of the system with respect to the axis of rotation is called a physical quantity that is equal to the sum of the product of the masses n material points of the squares of their distances from the axis of (Figure 1.42).

$$I = \sum_{i=1}^{n} m_i r_i^2 , \quad (1.134)$$

$$I = m_1 r_1 + m_2 r_2 + m_3 r_3 + m_4 r_4 . \quad (1.135)$$

The moment of inertia of the body in the event of a continuous distribution of mass is

$$I = \int_V r^2 dm = \int_V r^2 \rho dV \quad (1.136)$$

-integrated throughout.

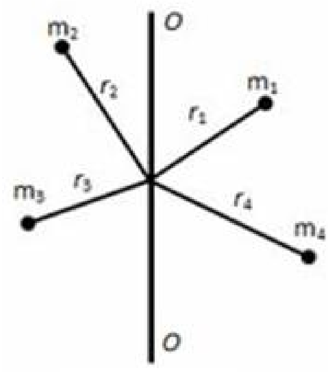


Figure 1.42

1. We find the moment of inertia of a uniform disk about an axis perpendicular to the plane of the disk and through its center. We divide the disk into annular layers of thickness dr. All points of the layer will be the same distance from the axis equal to r. The volume of such a layer is

$$dV = S \cdot \pi \cdot 2r \cdot dr \cdot b$$

Square ring $S = \pi((r + dr)^2 - r^2) = \pi(r^2 + 2rdr + dr^2 - r^2) = \pi \cdot 2rdr$

$$I = \int_0^1 2\pi b r \rho r^2 = 2\pi b \rho \int_0^1 r^3 dr = 2\pi b \rho \frac{R}{4}$$

$$m = \rho \cdot V = \rho \pi R^2 b$$

$$I_d = \frac{mR^2}{2} . \quad (1.137)$$

2. Walled hollow cylinder of radius R (a hoop, a bicycle wheel, and the like).

$$I = mR^2 . \quad (1.138)$$

3. Solid cylinder or disk of radius R (Figure 1.45)

$$I = \frac{1}{2} mR^2 . \quad (1.139)$$

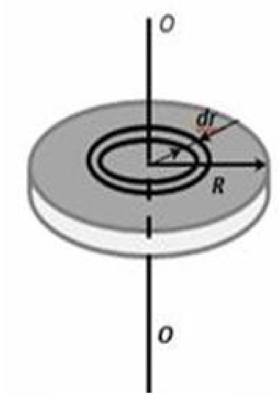


Figure 1.43

4. Moment of inertia of the balls (Figure 1.43):

$$I = \frac{2}{5} mR^2 . \quad (1.140)$$

5. Direct thin long rod axis is perpendicular to the rod and passing through its middle.

$$I = \frac{1}{12} ml^2 . \quad (1.141)$$

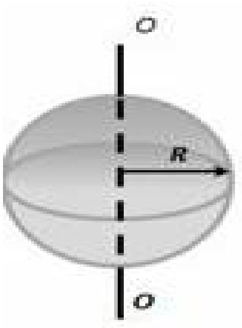


Figure 1.43

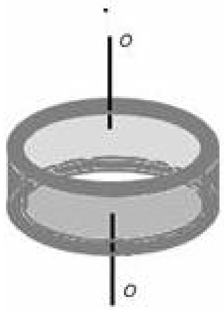


Figure 1.44

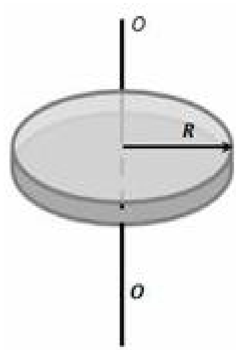


Figure 1.45

If you know the moment of inertia about an axis passing through its center of mass, moment of inertia about any axis parallel to this, is determined with the help of Theorem Steiner: the moment of inertia with respect to the I axis of rotation is parallel to the moment of inertia I_C relatively parallel to the axis passing through the center of mass C body, with a folded piece of body mass m and the square of the distance between the axes of the

$$I = I_c + ma^2. \qquad (1.142)$$

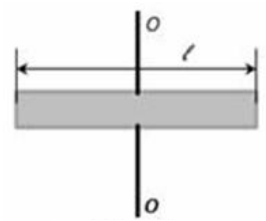


Figure 1.46

6. The moment of inertia of a straight rod length, the axis perpendicular to the rod and passing through its end .

$$I = I_c + ma^2 = \frac{1}{12}ml^2 + m\left(\frac{l}{2}\right)^2 = ml^2\left(\frac{1}{12} + \frac{1}{4}\right) = \frac{1}{3}ml^3 \cdot$$

1.8.2 The kinetic energy of rotation

Consider a rigid body rotating around a fixed axis Z, passing through it with angular velocity ϖ. because the body is absolutely rigid, therefore, all of the body will rotate at the same angular velocity.

If we break the body in small amounts to the elementary masses m_1, m_2 ... at a distance r_1, r_2 ..., from the axis of rotation, the kinetic energy of the body can be written as (Figure 1.47)

$$E_k = \frac{m_1\upsilon_1}{2} = \frac{m_2\upsilon_2}{2} + \ldots + \frac{m_n\upsilon_n^2}{2} = \sum_{i=1}^{n}\frac{m_i\upsilon_i^2}{2}. \qquad (1.143)$$

It is known that $\omega = \dfrac{\upsilon}{r_H}$ or $\upsilon = \omega \cdot r$ then $E_{krot} = \dfrac{I\omega^2}{2}$

$$E_k = \sum_{i=1}^{n} \frac{m_i \omega^2 r_i^2}{2} = \frac{\omega^2}{2} \sum_{i=1}^{n} m_i r_i^2 = \frac{I\omega^2}{2}. \qquad (1.144)$$

From a comparison of $E_{k\,\mathrm{rot}}$ with E_k translational motion $E_k = \left(\dfrac{m\upsilon^2}{2}\right)$ that the moment of inertia of the rotary motion replaces mass in the rotational motion and is a measure of the inertia of the body.

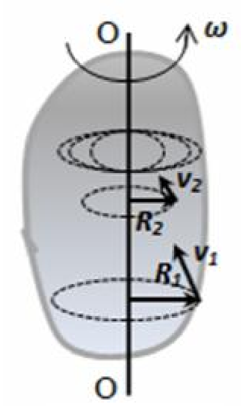


Figure 1.47

If the body is involved in translational and rotational motions at the same time, it

$$E_k = E_{koransi} + E_{krot} = \frac{m\upsilon^2}{2} + \frac{I\omega^2}{2}. \qquad (1.145)$$

For example, a cylinder rolling without slipping on a plane.

$$E_k = \frac{m\upsilon^2}{2} + \frac{I\omega^2}{2} = \frac{m\upsilon^2}{2} + \frac{mR^2\upsilon^2}{2R^2} = m\upsilon^2. \qquad (1.146)$$

1.8.3 Torque. The dynamic equation of rotational motion of a rigid body

The **moment of a force (torque)** $\vec{F}$ about a fixed point *O* is called pseudovector $\vec{M}$ value equal to the vector product of the radius vector $\vec{r}$ from the point *O* to the point of application of force, the force $\vec{F}$ (Figure 1.48).

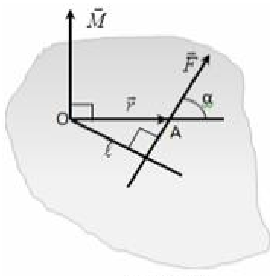


Figure 1.48

$\vec{M}$ **-** pseudovector its direction coincides with the plane of motion of the right screw as it rotates from $\vec{r}$ to $\vec{F}$. **The direction of the torque** can also be defined by the rule of his left hand, four fingers of his left hand to put in the direction of the first factor $\vec{r}$, the second factor $\vec{F}$ is in the palm, bent at right angles to the thumb indicates the direction of the torque $\vec{M}$. Moment of the force vector is always perpendicular to the plane in which the vectors $\vec{r}$ and $\vec{F}$

$$r\sin\alpha = l$$

- where l the shortest distance between the line of action of the force and the point 0-called shoulder strength.

$$M = l \cdot F$$

-strength shoulder on force.

The moment of a force $\vec{F}$ about a fixed axis Z is called a scalar quantity equal to the projection on the axis of the torque $\overrightarrow{M}$ definedrelative to an arbitrary point O of the axis Z. If the Z-axis is perpendicular to the plane in which the vectors $\vec{r}$ and $\vec{F}$, i.e. coincides with the direction $\overrightarrow{M}$, then a moment of force represented as a vector coincides with the axis.

$$\overrightarrow{M}_z = \left[\vec{r}\vec{F}\right]_z . \tag{1.147}$$

Axis, whose position in space remains unchanged during the rotation around the body in the absence of external forces, called the free axis of the body.

For a body of any shape and with an arbitrary distribution of mass, there are 3 mutually perpendicular passing through the center of mass of the body axis, which can serve as free axes: they are called the principal axes of inertia.

We find an expression for rotational motion of the body. Let the mass m solid external force $\overrightarrow{F_i}$. Then the work of this force for the time dt is

$$dA_i = F_i \upsilon_i dt = F_i \left[\overrightarrow{\omega_i} \times \vec{r_i}\right] dt . \tag{1.148}$$

Feasible in the mixed product of vectors a cyclic permutation of the factors

$$\vec{a}\left[\vec{b} \times \vec{c}\right] = \vec{b}\left[\vec{c} \times \vec{a}\right] = \vec{c}\left[\vec{a} \times \vec{b}\right], \tag{1.149}$$

$$dA_i = \overrightarrow{\omega}_i \left[\vec{r_i} \times \overrightarrow{F_i}\right] dt = \vec{\omega} \cdot \overrightarrow{M_i} \cdot dt = \omega M_{zi} dt , \tag{1.150}$$

$$d\varphi = \omega\ dt , \tag{1.151}$$

$$dA = \sum_i \Delta A_i = \omega \cdot M_z \cdot dt = M_z d\varphi , \tag{1.152}$$

$$dA = M_z d\varphi . \tag{1.153}$$

Work rotation of the body is the product of the moment of the force on the angle of rotation of the body dφ. Work is to increase its kinetic energy:

$$dA = dE_K , \tag{1.154}$$

$$dE_K = d\left(\frac{I_Z \omega^2}{2}\right) = I_z \omega d\omega \cdot \tag{1.155}$$

Then

$$M_z d\varphi = I_z \omega d\omega , \tag{1.156}$$

Or

$$M_z \frac{d\varphi}{dt} = I_z \omega \frac{d\omega}{dt}, \tag{1.157}$$

$$M_z = I_z \frac{d\varphi}{dt} = I_z \varepsilon, \tag{1.158}$$

$$\frac{d\varphi}{dt} = \omega \Rightarrow M_z = I\varepsilon \cdot \tag{1.159}$$

the basic equation of the dynamics of rotational motion.

If the axis of rotation coincides with the main axis of inertia through the center of mass, then the vector equality.

$$\vec{M} = \vec{I}\vec{\varepsilon}. \tag{1.160}$$

I - principal moment of inertia (moment of inertia with respect to the main axis).

1.8.4 The angular momentum. The law of conservation of angular momentum

Angular momentum of a particle *A* relative to a fixed point *0* is a physical quantity, defined by the vector product

$$\vec{L} = \left[\vec{r} \times \vec{p}\right] = \left[\vec{r} \times m\vec{\upsilon}\right], \tag{1.161}$$

$$\left[\vec{L}\right] = r \cdot p \cdot \sin(\vec{r}\,\vec{p}) = m\upsilon r \sin(\vec{r}\vec{\upsilon}) = pl, \tag{1.162}$$

$\vec{r}$ - radius vector from the point *O* to the point *A*.

$\vec{p} = m\vec{\upsilon}$ - the momentum of a particle.

$\vec{L}$ - pseudovector, its direction is determined by the left-hand rule.

Angular momentum of a rigid body about a fixed axis *Z* is called a scalar quantity equal to the projection on the axis of the angular momentum, defined relative to an arbitrary point *O* that axis. The value of the angular momentum L_z is independent of the point *O* on the axis *Z*.

The angular momentum of a rigid body about an axis is the sum of the angular momentum of individual particles:

$$L_z = \sum_{i=1}^{n} m_i \upsilon_i r_i = \sum_{i=1}^{n} m_i \omega_i r_1 r_2 = \sum_{i=1}^{n} m_i \omega_i r_i^2 = I_z \omega \tag{1.163}$$

Differentiate with respect to *dt*

$$L_z = I_z \omega, \tag{1.164}$$

$$\frac{dL_z}{dt} = I_z \varepsilon = M_Z, \tag{1.165}$$

$$\frac{dL_z}{dt} = M_Z, \tag{1.166}$$

- the basic equation of the dynamics of rotational motion.

Generally performed vector equality

$$\frac{d\vec{L}}{dt} = \vec{M}, \tag{1.167}$$

In a closed system the moment of the external forces is zero

$$\frac{d\vec{L}}{dt} = 0 \Rightarrow const \quad I_1\omega_1 = I_2\omega_2, \tag{1.168}$$

The law of conservation of angular momentum: the angular momentum of a closed system is conserved; i.e does not change over time.

Table 3

The values characterizing the translational and rotational motion, and the relationship between them

Liner		**Angular**		**RelationShip**	**The time dependence of**
Path (displacement)	$S(\vec{r})$	Rotation vector	$\vec{\varphi}$	$\vec{r} = [\vec{\varphi} \times \vec{R}]$ $\varphi = \frac{S}{R}$	$S = S_0 + \upsilon_0 t + \frac{at^2}{2}$ $\varphi = \varphi_0 + \omega_0 t + \frac{\varepsilon t^2}{2}$
Liner speed	$\vec{\upsilon} = \frac{d\vec{S}}{dt}$	Angular velocity	$\vec{\omega}$	$\vec{\upsilon} = [\vec{\omega} \times \vec{R}]$	$\upsilon = \upsilon_0 + at$ $\omega = \omega_0 + \varepsilon t$
Liner acceleration	$\vec{a} = \frac{d\vec{\upsilon}}{dt}$	Angular acceleration	$\vec{\varepsilon}$	$\vec{a_\tau} = [\vec{\varepsilon} \times \vec{R}]$ $\vec{a_n} = -\omega^2 \vec{R}$ $\vec{a} = \vec{a_\tau} + \vec{a_n} =$ $[\vec{\varepsilon} \times \vec{R}] + \omega^2 \vec{R}$	$a = \frac{\upsilon_2 - \upsilon_1}{t_2 - t_1}$ $\varepsilon = \frac{\omega_2 - \omega_1}{t_2 - t_1}$
mass	m	Moment of inertia	I	$I = mR^2$	
Momentum, Liner momentum	$\vec{p}$	Angular momentum	$\vec{L}$	$\vec{L} = [\vec{R} \times \vec{p}] =$ $= I\omega$	
force	$\vec{F}$ $\vec{F} = m\vec{a}$	Moment of force, torque	$\vec{M}$	$\vec{M} = [\vec{R} \times \vec{F}] =$ $= I\varepsilon$	
Kinetic energy	E_K $E_K = \frac{m\upsilon^2}{2}$	The rotation kinetic energy	E_{Krot} $E_{Krot} = \frac{m\omega^2}{2}$		
Elementary work	dA $dA = F_s dS =$ $= \vec{F} d\vec{S}$	Elementary work of the rotation motion	dA $dA = \vec{M} d\vec{\varphi} =$ $= M d\varphi$		

In the absence of an applied torque, angular momentum is conserved.

An interesting example of torque and angular momentum is a ***gyroscope*** (Figure 1.49).

This is a spinning object balanced on a pivot.

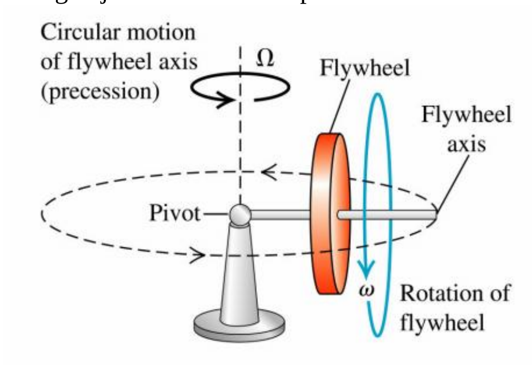


Figure 1.49

The object is set to spin about its axis, and instead of falling down as you would expect, it “precesses” about the pivot.

Consider first a gyroscope which is not spinning

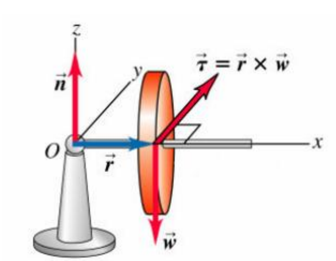


Figure 1.50

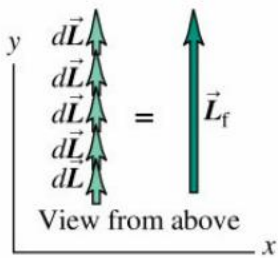


Figure 1.51

Gravity pulls the centre-of-mass down ($\vec{\omega}$), applying a torque $\vec{\tau} = \vec{r} \times \vec{\omega}$ at right angles to both $\vec{r}$ and $\vec{\omega}$. (Figure 1.50)	The change in the angular momentum in a time dt is $d\vec{L} = \vec{\tau} dt$ Which is also at right angels to both $\vec{r}$. (Figure 1.51)

With each *dt*, the changes in angular momentum add up, the angular momentum gets bigger and the gyroscope falls more quickly.

Now consider a gyroscope which is spinning

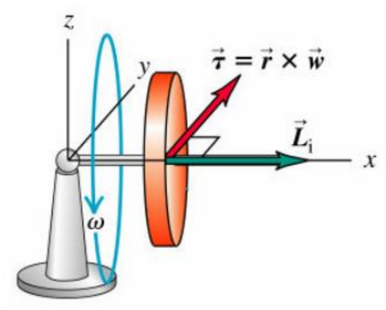


Figure1.52

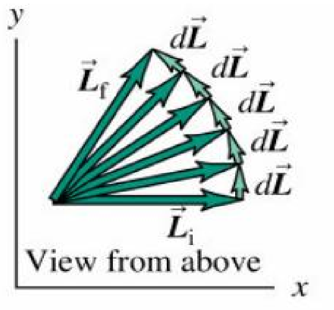


Figure 1.53

As before, Gravity pulls the centre-of-mass down ($\vec{\omega}$), applying a torque $\vec{\tau} = \vec{r} \times \vec{\omega}$ at right angles to both $\vec{r}$ and $\vec{\omega}$. (Figure 1.52)	Now $d\vec{L} = \vec{\tau} dt$, as before, but this time the torque only changes the direction of the angular momentum, not its magnitude. (Figure 1.53)

The angular momentum is always in the same direction as so the gyroscope doesn't fall – it ***precesses***.

For a gyroscope of mass m:

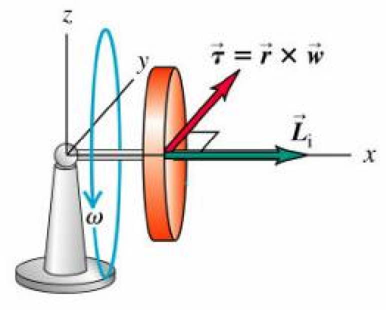


Figure 1.54

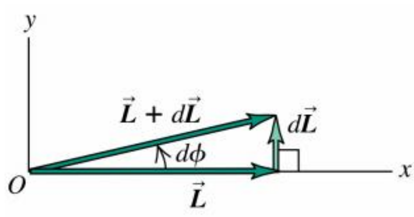


Figure 1.55

Downwards force on centre-of-mass $= mg$.

Torque (Figure 1.54) $\tau = mgr$.

Change of angular momentum (Figure 1.55) $dL = Ld\varphi$.

So $\dfrac{dL}{dt} = L\dfrac{d\varphi}{dt} = mgr \Rightarrow \dfrac{d\varphi}{dt} = \dfrac{mgr}{L}$.

The gyroscope precesses with an angular speed $\dfrac{mgr}{L}$.

1.9 CONTINUUM MECHANICS

1.9.1 Continuum model, leading to the problem model.

In the outside world there are movements that can not be described within the model we studied (particles of solids). This is the movement of liquids or gases flowing through the pipes (fuel and technical liquids in pipelines aircraft), flow around the ship (water), aircraft (air) and so on.

Physical Description motions of liquids and gases requires a separate mechanical model that has certain features. First, in substance, that moves are usually no fixed geometric dimensions and shape (an example is the air that flows around the airplane).

Second, its characteristics (speed, etc.). Not local and distributed throughout the material, and they can be different at different points and change over time. This condition satisfies the continuum model.

Continuum is a mechanical model that provides continuous spatial distribution of matter and its motion characteristics.

The **continuum** model ignores the real continuum discrete (atomic and molecular) structure of matter.

The key objectives of continuum models are:

a) The calculation of traffic characteristics continuum each point in space, depending on the time and environmental conditions;

b) calculate the forces acting on the body that moves relatively continuum, as well as the distribution of forces on the body surface.

Experience shows that the main types of motion (or flow types) continuum is laminar and turbulent flow. Laminar flow can be divided into separate flow layers do not mix with each other. Turbulent flow is unordered traffic environments that can not be suspended even stratified.

Turbulent flow is stochastic (non-deterministic), characteristics that can not be accurately calculated. Laminar and turbulent flow occur at particular air flow around the wings of the aircraft during flight (Figure 1.56).

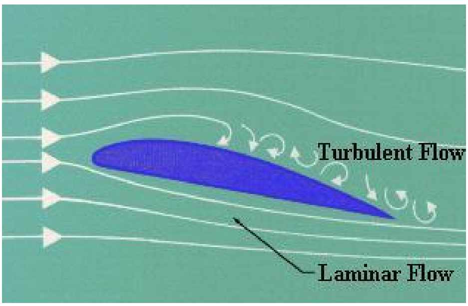


Figure 1.56

If the smooth flow of air is interrupted over a wing section, turbulence is created which results in a loss of lift and a high degree of drag. An airfoil designed for minimum drag and uninterrupted flow of the boundary layer is called a laminar airfoil [142].

1.9.2 The density of the medium. The rate of flow lines and flow tube. Fixed flow

Consideration model continuum we start with its basic concepts. First of all, note that instead of the name "solid medium" is often used term "liquid", we will also use it.

The density of the medium. Consider the movement of fluid in the system of coordinates (*x, y, z*) (Figure 1.57). Isolate a small amount of liquid ΔV Δt mass moving with fluid ("liquid particles"). Medium density ρ is defined as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}. \qquad (1.169)$$

Quantifies the density inert fluid properties. In SI units [ρ] = kg / m^3. The density generally depends on the coordinates of the space in which it is measured, and from time to time: ρ = ρ (*x, y, z, t*) [143].

An important partial model is stationary currents, which flow characteristics at different points in space may be different, but each point,

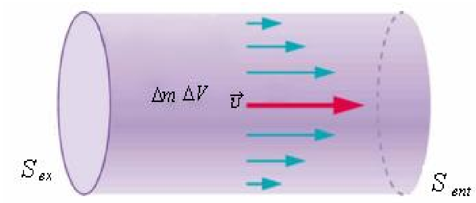


Figure 1.57

these characteristics do not depend on time. For stationary flow $\rho = \rho(x, y, z)$.

The rate of flow lines and flow tube υ flow velocity at the point space coordinates (x, y, z) at time t is the rate of liquid particles, which at the moment passes through this point (see. Figure 1.57). In general, $\upsilon = \upsilon(x, y, z)$, for stationary flows $\upsilon = \upsilon(x, y, z)$.

For laminar flow can introduce the concept of flow lines - an imaginary line tangent to the direction in which arbitrary point coincides with the direction of the flow velocity vector v at this point. The direction of the flow lines indicate flow direction (see. Figure 1.57). In steady flow pattern lines flow does not change with time, and they reflect the current line trajectory of fluid particles (in unsteady flows is not the case).

Tube current (or stream) is a tsylindropodibnyy imaginary volume of space bounded laterally flow lines. In laminar flow steady flow all can be divided into separate streams, each of which is fixed. The fluid flows inside of a tube flow only through the intersection S_{ex} its input, and comes out of it only through its initial intersection S_{ent} (see. Figure 1.57). In special disciplines to analyze trends sometimes used tube of infinitesimal current input and output section - elementary streams.

1.9.3 Equations of continuity of flow. A model of ideal incompressible
Mach fluid. Continuity equation for incompressible fluid flow

We turn to the study of the laws of motion continuum. Confine ourselves to these laws in the case of stationary laminar flows.

Consider the flow tube, through which the stationary fluid flow (Figure 1.58) [143]. Let S_1 and S_2 - square input and output sections of the tube, ρ_1 and ρ_2 - density of the liquid, υ_1 and υ_2 - flow velocity in these sections. Over a short period of time Δt in the receiver enters mass of liquid

$$\Delta m_1 = \rho_1 \upsilon_1 S_1 \Delta t\,, \tag{1.170}$$

and flows out of the tube during the same period Δt

$$\Delta m_2 = \rho_2 \upsilon_2 S_2 \Delta t \tag{1.171}$$

Figure 1.58

Since the flow is stationary, the mass of the fluid inside the tube can change over time, so $\Delta m_1 = \Delta m_2$.
Equating the right sides of equations (1.170) and (1.171) and reducing to Δt, we get

$$\rho_1 \upsilon_1 S_1 = \rho_2 \upsilon_2 S_2 \tag{1.172}$$

Since sections S_1 and S_2 can be carried out at random locations of the tube current, this equality means that for any flow tube

$$\rho \upsilon S = const. \tag{1.173}$$

In the stationary laminar flow continuum medium density product to flow velocity and the cross-sectional area of the tube current remains constant along any flow tube.

This law is called the law of *continuity of flow*. From equations (1.170) and (1.171) that the value ρvS, located on the left side (1.173) is Δm / Δt (kg/ s), that determines the mass of fluid passing through the tube flow per unit time (i.e, 1 s). In special disciplines, this value is often called mass flow or flow (liquid, air) Because of (1.173) is called the equation of sustainability costs.

Different problems hydro- and aerodynamics is often used continuum model for which ρ = const - incompressible fluid model. The analysis shows that compressibility of real fluids and gases is negligible provided $\upsilon_{sound} << \upsilon$, where υ - flow velocity and υ_{sound} - speed of sound in the medium under these conditions. An important characteristic of fluid flow Mach number is:

$$M = \frac{\upsilon}{\upsilon_{sound}}, \qquad (1.174)$$

Incompressible fluid model is valid in the case of M << 1 regardless of the actual physical nature of the medium (gas or liquid). For this model, the formulas (1.172) and (1.173) based $\rho_1 = \rho_2$ or ρ = const follows the *law of continuity for incompressible fluid flow*

$$\upsilon_1 S_1 = \upsilon_2 S_2 \text{ or } \upsilon S = const \qquad (1.175)$$

The value of $\upsilon S = \Delta V / \Delta t$ (m^3 / s) determines the volume of fluid passing through the tube flow per unit of time is called volumetric flow rate.

Since the law (1.175) that the incompressible fluid with decreasing cross-sectional area of the tube current, the flow velocity increases and vice versa. For compressible medium (also called compressible gas) this statement is generally not fair. The analysis shows that with decreasing cross-sectional area of the tube current flow rate will increase only when this rate is less than the speed of sound (M <1). In the case of M> 1 with decreasing flow cross-sectional area decreases as flow rate and with the increase - increases.

1.9.4 Equations and Bernoulli's law for ideal incompressible fluid

We now turn to the formulation of the law of the dynamics of continuous media - Bernoulli law. The law we have to formulate a simple model of a continuum - *a model of ideal incompressible fluid* [143].

Ideal incompressible fluid is a continuous medium in which the density is constant in time and equal in all its points (ρ = const) and, furthermore, no internal friction force between different parts of the liquid.

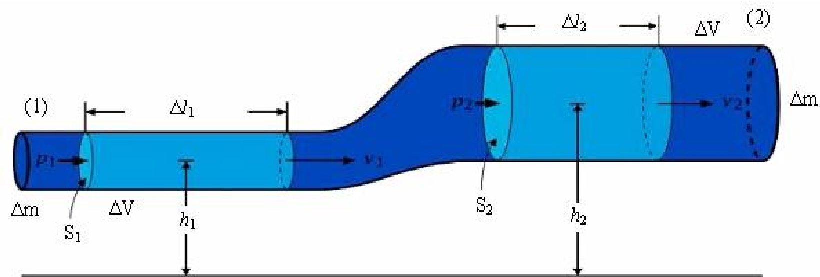


Figure 1.59

For a Bernoulli equation will allocate a certain amount of fluid that moves inside of a tube flow. Since the flow is stationary, this change in position can be reduced to the transition of a fluid volume mass Δm ΔV out (1) to position (2) (Figure 1.59). Changing the kinetic energy of the fluid

$$\Delta A = \frac{\Delta m \upsilon_2^2}{2} - \frac{\Delta m \upsilon_1^2}{2} \tag{1.176}$$

The value of ΔA equal to the mechanical work of all the forces acting on the selected portion of the fluid in its motion inside the tube current. As the fluid is ideal, the friction force between the selected part of the fluid and fluid surrounding the missing, as are left only gravity and pressure forces F_1 and F_2, which are normal to the tube sections flow so

$$\Delta A = A_{\text{atrac.}} + A_{\text{pr.}} \tag{1.177}$$

Work gravity formula to the change of potential energy mass Δm, which moved from the height h_1 to a height h_2. From formulas (1.176) and (1.177) we have

$$A_{\text{atrac.}} = \Delta m g h_1 - \Delta m g h_2. \tag{1.178}$$

Work pressure forces is $A_{\text{pr.}} = F_1\Delta l_1 - F_2\Delta l_2$. Given that $F_1 = r_1S_1$ and $F_2 = r_2S_2$ (where p_1, p_2 - pressure fluid respectively at the beginning and end of the selected portion of the tube current, S_1, S_2 - square of the respective sections) and subject nestyslyvosti liquid $S_1\Delta l_1 = S_2\Delta l_2 = \Delta V$, have

$$A_{\text{pr.}} = p_1\Delta V - p_2\Delta V. \tag{1.179}$$

Substituting formula (1.178) and (1.179) to (1.177) in view of (1.176), we obtain - $\frac{\Delta m \upsilon_2^2}{2} - \frac{\Delta m \upsilon_1^2}{2} = \Delta m g h_1 - \Delta m g h_2 + p_1\Delta V - p_2\Delta V$. Consider that $\Delta m = \rho\Delta V$. Substituting this in previous equality, reducing both its part and on ΔV reordering members have

$$p_1 + \frac{\rho\upsilon_1^2}{2} + \rho g h_1 = p_2 + \frac{\rho\upsilon_2^2}{2} + \rho g h_2. \tag{1.180}$$

Current velocity v_1 and v_2 can be registered in random sections of tube current, so (1.180) means that along the tube reaches the condition

$$p + \frac{\rho\upsilon^2}{2} + \rho g h = const. \tag{1.181}$$

This comprises three equal terms with the dimension of pressure (Pa): p - pressure at this point of the liquid (static pressure); $\frac{\rho\upsilon^2}{2}$ - dynamic pressure (or dynamic pressure); $\rho g h$ - hydrostatic pressure. Equation (1.181) means that

• *The amount of static, dynamic and hydrostatic pressure remains constant along any flow tube ideal incompressible fluid.*

This statement is called the law of Bernoulli and equation (1.181) or an equivalent (1.182) - Bernoulli equation for ideal incompressible fluid.

1.9.5 Laminar Flow

In fluid dynamics, laminar movement, as well as streamline movement, takes place when a fluid moves within equivalent levels one above another, without interruption between your layers. At reduced velocities, the actual fluid has a tendency to stream without having lateral blending, as well as adjacent layers, glide past each other just like actively playing cards. There aren't any cross-currents vertical with respect towards the path associated with movement, neither eddies or even whirls relating to fluids. In laminar movement, the particular motion belonging to the particles in the fluid is incredibly organised with all of the particles entering into direct lines synchronised towards the pipe walls. Laminar movement is really a flow routine observed as a substantial momentum diffusion and occasional momentum convection. Whenever a fluid is usually flowing by way of a closed funnel say for example a pipe or even in between a couple of flat plates, possibly associated with two kinds of movement can happen with respect to the velocity as well as viscosity in the fluid: laminar flow or turbulent flow. Laminar flow has a tendency to take place within lower velocities (Figure 1.60), beneath a threshold from which it will become turbulent. In non-scientific conditions, the laminar movement is actually consistent at the same time turbulent circulation is usually rough.

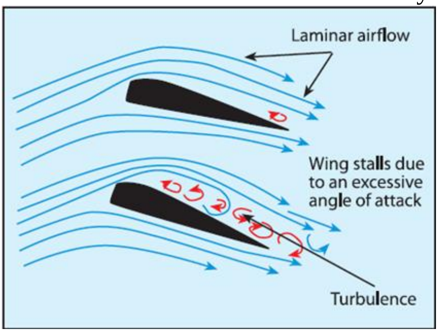


Figure 1.60

1.9.6 Turbulent Flow

In the field of the fluid dynamics, interrupted flow of the fluid is called turbulent flow. Turbulent flow, the kind associated with fluid (it could be gas as well as liquid) movement in which the fluid passes through unpredictable variances, as well as blending, as opposed to laminar flow, during which the fluid proceeds in sleek ways or even levels. When it comes to turbulent flow the rate belonging to the fluid with a level is constantly in the process of modifications in equal magnitude as well as the path. The movement associated with the wind and also estuaries and rivers is normally turbulent in this particular sense, whether or not the currents tend to be smooth. The environment or even water whirls as well as eddies at the same time it's general mass proceeds together a particular direction. Most kinds associated with fluid movement tend to be turbulent, besides laminar flow at the primary edge of solids going in accordance with fluids as well as incredibly near to solid areas, for example the inside wall associated with a pipe, or perhaps in cases associated with fluids involving substantial viscosity (comparatively terrific sluggishness) moving gradually by means of little channels. Typical examples of turbulent flow are usually blood circulation inside arteries, oil transportation inside pipelines, lava movement, atmosphere as well as sea currents, the particular movement by means of pumping systems as well as turbines, and also the movement when it comes to boat wakes as well as close to aircraft wing tips (Figure 1.61, Figure 1.62).

Figure 1.61

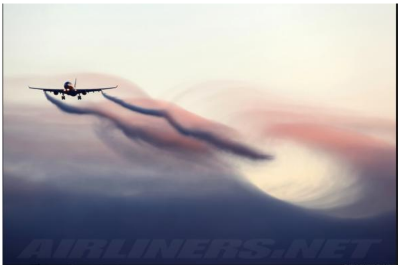


Figure 1.62

Chapter 2 MOLECULAR PHYSICS AND THERMODYNAMICS

2.1 MOLECULAR PHYSICS

2.1.1 On molecular physics and thermodynamics. Statistical and thermodynamic methods

Molecular physics and thermodynamics - the physical science that study macroscopic processes in bodies, due to the huge number of bodies contained in atoms and molecules.

Molecular physics is a branch of physics that studies the structure and properties of matter, based on the so-called molecular-kinetic concepts. According to these ideas:

1. Any body - solid, liquid or gas is made up of a large number of very small particles of isolated molecules.

2. The molecules of any substance is in an endless chaotic motion (eg., Brownian motion).

3. Used an idealized model of an ideal gas, which provides:

a) own volume of the gas molecules is negligible compared to the volume of the vessel (vacuum).

b) no forces between the molecules interact.

c) clash of the gas molecules with each other and with the walls of the vessel is elastic.

4. The macroscopic properties of bodies (pressure, temperature, etc.) are described by statistical methods, the basic concept of which is the statistical ensemble, i.e describes the behavior of a large number of particles through introduction of the average characteristics (average speed, energy) the whole ensemble, and not a single particle.

Thermodynamics in contrast to the molecular-kinetic theory studies the properties of macroscopic bodies, is not interested in their macroscopic picture.

Thermodynamics - the branch of physics that studies the general properties of macroscopic systems in thermal equilibrium, and the transition between these states.

At the heart of thermodynamics are three fundamental laws, called thermodynamics, established by summarizing a large set of experimental facts.

Molecular-kinetic theory and thermodynamics complement each other to form a whole, but to distinguish different methods of investigation.

Thermodynamic system - a set of macroscopic bodies that interact and exchange energy between themselves and with other bodies. State of the system is given by the thermodynamic parameters - a set of physical quantities characterizing the properties of a thermodynamic system, usually in a position to choose the parameters as temperature, pressure and specific volume.

Temperature - a physical quantity that characterizes the state of thermodynamic equilibrium macroscopic system.

[T] = K – thermodynamic scale, [t] = °C - International Practical Scale. Relationship thermodynamic and international practical temperatures:

T = t + 273, for example, at t = 20°C, T = 293 K.

Specific volume - this volume per unit mass. When the body is homogeneous, ie ρ = const, then the macroscopic properties of a homogeneous body can represent the body volume V.

2.1.2 Mole of substance. Avogadro's number. Avogadro's law

In physics, the **Avogadro constant** (named after the scientist Amedeo Avogadro) is the number of constituent particles, usually atoms or molecules, that are contained in the amount of substance given by one mole. Thus, it is the proportionality factor that relates the of a compound to the mass of a sample. Avogadro's constant, often designated with the symbol N_A or L, has the value $6.022140857(74)\times10^{23}$ mol^{-1} in the International System of Units (SI).

Previous definitions of chemical quantity involved **Avogadro's number**, a historical term closely related to the Avogadro constant, but defined differently: Avogadro's number was initially defined by Jean Baptiste Perrin as the number of atoms in onegram-molecule of atomic hydrogen, meaning one gram of hydrogen.

This number is also known as Loschmidt constant in German literature. The constant was later redefined as the number of atoms in 12 grams of the isotope carbon-12 (^{12}C), and still later generalized to relate amounts of a substance to their molecular weight. For instance, to a first approximation, 1 gram ofhydrogen element (H), having the atomic (mass) number 1, has 6.022×10^{23} hydrogen atoms. Similarly, 12 grams of ^{12}C, with the mass number 12 (atomic number 6), has the same number of carbon atoms, 6.022×10^{23}. Avogadro's number is adimensionless quantity, and has the same numerical value of the Avogadro constant given in base units. In contrast, the Avogadro constant has the dimension of reciprocal amount of substance.

Revisions in the base set of SI units necessitated redefinitions of the concepts of chemical quantity. Avogadro's number, and its definition, was deprecated in favor of the Avogadro constant and its definition. Changes in the SI units are proposed to fix the value of the constant to exactly $6.02214X\times10^{23}$ when it is expressed in the unit mol^{-1}, in which an "X" at the end of a number means one or more final digits yet to be agreed upon.

In chemistry, the **molar mass** *M* is a physical property defined as the mass of a given substance (chemical element orchemical compound) divided by the amount of substance. The base SI unit for molar mass is kg/mol. However, for historical reasons, molar masses are almost always expressed in g/mol.

As an example, the molar mass of water: $M(H_2O) \approx 18$ g/mol**.**

The molar mass of atoms of an element is given by the standard relative atomic mass of the element multiplied by the molar mass constant, *M*u = 1×10^{-3} kg/mol = 1 g/mol:

M(H) = 1.007 97(7) × 1 g/mol = 1.00797(7) g/mol.

M(S) = 32.065(5) × 1 g/mol = 32.065(5) g/mol.

M(Cl) = 35.453(2) × 1 g/mol = 35.453(2) g/mol.

M(Fe) = 55.845(2) × 1 g/mol = 55.845(2) g/mol.

The **ideal gas law** is the equation of state of a hypothetical ideal gas. It is a good approximation of the behavior of many gases under many conditions, although it has several limitations. It was first stated by Émile Clapeyron in 1834 as a combination of the empirical Boyle's law, Charles' law and Avogadro's Law. The ideal gas law is often written as

$$PV = nRT, \tag{2.1}$$

where:

P - the pressure of the gas, V - the volume of the gas, n - s the amount of substance of gas (in moles), R - is the ideal, or universal, gas constant, equal to the product of the Boltzmann constant and the Avogadro constant, T - is the absolute temperature of the gas.

It can also be derived microscopically from kinetic theory, as was achieved (apparently independently) by August Krönigin 1856 and Rudolf Clausius in 1857.

How much gas is present could be specified by giving the mass instead of the chemical amount of gas. Therefore, an alternative form of the ideal gas law may be useful. The chemical amount (*n*) (in moles) is equal to the mass of each molecule of the gas (*m*) (in grams) divided by the molar mass (*M*) (in grams per mole):

$$n = \frac{m}{M}\cdot$$

By replacing *n* with *m*/*M* and subsequently introducing density $\rho = m/V$, we get:

$$PV = \frac{m}{M}RT, \tag{2.2}$$

$$P = \rho\frac{R}{M}T\cdot \tag{2.3}$$

Defining the specific gas constant $R_{\text{specific(r)}}$ as the ratio R/M,

$$P = \rho R_{specific} T \ . \tag{2.4}$$

This form of the ideal gas law is very useful because it links pressure, density, and temperature in a unique formula independent of the quantity of the considered gas. Alternatively, the law may be written in terms of the specific volume *v*, the reciprocal of density, as

$$P\upsilon = R_{specific} T \ . \tag{2.5}$$

It is common, especially in engineering applications, to represent the **specific** gas constant by the symbol *R*. In such cases, the **universal** gas constant is usually given a different symbol such as $\vec{R}$ to distinguish it.

In any case, the context and/or units of the gas constant should make it clear as to whether the universal or specific gas constant is being referred to.

2.1.3 Molecular-kinetic theory (MKT) for ideal gases

In molecular - kinetic theory uses an idealized model of an ideal gas.

Ideal gas called gas whose molecules do not interact with each other at a distance and have a negligible own sizes.

In real gas molecules have the force of intermolecular interaction. However, H_2, *He*, O_2, N_2 with n. c. (T = 273 K, P = 1.01 x 10^3 Pa) can be roughly considered as an ideal gas.

The process by which one of the parameters (*p*, *V*, *T*, *S*) remain constant, called isoprocesses (Figure`s 2.1-2.3).

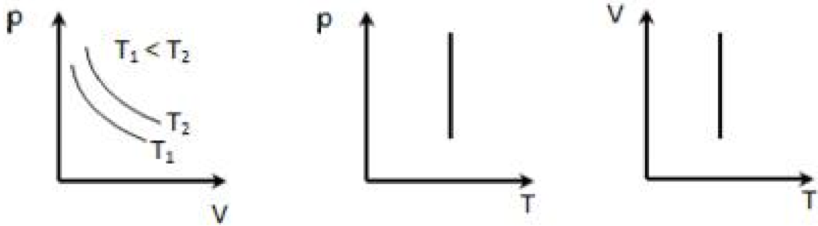


Figure 2.1

1. Isothermal process T = const, m = const, described by Boyle's law:

$$PV = const. \tag{2.6}$$

2. Isobaric process p = const is described by Gay-Lussac

$$V = V_0(1+\alpha t) \ . \tag{2.7}$$

$$V = V_0 \alpha T \tag{2.8}$$

α-thermal coefficient of volumetric expansion α = 1/273 K^{-1}.

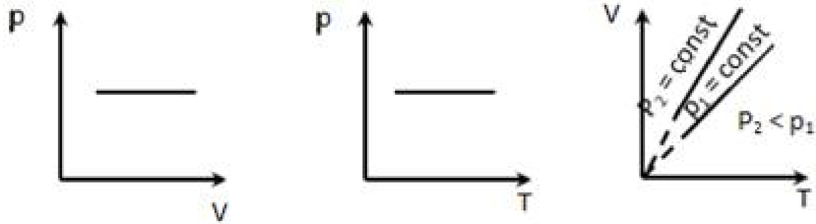


Figure 2.2

3. Isochoric process V = const is described by Charles law.

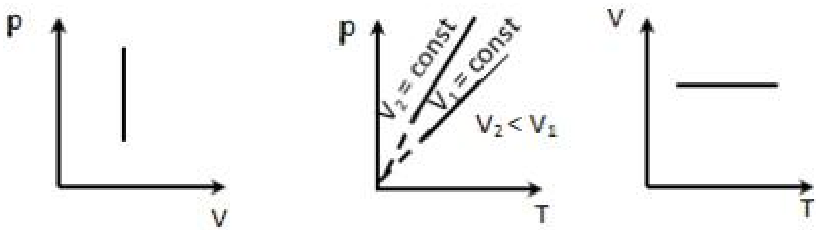


Figure 2.3

$$p = p_0(1+\alpha t) \tag{2.9}$$

$$p = p_0 \alpha T\ , \tag{2.10}$$

α - characterizes the dependence of the temperature. α is equal to the relative change in volume of a gas when heated at 1 K. The experience, α is the same for all gases and is α = 1/273 K^{-1}.

At random motions of gas particles collide with each other and with the walls of the vessel. The mechanical action of the collision, the vessel is seen as pressurn the walls. We distinguish on wall of the vessel some elementary area ΔS and find the pressure on this area.

The momentum obtained by concerned wall by the impact of a single molecule will be equal

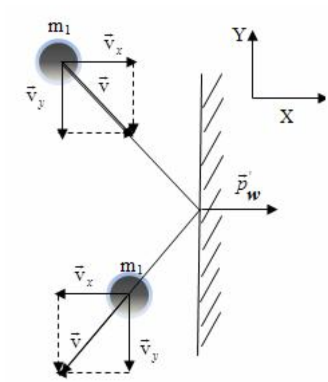


Figure 2.4

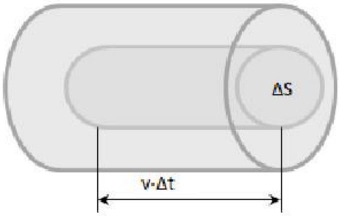


Figure 2.5

$$m_0\upsilon_x + p_w = -m_0\upsilon_x + p`_w \qquad (2.11)$$

$$p`_w = 2m_0\upsilon_x\,, \qquad (2.12)$$

m_0 - mass of a single molecule.

Newton's second law can be written as:

$$Fdt = d(m\upsilon_x)\,, \qquad (2.13)$$

$$F_i dt = F_{icp}\Delta t_i\,, \qquad (2.14)$$

Δt_i-time between two successive collisions of the i-th molecule with this wall

$$\Delta t_i = \frac{2l}{\upsilon_x}\,, \qquad (2.15)$$

$$f_i = \frac{2m_i\upsilon_{ix}^2}{2l}\,;\; m_1 = m_2 = \ldots = m_i = m\,, \qquad (2.16)$$

$$f_{iavx} = \frac{m\upsilon_{ix}^2}{l}\,, \qquad (2.17)$$

$$\vec{F} = \sum_{i=1}^{n} f_{iavx} = \sum_{i=1}^{n}\frac{m\upsilon_{ix}^2}{l}\,, \qquad (2.18)$$

All directions are equivalent:

$$\upsilon^2 = \upsilon_x^2 + \upsilon_y^2 + \upsilon_z^2\,, \qquad (2.19)$$

$$\upsilon_x^2 = \upsilon_y^2 = \upsilon_z^2 = \frac{1}{3}\upsilon_i^2\,, \qquad (2.20)$$

$$\vec{F} = \sum_{i=1}^{n}\frac{m\upsilon_i^2}{3l} = \frac{m}{3l}\sum_{i=1}^{n}\upsilon_i^2\,. \qquad (2.20,a)$$

We introduce the mean square velocity, which characterizes the whole set of molecules

$$\upsilon_{m.sq} = \sqrt{\frac{\upsilon_1^2 + \upsilon_2^2 + \ldots + \upsilon_i^2 + \ldots + \upsilon_N^2}{N}}\,. \qquad (2.21)$$

Pressure

$$p = \frac{F}{l^2} = \frac{m}{3l^3}\left(\frac{\upsilon_1^2 + \upsilon_2^2 + \ldots + \upsilon_n^2}{N}\right) = \frac{2mN}{3l^3}\cdot\frac{\upsilon_{m.sq.}^2}{2}\,, \qquad (2.22)$$

$$l^3 = V\,, \qquad (2.24)$$

$$p = \frac{2}{3}\frac{N}{V}\cdot\frac{m\upsilon^2_{m.sq.}}{2} = \frac{2}{3}\frac{N}{V}\varepsilon_0, \tag{2.25}$$

$$\varepsilon_0 = \frac{m\upsilon^2_{m.sq.}}{2}, \tag{2.26}$$

$p = \frac{1}{3}nm_0\upsilon^2_{m.sq.}$ - the basic equation of the MKT.

$$pV = \frac{2}{3}N\varepsilon_0, \tag{2.27}$$

$$E_K = N\varepsilon_0, \tag{2.28}$$

$$pV = \frac{2}{3}E_K\,. \tag{2.29}$$

From equation Mendeleev - Clapeyron:

$$pV_m = RT \Rightarrow RT = \frac{1}{3}\mu\upsilon^2_{m.sq.}, \tag{2.30}$$

$$\upsilon^2_{m.sq.} = \sqrt{\frac{3RT}{\mu}}, \tag{2.31}$$

$$\mu = m_0N_A \Rightarrow \upsilon_{m.sq.} = \sqrt{\frac{3RT}{m_0N_A}} = \sqrt{\frac{3kT}{m_0}}\,. \tag{2.32}$$

2.1.4 Maxwell's law of distribution of velocities and energies

The distribution law of ideal gas molecules in velocity, theoretically derived by Maxwell in 1860 determines how many molecules d*N* homogeneous (p = const) monatomic ideal gas of the total number *N* of molecules per unit volume at a given temperature *T* speeds in the range of inmates υ to $\upsilon + d\upsilon$.

To derive the function of the velocity distribution *f*(*v*) equal to the ratio of the number of molecules *dN*, the speed of which lie in the interval $\upsilon \div \upsilon + d\upsilon$ to the total number of molecules *N* and the size of the interval $d\upsilon$ (Figure 2.6)

$$f(\upsilon) = \frac{dN(\upsilon)}{Nd\upsilon}. \tag{2.33}$$

Maxwell used two sentences:

a) all directions in space are equivalent and therefore any direction of the particle, ie, any direction of the velocity is equally likely. This property is sometimes called the property of the isotropic distribution function.

b) moving along three mutually perpendicular axes that are independent, i.e x-component of velocity υ_x does not depend on what the meaning of its components or υ_y, υ_z. And then the output *f*(*v*) is initially for one component υ_x , and then extended to all the coordinates of speed.

It is also believed that the gas consists of a large number *N* of identical molecules are in a state of random thermal motion at the same temperature. Force fields do not apply to gas.

Function $f(v)$ determines the relative number of molecules $\frac{dN(\upsilon)}{N}$ speeds are in the interval from υ to $\upsilon + d\upsilon$ (eg gas has $N = 10^6$ molecules, with d*N* = 100 molecules have a velocity of υ = 100 and $\upsilon + d\upsilon$ = 101 *m /s* (d*v* = 1 m/s) then

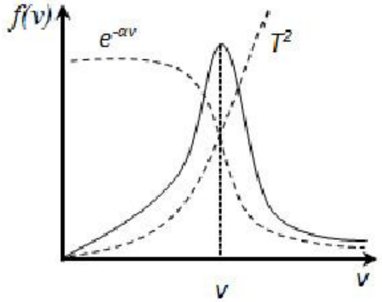


Figure 2.6

$$f(\upsilon) = \frac{100}{10^6 \cdot 1} = 10^{-4}$$

Using the methods of probability theory, Maxwell found the function $f(v)$ - the distribution of ideal gas molecules in velocity:

$$f(\upsilon) = 4\pi \left(\frac{m_0}{2\pi kT} \right)^{\frac{3}{2}} \upsilon^2 e^{-\frac{m_0 \upsilon^2}{2kT}}, \tag{2.34}$$

$f(v)$ depends on the type of gas (the variable (temperature *T*)

$f(v)$ depends on the ratio of the kinetic energy of a molecule corresponding to selected speed υ $\left(\frac{m_0 \upsilon^2}{2} \right)$ to the value of *kT* characterizes the average thermal energy of the gas molecules.

For small υ $\upsilon >> \Delta e^{-2\upsilon^2}$ and the function *f*(*v*) changes almost on a parabola υ^2. With an increase in the factor $\upsilon \cdot e^{-2\upsilon^2}$ decreases faster than the multiplier υ^2, i.e a max function $f(v)$. The speed at which the distribution function of molecules of an ideal gas at the maximum speed is called the most probable velocity υ_{mp} from the condition:

$$f_0^{`}(\upsilon) = 0, \tag{2.35}$$

$$\frac{d}{d\upsilon}(f(0)) = 0, \tag{2.36}$$

$$\frac{d}{d\upsilon}\left(\upsilon^2 e^{\frac{-m_0\upsilon^2}{2kT}} \right) = 2\upsilon e^{\frac{-m_0\upsilon^2}{2kT}} - 2\upsilon^2 \frac{m_0\upsilon^2}{2kT} e^{\frac{-m_0\upsilon^2}{2kT}} = 2\upsilon e^{\frac{-m_0\upsilon^2}{2kT}} \left(1 - \frac{m_0\upsilon^2}{2kT} \right) = 0, \tag{2.37}$$

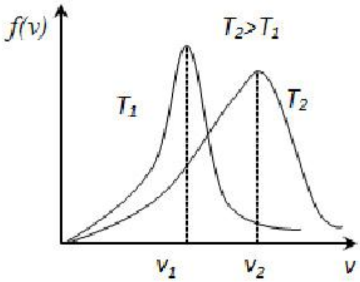


Figure 2.7

$$\upsilon_{mp} = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{\mu}}, \tag{2. 38}$$

$\upsilon \sim \sqrt{T}$ - therefore, with increasing temperature the most probable speed increases, but the area of ΔS, bounded by the curve of the distribution function remains unchanged, since the normalization condition $\int_0^\infty f(\upsilon)d\upsilon = 1$ since the probability of a certain event is 1, so that the temperature distribution curve $f(v)$ will stretch and fall.

In statistical physics, the average value of any quantity is defined as the integral from 0 to infinity, the product of the probability density for this value (statistical weight) (Figure 2.7)

$$\langle x \rangle = \int_0^\infty x f(x) dx, \tag{2.39}$$

Then the arithmetic average velocity of the molecules

$$\upsilon_{av} = \int_0^\infty \upsilon f(\upsilon) d\upsilon = \int_0^\infty \upsilon \cdot 4\pi \left(\frac{m_0}{2\pi kT} \right)^{\frac{3}{2}} \upsilon^2 e^{\frac{-m_0 \upsilon^2}{2kT}} d\upsilon, \tag{2.40}$$

and integrating by parts received

$$\upsilon_{av} = \sqrt{\frac{8kT}{\pi m}} = \sqrt{\frac{8RT}{\pi \mu}}, \tag{2.41}$$

Speed, characterizing the state the gas

$$\upsilon_{mp} = \sqrt{\frac{2RT}{\mu}}, \tag{2.42}$$

$$\upsilon_{av} = \sqrt{\frac{8RT}{\pi \mu}}, \tag{2.43}$$

$$\upsilon_{m.sq.} = \sqrt{\frac{3RT}{\mu}}. \tag{2.44}$$

2.1.5 Experimental verification of the Maxwell distribution of law – Stern experience

Along the axis of the inner cylinder tight platinum wire, covered with a layer of silver, which is heated by the current. When heated, the silver evaporates; the silver atoms are emitted through the gap and onto the inner surface of the second cylinder. If both cylinders are fixed, all atoms regardless of their speed fall in the same place *B* (Figure 2.8).

When rotating cylinder with an angular velocity ω silver atoms get into the points *B*', *B*" and so on. The magnitude of ω, the distance and displacement $x = BB'$can calculate the velocity of the atoms belonging to a point B'.

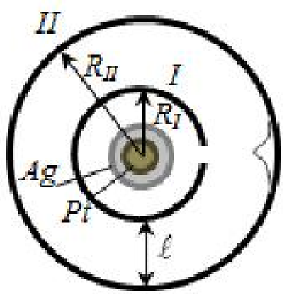


Figure 2.8

$$x = \omega \frac{l}{V} R_{II}, \tag{2.45}$$

$$\upsilon = \frac{\omega\, l}{x} R_{II}. \tag{2.46}$$

Slit image getting blurred. Exploring the thickness of the deposited layer can estimate the distribution of the velocity, which corresponds to a Maxwellian distribution.

2.1.6 Barometric formula. The Boltzmann distribution

Up to now, we have considered the behavior of an ideal gas not liable to attack to external force fields. From experience it is well known that the action of external forces, a uniform distribution of particles in space can be broken. Since gravity molecules tend to fall to the bottom of the vessel.

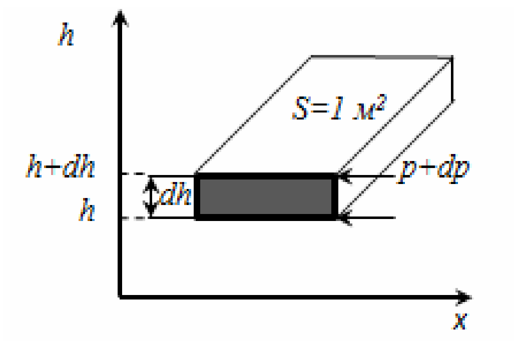


Figure 2.9

Intense thermal motion prevents precipitation of, and the molecules are distributed so that their concentration gradually decreases with increasing height.

We derive the variation of pressure with height assuming that the gravitational field is uniform, the temperature is constant and the mass of all the molecules of the same. If the atmospheric pressure at a height *h* equal to *p*, then at a height $h +$ dh is equal to $p +$ dp (with d$h > 0$, d$p < 0$, since *p* decreases with increasing *h*) (Figure 2.9).

The pressure difference at the heights *h* and *h* + d*h*, we can determine as the weight of the air molecules enclosed in a volume with a base area equal to 1 and a height d*h*.

$$p-(p+dp)=\rho g dh, \tag{2.47}$$

ρ – density at the height *h*, and since $dh \to 0$, then ρ = const.
Then

$$dp=-\rho g dh, \tag{2.48}$$

Of Mendeleev-Clapeyron equation.

$$pV=\frac{m}{\mu}RT \Rightarrow \rho=\frac{m}{V}=\frac{p\mu}{RT}, \tag{2.49}$$

Then

$$dp=-\frac{\mu g}{RT}p dh, \tag{2.50}$$

$$\frac{dp}{p}=-\frac{\mu g}{RT}dh. \tag{2.51}$$

With the change in height from h_1 to h_2 pressure changes from p_1 to p_2

$$\int_{p_1}^{p_2}\frac{dp}{p}=-\int_{h_1}^{h_2}\frac{\mu g}{RT}dh, \tag{2.52}$$

$$\ln\frac{p_2}{p_1}=-\frac{\mu g}{RT}(h_2-h_1). \tag{2.53}$$

Potentiated this expression

$$\frac{p_2}{p_1}=e^{-\frac{\mu g}{RT}(h_2-h_1)}, \tag{2.54}$$

$$p_2=p_1e^{-\frac{\mu g}{RT}(h_2-h_1)}. \tag{2.55}$$

Barometric formula shows how the pressure changes with altitude
At $h_2=h_1=0$ $p_2=p_3=p_{0\,atm.}$
Then

$$p=p_0e^{-\frac{\mu gh}{RT}}, \tag{2.56}$$

Because the

$$p=nkT, \tag{2.57}$$

and

$$p_0=n_0kT, \tag{2.58}$$

$$n=n_0e^{-\frac{\mu gh}{RT}}, \tag{2.59}$$

n - the concentration of molecules at a height

n_0 - the concentration of molecules at a height h = 0.
Because the

$$\mu = m_0 N_A,\ R = kN_A, \qquad (2.60)$$

then

$$n = n_0 e^{-\frac{\mu gh}{RT}}, \qquad (2.61)$$

$$U = m_0 gh. \qquad (2.62)$$

the potential energy of the molecules in a gravitational field

$$n = n_0 e^{-\frac{\mu gh}{RT}}. \qquad (2.63)$$

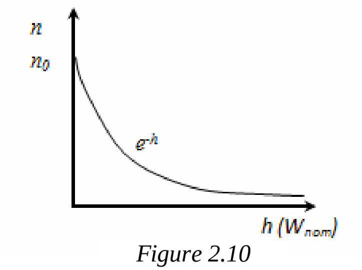


Figure 2.10

Boltzmann distribution law in an exterior potential field. From it follows that at T = const the density of gas is more there where potential energy of molecules is less.

2.1.7 The experimental determination of the Avogadro`s constant

J. Perrin (French scientist) in 1909, studied the behavior of Brownian particles in the emulsion gamboge (tree sap) with dimensions were examined with a microscope, which had a depth of field - 1 mm.

Moving the microscope in the vertical direction could be to investigate the distribution of Brownian particles in height.

Having applied to them a Boltzmann distribution law it is possible to write down

$$n = n_0 e^{-\frac{(m-m_1)gh}{RT}}, \qquad (2.64)$$

where m- particle mass

m_1 - weight of the displaced fluid.

If n_1 and n_2 concentration of particles at levels h_1 and h_2, and

$$k = \frac{R}{N_A}, \qquad (2.65)$$

then

$$N_A = \frac{3RT \ln \frac{n_1}{n_2}}{4\pi r^3 (\rho - \rho_1) g (h - h_1)}. \qquad (2.66)$$

Value $N_A = 6{,}5 \div 7{,}2 \cdot 10^{23} \frac{1}{mole}$ is in good agreement with the reference value $N_A = 6{,}02 \cdot 10^{23} \frac{1}{mole}$, which confirms the Boltzmann distribution of particles.

2.1.8 The mean free path of the molecules. Effective diameter

Gas molecules are in a state of chaotic motion continuously bump into each other. Between two successive collisions of molecules move uniformly in straight lines, passing with a path, which is called the mean free path. In general, the length of the path between successive collisions is different but since we are dealing with a large number of molecules and they are in random motion, we can speak of the **mean free path** (Figure 2.11):

$$\lambda = \frac{(\lambda_1 + \lambda_2 + ... + \lambda_n)}{n}. \tag{2.67}$$

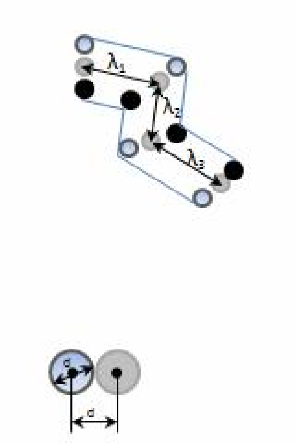


Figure 2.11

The minimum distance that converge in a collision centers of two molecules, called the effective diameter of the molecule.

It depends on the speed of the colliding molecules, that is, the temperature (the effective diameter decreases with increasing. For a second (t = 1 s) molecule transits on the average parth equal in magnitude to the average velocity.

If for one second, she undergoes an average z collision, then

$$\bar{\upsilon} = \frac{1}{t_S}, \tag{2.68}$$

$$\lambda = \frac{\bar{\upsilon}}{\bar{z}}. \tag{2.69}$$

To determine ν believe that the molecule has the shape of a ball, and moves among other fixed molecules. This molecule is confronted only with those molecules whose centers are at a distance *d*, i.e, lie within the "broken" a cylinder of radius d.

The average number of collisions per second is equal to one the number of molecules in the volume of "broken" cylinder.

$$\bar{z} = nV, \tag{2.70}$$

where n - the concentration of the molecules, and

$$V = \pi d^2 \bar{\upsilon}, \tag{2.71}$$

$\bar{\upsilon}$ - average speed of the molecules, or the path traveled by it in 1 second.

$\bar{z} = n\pi d^2 \bar{\upsilon}$ - the average number of collisions.

Taking into account a motion of other molecules:

$$\bar{z} = \sqrt{2} n\pi d^2 \bar{\upsilon}, \tag{2.72}$$

$$\overline{\lambda} = \frac{1}{\sqrt{2}\pi d^2 n}, \qquad (2.73)$$

i.e

$$\overline{\lambda} \sim \frac{1}{n}, \qquad (2.74)$$

$$n \sim \rho \Rightarrow \overline{\lambda} \sim \frac{1}{n}, \qquad (2.75)$$

$$\frac{\overline{\lambda_1}}{\lambda_1} = \frac{n_2}{n_1} = \frac{p_2}{p_1}. \qquad (2.76)$$

2.1.9 Transport Phenomena

Transport phenomena combine a group of processes associated with the irregularities of density, temperature and velocity of the orderly movement of individual layers of material. Alignment leads to inhomogeneities in transport phenomena.

Transport phenomena in gases and liquids consist in the fact that these substances an ordered, directed mass transport (diffusion), momentum (internal energy) and internal energy (thermal conductivity).

In the gas breaks complete randomness of the molecules and the distribution of molecular velocities. Deviations from the law of Maxwell explained directional transfer of physical characteristics of the material in the transport phenomena.

We only consider the one-dimensional phenomena, in which the physical quantities determining these phenomena depend only on one coordinate

1. Thermal conductivity

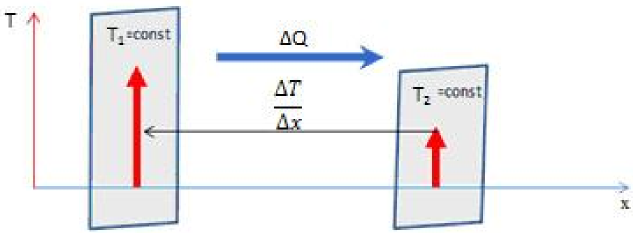


Figure 2.12

The phenomenon of thermal conductivity observed in the different parts of the considered gas temperature are different. Consideration of the effects of heat conduction from the microscopic point of view, shows that the amount of heat transported through the area ΔS, perpendicular to the direction of transfer is directly proportional to the thermal conductivity χ, which depends on the type of substance or gas, the temperature gradient $\frac{\Delta T}{\Delta x}$, the value area ΔS and observation time Δ*t* (Figure 2.12).

Fourier law:

$$\Delta Q = -x\frac{\Delta T}{\Delta x}\cdot \Delta S\Delta t, \qquad (2.77)$$

The minus sign in the Fourier law shows that heat is transferred in the direction of decreasing temperature *T*.

With molecular-kinetic phenomena in terms of thermal conductivity is explained as follows. In the area of gas, where the temperature is higher, the kinetic energy of the random thermal motion of the molecules is greater than the area where the temperature is lower. As a result of random thermal motion of the molecules move from the area where the region above *T*, where *T* is less.

However, they suffer from a kinetic energy greater of the average kinetic energy possessed by the molecules in the field of lower energy. Due to continuous collisions of molecules over time the process of alignment of the mean kinetic energy, that is, temperature equalization. The thermal conductivity χ is equal

$$\chi = \frac{1}{3}c_V\rho\overline{\upsilon}\overline{\lambda}, \qquad (2.78)$$

$$[\chi] = \frac{J}{K\cdot kg}\frac{kg}{m^3}\frac{m}{s}m = \frac{J}{K\cdot m\cdot s} = \frac{W}{K\cdot m}. \qquad (2.79)$$

Where c_V - specific heat capacity of gas at constant volume (a quantity of heat necessary for heating 1 *kg* of gas on 1 *K* at constant volume).

ρ - density of gas, $\overline{\upsilon}$ - average thermal velocity of the molecules, $\overline{\lambda}$ - medial free length.

Physical sense χ: the thermal conductivity χ is numerically equal to density of a thermal stream $f_E = \frac{Q}{\Delta S\Delta t}$ at the temperature gradient $\frac{\Delta T}{\Delta x}$ equal.

2. Diffusion

The phenomenon of diffusion is the spontaneous mixing of molecules of different gases or liquids. Diffusion phenomenon is observed in solids. In those cases where a chemically pure homogeneous gas concentration of molecules will be different, there is a transfer of molecules, leading to equalize (or concentration) of molecules. This phenomenon is of self-diffusion. For simplicity we assume that the density is inhomogeneous along the x axis.

Of the phenomenon of self-diffusion from the macroscopic point of view was Fick, who established the following law: the mass of the gas transported through the area ΔS, perpendicular to the direction of the transfer during Δt is proportional to the self-diffusion coefficient D, which depends on the type of gas, the density gradient $\frac{\Delta\rho}{\Delta x}$, the value of the site ΔS and the observation time Δt (Figure 2.13).

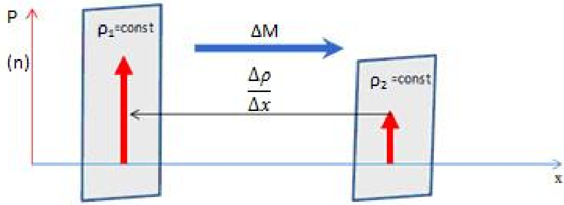


Figure 2.13

$$\Delta M = -D\frac{\Delta\rho}{dx}\Delta S\Delta t \quad \textbf{- Fick's law.} \tag{2.80}$$

The minus sign indicates that the mass of the gas transported in the direction of decreasing density. Self-diffusion coefficient D is numerically equal to the mass of gas transferred per unit time through a unit area perpendicular to the direction of transport, with a gradient of density equal to one

$$D = \frac{\Delta M}{\frac{\Delta\rho}{\Delta x}\Delta S\Delta t} = \frac{j_m}{\frac{\Delta\rho}{\Delta x}}, \tag{2.81}$$

$$j_m = \frac{\Delta M}{\Delta S\Delta t} \quad \text{- fluence.} \tag{2.82}$$

According to the kinetic theory of gases

$$D = \frac{1}{3}\bar{\upsilon}\bar{\lambda}, \tag{2.83}$$

$$[D] = \frac{m}{s}m = \frac{m^2}{s}. \tag{2.84}$$

3. The internal friction (viscosity)

The phenomenon of internal friction is observed in the case where the different layers of gas are moving at different speeds. In this case, the layers are decelerated more rapidly moving slowly. At the macroscopic motion of the gas layers (i.e, wall motion as a whole) has an impact microscopic thermal motion of the molecules (Figure 2.14).

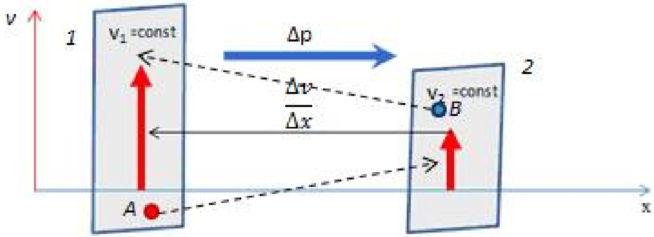


Figure 2.14

Consider one layer of gas moving at a speed v_1 and gas layer 2, moving at a speed v_2 $v_1 > v_2$. As a result of the thermal random motion of molecule A from layer 1 to layer 2 switch and change its momentum from the value mv to any value mv' ($v_2 < v' < v_1$).

The molecules B in a layer 2 as a result of the heat goes into the random motion of the layer 1 and change its momentum from the value mv_2 to the value of mv'' ($v_2 < v'' < v_1$), that is, the molecules in the layer above the former two, once in the layer 1, collisions with molecules it accelerates its orderly movement, and ordered the moving molecules of the layer 1 is slowing. On the contrary, the transition of molecules from a fast-moving layer 1 to layer 2, they carry large momenta and intermolecular collisions at layer 2 speed motion of the molecules of this layer.

The phenomenon of internal friction is described by Newton's viscous force F, acting between two layers of gas is directly proportional to the internal friction coefficient η, the velocity gradient and the size of square of ΔS.

(Impulse dp, carried through the area dS in the time Δt, is directly proportional to the internal friction coefficient η, the velocity gradient $\frac{d\upsilon}{dx}$, the value of the area dS and observation time dt).

$$dp = -\eta \frac{d\upsilon}{dx} dSdt \quad \textbf{- Newton's law.} \tag{2.85}$$

$$F = \frac{dp}{dt} = -\eta \frac{d\upsilon}{dx} dSdt \cdot \tag{2.86}$$

The minus sign indicates that the viscous force is opposite to the velocity gradient, that is, the momentum transferred in the direction of decreasing velocity.Coefficient of internal friction is given by

$$\eta = \frac{1}{3} \rho \overline{\upsilon} \overline{\lambda} \cdot \tag{2.87}$$

Relation between the coefficients for the transport phenomena

$$D = \frac{1}{3}\overline{\upsilon}\overline{\lambda}, \tag{2.88}$$

$$\chi = \frac{1}{3}\rho\overline{\upsilon}\overline{\lambda} = \rho D, \tag{2.89}$$

$$\chi = \frac{1}{3}c_V \rho\overline{\upsilon}\overline{\lambda} = c_V \eta = c_V \rho D \cdot \tag{2.90}$$

2.2 THERMODYNAMICS

System is a restricted region of space or a finite portion of matter one has chosen to study. Or the part of the universe, with well-defined boundaries, one has chosen to study.

Surrounding is the rest of the universe outside the region of interest (i.e. the rest of space outside the system).

Boundary or **Wall** is the surface that divides the system from the surroundings.

This wall or boundary may or may not allow interaction between the system and the surroundings.

Open System: This is a system that its boundary allows transfer of mass and energy into or out of the system. In other words, the boundary allows exchange of mass and energy between the system and the surrounding.

Closed System: This is a system that its boundary allows exchange of energy alone (inform of heat) between the system and its surrounding (i.e. the boundary allows exchange of energy alone). This type of boundary that allows exchange of heat is called diathermal boundary.

Isolated System: This is a system that its boundary allows neither mass nor energy between it and the surrounding. In other words, the boundary does not allow exchange of mass nor energy.

Isochoric process: This is a thermodynamic process that occurs at constant volume (i.e. $\Delta V = 0$ during this process). This implies that during this process no work is done on or by the system.

Isobaric process: This is a thermodynamic process that occurs at constant pressure (i.e. $\Delta p=0$ during this process).

Isothermal process: This is a thermodynamic process that takes place at constant temperature (i.e. $\Delta T = 0$ during this process).

Adiabatic process: This is a thermodynamic process in which there is no heat transfer into or out of the system. For this process, change in quantity of heat is zero (i.e. $\Delta Q = 0$ during this process).

It is possible to have multiple processes within a single process. A good example would be a case where volume and pressure change during a process, resulting in no change in temperature and no heat transfer. This kind of a process would be both adiabatic and isothermal.

Cyclic Processes: These are series of processes in which after certain interchanges of heat and work, the system is restored to its initial state. For a cyclic process ΔU =0, and if this is put into the first law

$$Q = W. \qquad (2.91)$$

This implies that the net work done during this process must be exactly equal to the net amount of energy transferred as heat; the store of internal energy of the system remains unchanged.

Reversible Process: A reversible process can be defined as one which direction can be reversed by an infinitesimal change in some properties of the system.

Irreversible Process: An irreversible process can be defined as one which direction cannot be reversed by an infinitesimal change in some properties of the system.

Quasi-static Process: This is a process that is carried out in such a way that at every instant, the system departs only infinitesimal from an equilibrium state (i.e. almost static). Thus a quasi-static process closely approximates a succession of equilibrium states.

Non-quasi-static Process: This is a process that is carried out in such a way that at every instant, there is finite departure of the system from an equilibrium state.

2.2.1 First law of thermodynamics. The internal energy

Every thermodynamic system in any state has the energy, which is called the total energy. The total energy of the system consists of the kinetic energy of the system as a whole, the potential energy of the system as a whole and of the internal energy (Figure 2.15)

$$E_k = \frac{m\upsilon^2}{2}, \; E_p = mgh \cdot \qquad (2.92)$$

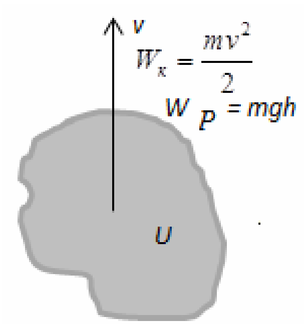


Figure 2.15

The internal energy of a system is the sum of all kinds of random (thermal) motion of molecules: the potential energy of theintratomic and intranuclear movements. The internal energy is a function of the gas state. For the internal energy of the gas state is uniquely determined, that is, a certain function.

$$U = f(p,V,T), \tag{2.93}$$

In the transition from one state to another internal energy changes. But the internal energy of the new state does not envy the process by which the system passed in this state.

There are two different ways to change the internal energy of a thermodynamic system. The internal energy of the system can change as a result of the work and as a result of heat transfer system. The work is a measure of change in the mechanical energy of the system. When the work is a movement system or individual macroscopic parts relative to each other. For example, fitting-in piston in the cylinder, which houses the gas, we compress the gas, causing its temperature increases, ie changes the internal energy of the gas.

The internal energy can be varied as a result of heat transfer, ie impart to gas certain amount of heat *Q*.

The difference between the heat and the work is that the heat is transferred from a large number of microscopic processes in which the kinetic energy of the molecules of a heated body in collisions of molecules transferred less heated body. Common between heat and work, that they are functions of the process, that is, we can talk about the value of warmth and work, when the transition of the system from the state the first in the state of the second. The warmth and the work is not a function of the state, as opposed to the internal energy. We can not say what is the work and heat the gas in state 1, but the internal energy in the state 1 can talk.

Suppose that a system (gas enclosed in a **cylinder under the piston), having** internal energy, get some heat *Q*, going into a new state, characterized by the internal energy of U_2, has made the work of *A* over the environment, that is, against the external forces. The amount of heat is positive when it is applied to the system, and negative when taken from the system. Work is positive when it is done with gas against external forces, and negative when it is done on the gas.

First Law of Thermodynamics: The amount of heat (ΔQ), the system is given to increase the internal energy of the system and to perform system work (*A*) against external forces.

$$\Delta Q = \Delta U + A, \tag{2.94}$$

$$A = p\Delta V, \tag{2.95}$$

$$[Q] = [U] = [A] = J. \tag{2.96}$$

Record the *I* thermodynamics beginning differentiall form

$$\delta Q = dU + \delta A, \tag{2.97}$$

dU - infinitesimal change of an internal energy of system, δA - elementary work, δQ - infinitesimal amount of heat.

If the system periodically returns to its original state, the change in the internal energy is equal to zero. Then $A_{1-1} = Q$ that is, I kind of perpetual motion machine, batch engine, which would have made a great job of it than the message from outside the energy is not possible.

2.2.2 The degrees of freedom of the molecule. The Law on the uniform energy distribution of the molecule. Energy distribution on molecule degrees of freedom

The number of degrees of freedom of the mechanical system is the number of independent variables, by which e can be specified at the system.

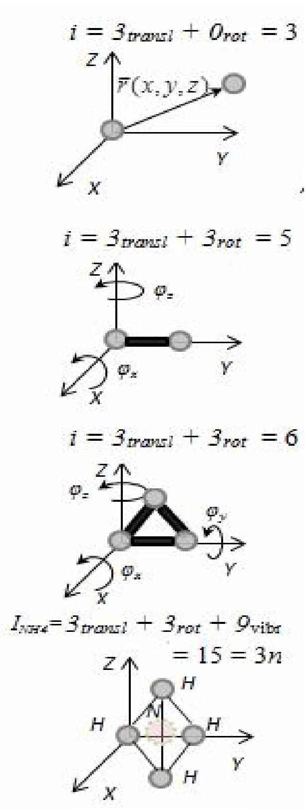


Figure 2.16

Monatomic gas has three translational degrees of freedom $i = 3$, because to describe the position of the gas in the space of only three coordinates (x, y, z) (Figure 2.16).

Rigid connection called a relationship in which the distance between the atoms is not changed. Diatomic molecule with a rigid connection (N_2, O_2, H_2) have three translational degrees of freedom and two rotational degrees of freedom: $i = i_{transl} + i_{rot} = 3 + 2 = 5$.

Translational degrees of freedom associated with the motion of the molecule as a whole in space, rotational - with rotation of the molecule as a whole. Rotation about the axes x and z by the angle φ_x and φ_z to changes in the position of the molecules in space, the rotation axis of the molecule does not change its position, therefore, coordinate φ_y in this case is not necessary. Triatomic molecule with a rigid connection has 6 degrees of freedom

$$i = i_{transl} + i_{rot} = 3 + 3 = 6$$

If the bond between the atoms is not tough, it adds the vibrational degrees of freedom. For non-linear molecules $i_{vibr} = 3N - 6$, where N - number of atoms in the molecule.

Regardless of the total number of degrees of freedom of the molecules of 3 degrees of freedom is always progressive. None of the translational has no advantage over the other, so each of them is on average the same energy, equal to 1/3 of $\vec{\varepsilon}$

$$p = \frac{2}{3} n \overrightarrow{\varepsilon_{3\,transi}} \quad p = nkT \Rightarrow$$

$$\overrightarrow{\varepsilon_{3\,transi}} = \frac{3}{2} kT \quad \overrightarrow{\varepsilon_1} = \frac{\overrightarrow{\varepsilon_{3\,transi}}}{3} = \frac{1}{2} kT \cdot \tag{2.98}$$

Boltzmann law has established that in order for the statistical system (i.e, for a system in which large number of molecules), which is in thermal equilibrium at each translational and rotational degrees of freedom have an average kinematic energy equal to 1/2 *kT*, and for each vibrational degree of freedom - the average energy equal to *kT*.

Vibrational degree of freedom "has" twice as much energy because it accounts for not only the kinetic energy (as in the case of the translational and rotational motion), but the potential energy, and $E_k = E_p$, so the average energy of a molecule

$$\overline{E} = \frac{i`}{2} kT, \tag{2.99}$$

$$i` = i_{transl} + i_{rot} + 2i_{vibr} \,. \tag{2.100}$$

We will consider a molecule with a rigid connection, so

$$i = i_{transl} + i_{rot}, \tag{2.101}$$

$$\overline{E} = \frac{i}{2} kT \cdot \tag{2.102}$$

as in an ideal gas the mutual potential energy of the molecules is zero (the molecules do not interact with each other), the internal energy is the product of 1 mol of the mean energy of one molecule to the number of molecules in a mole of substance, that is the number of Avogadro

$$U = \frac{i}{2} kTN_A = \frac{i}{2} RT \cdot \tag{2.103}$$

2.2.3 Heat capacity. Work gas

Specific heat capacity of a substance - the value of which is equal to the amount of heat required to heat 1 *kg* of matter at 1*K*.

$$c = \frac{\delta Q}{mdT}, \tag{2.104}$$

$$c = \frac{Q}{m\Delta T}, \tag{2.105}$$

$$[c] = \frac{J}{kg \cdot K} \,. \tag{2.106}$$

Molar heat capacity - the value of which is equal to the amount of heat needed to heat one mole of a substance by 1*K*.

$$C_\mu = \frac{\delta Q}{\nu dT}, \qquad (2.107)$$

$$C_\mu = \frac{Q}{\nu \Delta T}, \qquad (2.108)$$

$$[C_\mu] = \frac{J}{mole \cdot K}. \qquad (2.109)$$

Relationship molar and the specific heat

$$C_\mu = c \cdot \mu. \qquad (2.110)$$

Distinguish the specific heat at constant volume C_V (V = const) and constant pressure C_p (p = constif in the process of heating a substance to the volume or the pressure is kept constant.

Work of gases at change of its volume

Consider a gas in a piston and a cylinder

If the gas expands, the piston moves to the infinitesimal distance d*l*, the gas produced on the piston work (Figure 2.17).

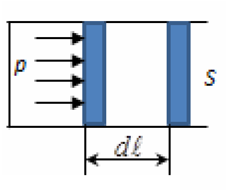


Figure 2.17

$$\delta A = Fdl = pSdl = pdV, \qquad (2.111)$$

where S - the piston area. $Sdl = dV$ – change the volume of the system

$$\delta A = pdV. \qquad (2.112)$$

A total work done by the gas at the change in volume fromV_1 to V_2 is equal

$$A = \int_{V_1}^{V_2} pdV. \qquad (2.113)$$

2.2.4 C_p, C_V and the relationship between them (Mayer's equations). First law of thermodynamics, Q, U, A to izoprocesses

Let's write down expressions *I* of the *I law* of thermodynamics for 1 mole gas

$$\delta Q = dU_\mu + \delta A, \qquad (2.114)$$

$$\delta Q = C_\mu dT, \qquad (2.115)$$

$$\delta A = pdV, \qquad (2.116)$$

$$C_\mu dT = dU_\mu + pdV. \qquad (2.117)$$

If the gas is heated at constant volume (V = const, dV = 0) A = 0, and imparted to the gas heat goes only to increase its internal energy

$$C_\mu dT = dU_\mu, \qquad (2.118)$$

$$C_V = \frac{dU_\mu}{dT}. \qquad (2.119)$$

that is, the molar heat of the gas at constant volume equals the change in internal energy of one mole of gas at temperatures hanging 1*K*.

Because

$$dU = \frac{i}{2}RdT, \tag{2.120}$$

$$C_V = \frac{i}{2}R \cdot \tag{2.121}$$

If the gas is heated at constant pressure p = const

$$C_p = \frac{dU_\mu}{dT} + \frac{pdU_\mu}{dT}, \tag{2.122}$$

because $\frac{dU_\mu}{dT}$ it does not depend on the type of process (the internal energy does not depend on p and V, is determined only by the temperature T) and $\frac{dU_\mu}{dT} = C_V \cdot$

From equation Mendeleev-Clapeyron

$$pV = RT, \tag{2.123}$$

$$pdV = RdT \Rightarrow R = \frac{pdV}{dT}, \tag{2.124}$$

$C_p = C_V + R$ - Mayer equation.

Mayer equation shows that C_p is always greater than the value of C_v on universal gas constant R, since at p = const requires an additional amount of heat to perform the work of expansion of gas, as constant pressure to ensure increased gas

$$C_p = C_V + R = \frac{i}{2}R + R = \frac{i+2}{2}R, \tag{2.125}$$

$$C_p = \frac{i+2}{2}R, \tag{2.126}$$

Value

$$\gamma = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} = \frac{i+2}{2}, \tag{2.127}$$

is a characteristic value for each gas. For monatomic gases $\gamma = 5/3$, for biatomic - 7/5, for threeatomic – 4/3.

1. *Isothermal process* T = const, m = const.

Boyle's law

$$p_1V_1 = p_2V_2 = const \Rightarrow \frac{V_2}{V_1} = \frac{p_1}{p_2}, \tag{2.128}$$

$$A=\int_{V_1}^{V_2} pdV=\int_{V_1}^{V_2}\frac{m}{\mu}RT\frac{dV}{V}=\frac{m}{\mu}RT\ln\frac{V_2}{V_1}=\frac{m}{\mu}RT\ln\frac{p_1}{p_2}, \qquad (2.129)$$

$$p=\frac{m}{\mu}\frac{RT}{V}, \qquad (2.130)$$

$$p=\frac{m}{\mu}\frac{RT}{V}, \qquad (2.131)$$

$$dU=-C_V\cdot dT=0\Rightarrow dU=0, \qquad (2.132)$$

$$\delta Q=dU+\delta A\Rightarrow \delta Q=\delta A. \qquad (2.133)$$

i.e the total amount of heat imparted to the gas is consumed in the performance of his work against external forces

$$Q=A=\frac{m}{\mu}RT\ln\frac{V_2}{V_1}=\frac{m}{\mu}RT\ln\frac{p_1}{p_2}. \qquad (2.134)$$

that at work expanding the temperature did not change to gas during the isothermal process is necessary to sum up the amount of heat equivalent to the work of foreign expansion.

2. *Isobaric process*. p = const m = const

$$\int_{V_1}^{V_2} pdV=p(V_2-V_1), \qquad (2.135)$$

$$A=p(V_2-V_1). \qquad (2.136)$$

The Mendeleev-Clapeyron equation for state 1 and 2 (Figure 2.18):

$$p_1V_1=\frac{m}{\mu}RT_1, \qquad (2.137)$$

$$p_1V_2=\frac{m}{\mu}RT_2, \qquad (2.138)$$

$$p_1(V_2-V_1)=A=\frac{m}{\mu}R(T_2-T_1), \qquad (2.139)$$

$$R=\frac{A}{\frac{m}{\mu}(U_2-U_1)}. \qquad (2.140)$$

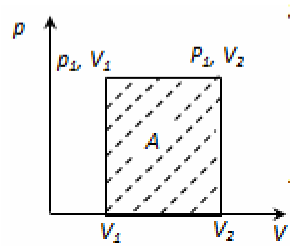


Figure 2.18

The physical meaning of R: R is numerically equal to the work by heating 1 mole of gas at 1K (T_2 - T_1 = 1 K) in the isobaric process.

$$\delta U=\frac{m}{\mu}C_V\cdot dT, \qquad (2.141)$$

$$\delta Q=\frac{m}{\mu}C_p\cdot dT, \qquad (2.142)$$

$$\delta Q=dU+\delta A, \qquad (2.143)$$

$$Q=\Delta U+A. \qquad (2.144)$$

heat supplied to the gas, is the change in its internal energy and the work of the commission.

3. Isochoric process. V = const, m = const

$$\delta A = p \cdot dV = 0, \tag{2.145}$$

because V = const and $dV = 0$

$$dU = \frac{m}{\mu} C_V dT, \tag{2.146}$$

$$A = 0 \quad \delta A = dU, \tag{2.147}$$

$$Q = \Delta U, \tag{2.148}$$

$$\delta Q = dU = \frac{m}{\mu} C_V \cdot dT. \tag{2.149}$$

All the warmth imparted by gas, is the change in its internal energy.

2.2.5 The adiabatic process. Polytropic process

Called **adiabatic process** that takes place without heat exchange with the environment. To include all the adiabatic fast processes. For example, an adiabatic process can be regarded as the propagation of sound in the medium, as speed of sound is so high that the energy exchange between the wave and the medium does not have time to happen.

Adiabatic processes are used in internal combustion engines, refrigeration, etc. We find the equation relating the parameters of ideal gas at an adiabatic process.

We write I law of thermodynamics.

$$\delta Q = dU + \delta A, \tag{2.150}$$

For an adiabatic process

$$\delta Q = 0 \Rightarrow \delta A = -dU. \tag{2.151}$$

i.e external work is done by changing the internal energy of the system

$$\delta A = pdV, \tag{2.152}$$

$$dU = \frac{m}{\mu} C_V dT, \tag{2.153}$$

$$\frac{m}{\mu} C_V dT + pdV = 0. \tag{2.154}$$

From equation Mendeleev-Clapeyron express p

$$pV = \frac{m}{\mu} RT \quad p = \frac{m}{\mu} \frac{RT}{V}, \tag{2.155}$$

$$\frac{m}{\mu} C_V dT + \frac{m}{\mu} \frac{RT}{V} dV = 0, \tag{2.156}$$

$$\frac{dT}{T} + \frac{R}{C_V} \frac{dV}{V} = 0. \tag{2.157}$$

Rewritten as:

$$d\left(\ln T + \frac{R}{C_V}\frac{dV}{V}\right) = 0, \tag{2.158}$$

i.e

$$\ln T + \frac{R}{C_V}\ln V = const, \tag{2.159}$$

$$\gamma = \frac{C_p}{C_V} = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V} \Rightarrow \frac{R}{C_V} = \gamma - 1. \tag{2.160}$$

Potentiated

$$(e^{\ln x} = X), \tag{2.161}$$

$$TV^{\gamma-1} = const. \tag{2.162}$$

- Adiabatic equation in the coordinates *T* and *V*.

$$pV = \frac{m}{\mu}RT \Rightarrow T = \frac{\mu}{m}\frac{PV}{R}, \tag{2.163}$$

$$\frac{\mu p}{mR}VV^{\gamma-1} = const. \tag{2.164}$$

- The Poisson equation (adiabatic equation in the coordinates *p* and *V*) (Figure 2.19):

$$pV^{\gamma} = const, \tag{2.165}$$

$\gamma = \dfrac{C_p}{C_V}$ - the adiabatic (or Poisson's ratio).

$pV = const$ - isotherm equation as $\gamma > 1$, the adiabatic curve is steeper than the isotherm. This is explained by the fact that the

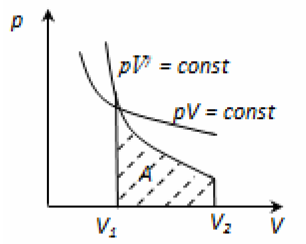


Figure 2.19

adiabatic compression 1 - 3 increase in gas pressure due not only to a decrease in its volume as an isothermal compression, but also increase in temperature

$$pV = \frac{m}{\mu}RT \Rightarrow V = \frac{m}{\mu}\frac{RT}{p}, \tag{2.166}$$

$$p\left(\frac{m}{\mu}\frac{RT}{p}\right)^{\gamma} = const, \tag{2.167}$$

$$Tp^{\frac{-\gamma+1}{\gamma}} = const. \tag{2.168}$$

- adiabatic equation in the coordinates *p*, *T*.

Compute the work is done by the gas in an adiabatic process.

First law of thermodynamics for an adiabatic process

$$\delta A = -dU \Rightarrow \delta A = -\frac{m}{\mu} C_V \cdot dT \cdot \tag{2.169}$$

If the gas adiabatically expands from volume V_1 to V_2, its temperature decreases from T_1 to T_2, and the work of expansion of ideal gas

$$A = -\frac{m}{\mu} C_V \int_{T_1}^{T_2} dT = \frac{m}{\mu} C_V (T_1 - T_2) \ \ (T_1 > T_2), \tag{2.170}$$

$$T_1 = \frac{\mu}{m} \frac{p_1 V_1}{R} \ \ \frac{C_V}{R} = \frac{1}{\gamma - 1}, \tag{2.171}$$

$$TV^{\gamma-1} = const, \tag{2.172}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} = const, \tag{2.173}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{\mu}{m} \frac{p_1 V_1}{R} \left(\frac{V_1}{V_2}\right)^{\gamma-1}, \tag{2.173,a}$$

$$A = \frac{m}{\mu} C_V \frac{\mu}{m} \frac{p_1 V_1}{R} \left(1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right), \tag{2.174}$$

$$A = \frac{p_1 V_1}{\gamma - 1} \left(1 - \left(\frac{V_1}{V_2}\right)^{\gamma-1}\right). \tag{2.175}$$

The work done by the gas during the adiabatic expansion 1- 2 is equal to square, shaded in the figure and it is less than the work of an isothermal expansion. This is explained by the fact that the adiabatic expansion cools the gas, whereas the isothermal expansion temperature is kept constant by the influx from the outside of the equivalent amount of heat.

Considered isochoric, isobaric, isothermal and adiabatic processes have in common - they occur at constant heat capacity (C_V, C_P, $C_T = \infty$, $C_A = 0$). In the first two processes are equal to the heat capacity C_v and C_p in an isothermal process ($dT = 0$) $C_T = \infty$, in an adiabatic process $\delta Q = 0$ and $C_A = 0$.

The process in which heat is constant is called **polytropic** (C = const). Started on the basis of I law thermodynamics at a constant heat capacity (C = const) can be derived polytropic equation

$$pV^n = const, \tag{2.176}$$

where

$$n = \frac{C - C_p}{C - C_V}; \ \ C \neq C_V, \tag{2.177}$$

n - adiabatic index.

When $C = 0$ $n = \gamma$ pV^γ=const adiabatic equation

When $C = \infty$ n = 1 pV = const - isotherm equation

When $C = C_p$ $n = 0$ p = const, $p = \frac{RT}{V} TV^{n-1} = const$

$\frac{T}{V} = const$ isobar equation.

When $C = C_V$ $n = \pm\infty$ $C - C_p \ln V + 0 \ln p = const \Rightarrow V = const$.

Thus, all the processes are special cases of a polytropic process.

2.2.6 Entropy. Second law of thermodynamics

Usually, any process in which the system moves from one state to another, occurs in a way that can not be done this process in the opposite direction so that the system passed through the same intermediate states, while in others the bodies are not any changes. This is due to the fact that in the process of the energy is dissipated, for example, due to friction, radiation, etc. Thus almost all processes in nature are irreversible.

In any process of the energy is lost. To characterize the energy dissipation is introduced the concept of entropy. (The entropy characterizes the thermal state of the system and determines the probability of a given state of the body. The more likely this condition, the greater the entropy.) All naturel processes are accompanied by an increase in entropy.

Entropy is constant only in the case of an idealized reversible process that occurs in a closed system, ie a system in which there is an exchange of energy with the external to this system bodies.

Thermodynamic entropy and its meaning:

$$dS = \frac{\delta Q}{T}. \quad (2.178)$$

Entropy is a function of the system, an infinitesimal change in a reversible process which is the ratio of the infinitesimal amount of heat introduced into the process, to the temperature at which it was introduced.

All in a reversible process the entropy change can be calculated by the formula:

$$\Delta S = S_2 - S_1 = \int_1^2 dS = \int_1^2 \frac{\delta Q}{T}, \quad (2.179)$$

where the integration is one of the initial state 1 of the system to the final state 2.

Since entropy is a state function, the integral $\int \frac{\delta Q}{T}$ is the property of its independence from the shape of the contour (path), on which it is calculated, so the integral is determined only by the initial and final states of the system.

In any reversible process the change in entropy is equal to 0

$$\Delta S = \int_1^2 \frac{\delta Q}{T} = 0 \qquad (2.180)$$

In thermodynamics, it is proved that the system undergoes irreversible scycle increases

$$\Delta S > 0, \qquad (2.181)$$

Equations (2.180) and (2.181) apply only to closed systems, but if the system is exchanged heat with the environment, its S can behave in any way.

Of (2.180) and (2.181) can be written as the Clausius inequality

$$\Delta \mathrm{S} \geq 0, \qquad (2.182)$$

i.e entropy of a closed system can either increase (in the case of irreversible processes) or remain constant (in the case of reversible processes).

If the system performs the equilibrium transition from state 1 to state 2, the entropy change

$$\Delta S_{1\to 2} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} = \int_1^2 \frac{dU + \delta A}{T}, \qquad (2.183)$$

where dU and δA is recorded for a particular process. According to this formula ΔS is determined up to an additive constant. The physical meaning is not the entropy, entropy difference. We find the entropy change in the process of an ideal gas.

$$dU = \frac{m}{\mu} C_V dT, \qquad (2.184)$$

$$\delta A = pdV = \frac{m}{\mu} RT \frac{dV}{V}, \qquad (2.185)$$

$$\Delta S_{1\to 2} = S_2 - S_1 = \int_{T_1}^{T_2} \frac{m}{\mu} C_V \frac{dT}{T} + \int_{V_1}^{V_2} \frac{m}{\mu} R \frac{T}{T}\frac{dV}{V} =, \qquad (2.186)$$

$$= \frac{m}{\mu}(C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1})$$

i.e changes in the entropy S $\Delta S_{1\to 2}$ ideal gas at its transition from state 1 to state 2 is independent of the process.

Because for an adiabatic process δQ = 0, ΔS = 0 => S = const, ie adiabatic reversible process takes place at constant entropy. So it is called isentropic.

An isothermal process (T = const; $T_1 = T_2$: $\ln 1 = 0$)

$$\Delta S = \frac{m}{\mu} R \ln \frac{V_2}{V_1}. \qquad (2.187)$$

When isochoric process (V = const; $V_1 = V_2$; $\ln 1 = 0$)

$$\Delta S = \frac{m}{\mu} C_V \ln \frac{T_2}{T_1}. \qquad (2.188)$$

Entropy is additive: the entropy of the system is the sum of the entropies of bodies in the system. $S = S_1 + S_2 + S_3 + \ldots$ Qualitative difference of the thermal motion of molecules from other forms of movement is its state of chaos, randomness. Therefore, to characterize the thermal motion to introduce a quantitative measure of the degree of molecular disorder.

If we consider any given macroscopic state of the body with some average values, then it is nothing but the continuous change of close microstates differing distribution of molecules in different parts of the volume and the energy is distributed between the molecules.

These continuous successive microstates characterizes the degree of disorder of the macroscopic state of the entire system, ϖ is called thermodynamic probability of the microstate. Thermodynamic probability ϖ of the system - is the number of ways that this can be done state macroscopic system, or the number of microstates carrying this microstate ($\varpi \geq 1$, and the mathematical probability of ≤ 1).

According to Boltzmann, the entropy S of the system and the thermodynamic probability linked as follows:

$$S = k \ln \omega , \tag{2.189}$$

where k - Boltzmann constant, $k = 1{,}38 \cdot 10^{-23} \frac{J}{K}$.

Thus, the entropy is defined logarithm of the state in which this can be achieved microstate. Entropy can be considered as a measure of the probability of the state thermodynamic system. Boltzmann formula allows us to give the following statistical interpretation of entropy. Entropy is a measure of disorder in a system. In fact, the greater the number of microstates realizing this microstate, the greater the entropy.

In the equilibrium state of the system - the most probable state of the system - the maximum number of microstates, with the maximum and entropy.

Because real processes are irreversible, it can be argued that all the processes in a closed system leads to an increase in its entropy - the principle of entropy increase.

In the statistical interpretation of entropy, this means that the process in a closed system are to increase the number of microstates, in other words, from less probable to more probable, as long as the probability of the state will not be maximized.

The first law of thermodynamics, expressing energy conservation and transformation of energy, does not establish the direction of the flow thermodynamic processes. You can also submit a set of processes that do not contradict the beginning First law thermodynamic, in which energy is conserved, but in nature they are not implemented.

Possible formulation of the **Second law of thermodynamics**:

1) the law of increasing entropy of a closed system in irreversible processes: any irreversible process in a closed system is such that the entropy of the system is on the increase $\Delta S \geq 0$.

2) $\Delta S \geq 0$ ($S = 0$ for a reversible and $\Delta S \geq 0$ for an irreversible process). The processes that take place in a closed system, entropy does not decrease.

3) From the Boltzmann $S = k \ln \omega > 0$, and consequently, an increase in the entropy of the system means the transition from a less probable to a more probable state.

4) According to Kelvin: circular process is not possible, the only result of which is the conversion of heat received from the heater into an equivalent work.

5) In the Clausius: circular process is not possible, the only result is to transfer heat from the less heated body to a warmer.

To describe the thermodynamic systems at 0 K using Theorem Nernst-Planck (third thermodynamics): the entropy of all bodies in equilibrium tends to zero as the temperature approaches 0 K

$$\lim_{T \to 0} S = 0 \cdot \tag{2.190}$$

From Theorem Nernst-Planck equation, it follows that $C_p = C_v = 0$ at 0 K.

2.2.7 Thermal and refrigerators. Carnot Cycle and its efficiency

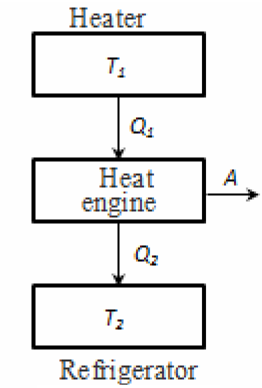


Figure 2.20

From the wording of the second law of thermodynamics on Kelvin that a perpetual motion machine of the second kind is impossible. (Perpetual motion - this batch engine does work by cooling a heat source.)

Thermostat is thermodynamic system that can exchange heat with the bodies without a change in temperature.

The principle of operation of the heat engine: the thermostat with temperature T_1 **-** heater for a **cycle** is subtracted the amount of heat Q_1, a thermostat with temperature T_2 ($T_2 < T_1$), refrigerator, for a series of heat transferred to Q_2, while work is done $A = Q_1 - Q_2$ (Figure 2.20)

Circular process or cycle is a process by which the system is going through a number of states, is reset. Cycle in the state diagram depicted a closed curve. Cycle, performed by an ideal gas can be divided into processes of expansion (1-2) and compression (2-1), the work of expansion is positive $A_{1\text{-}2} > 0$, because $V_2 > V_1$, the work of compression is negative $A_{1\text{-}2} < 0$, because $V_2 < V_1$. Hence, the work done by the gas per cycle, determined by the area covered by a closed curve 1-2-1. If the cycle is done positive work $A = \oint pdV > 0$ loop clockwise), then the cycle is called direct if $A = \oint pdV < 0$ - reverse cycle (cycle occurs in a counter-clockwise).

Direct cycle is used in heat engines - periodically a motor does the work by producing heat from the outside. Reverse cycle is used in refrigerators - periodically existing installations, in which through the work of external forces, heat is transferred to the body with a higher temperature.

As a result of the circular process, the system returns to its initial state, and therefore, the total change in internal energy is zero. Then start the I law of thermodynamics for a circular process

$$Q = \Delta U + A = A, \tag{2.191}$$

that is, the work done per cycle is the amount of heat received from the outside, but

$$Q = Q_1 - Q_2, \tag{2.192}$$

where Q_1 - the amount of heat received by the system, Q_2 - the amount of heat given system.

Thermal efficiency for cyclic process is the ratio of the work done by the system, to the amount of heat supplied to the system:

$$\eta = \frac{A}{Q_1} = \frac{Q_2 - Q_1}{Q_1} = 1 - \frac{Q_2}{Q_1}. \tag{2.193}$$

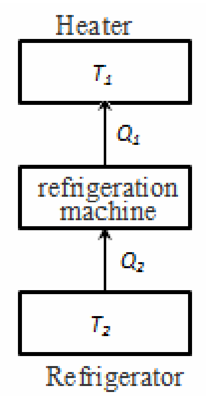


Figure 2.21

To $\eta = 1$, the condition $Q_2 = 0$, ie, heat engine should have one heat source Q_1, but this contradicts the second law of thermodynamics.

The reverse process is happening in the heat engine is used in the refrigeration machine (Figure 2.21).

The thermostat with temperature T_2 deducted the amount of heat Q_2 transferred to the thermostat and the temperature T_1, the amount of heat Q_1.

$Q = Q_2 - Q_1 < 0$ so $A < 0$.

Without doing the work can not take away the heat from a hot body, and at least give it a warmer. Based on the second law of thermodynamics, Carnot led theorem.

Carnot's theorem: From time to time all heat engines operating with the same heater temperature (T_1) and refrigerators (T_2), the highest efficiency have a reversible machine. Efficiency reversible machine with equal T_1 and T_2 are equal and do not depend on the nature of the working fluid.

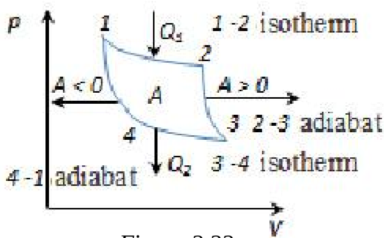


Figure 2.22

Working body - the body to make circular process to exchange energy to other bodies.

Carnot cycle - the most economical reversible cycle consisting of 2 isotherms and 2 adiabats (Figure 2.22).

1-2-isothermal expansion at T_1 heater, gas is supplied to the heat Q_1 and work is done

$$A_{12} = \int_1^2 pdV > 0 \cdot \tag{2.194}$$

2 - 3 - adiabats expansion, the gas does work $A_{2\text{-}3} > 0$ on external bodies.

3 - 4-isothermal compression at T_2 refrigerator, heat is taken Q_2 and work is done

$$A_{34} = \int_3^4 pdV < 0 \cdot \tag{2.195}$$

4-1-adiabatic compression work is done on the gas $A_{4\text{-}1} < 0$ external bodies.

An isothermal process U = const, so $Q_1 = A_{12}$

$$A_{12} = \frac{m}{\mu} RT_1 \ln\frac{V_1}{V_2} = Q_1 \cdot \tag{2.196}$$

Adiabatic expansion $Q_{2\text{-}3} = 0$, and the work done A_{23} gas by the internal energy $A_{23} = -U$

$$A_{23} = \frac{m}{\mu} C_V (T_3 - T_1) \cdot \tag{2.197}$$

The amount of heat Q_2, uploaded from gas to refrigerator at isothermal compression equal to the work of compression $A_{3\text{-}4}$

$$A_{34} = \frac{m}{\mu} RT_2 \ln\frac{V_4}{V_3} = -Q_2 \cdot \tag{2.198}$$

The work of the adiabatic compression

$$A_{41} = -\frac{m}{\mu} C_V (T_3 - T_1) = -A_{23} \cdot \tag{2.199}$$

The work performed by a circular process

$$A = A_{12} + A_{23} + A_{34} + A_{41} = Q_1 + A_{23} - Q_2 - A_{23} = Q_1 - Q_2, \tag{2.200}$$

and equal to the area of the curve 1-2-3-4-1.

Thermal efficiency Carnot cycle

$$\eta = \frac{A}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}. \tag{2.201}$$

From the equation for adiabatic processes 2-3 and 3-4 we obtain

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}; T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}, \tag{2.202}$$

then

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{\frac{m}{\mu} RT_1 \ln \frac{V_2}{V_1} - \frac{m}{\mu} RT_2 \ln \frac{V_3}{V_4}}{\frac{m}{\mu} RT_1 \ln \frac{V_2}{V_1}} = \frac{T_1 - T_2}{T_1}, \tag{2.203}$$

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}. \tag{2.204}$$

i.e efficiency Carnot cycle is determined only by the temperature heater and refrigerator. To increase the efficiency necessary to increase the difference T_1 -T_2.

Chapter 3 ELECTRICITY AND ELECTROMAGNETISM

3.1 ELECTROSTATICS. ELECTRIC FIELD IN VACUUM

3.1.1 Atomistic charge. The elementary charge. Law of conservation of electrical charge

Under the charge to understand the physical properties of elementary particles contaminated exerts a force on a charged particle Despite the huge variety of substances in nature, there are only two types of electric charges: positive, which arise, for example, the glass in sliding his skin, and negative - in Ebony, worn on the fur (Figure 3.1).

Like charges repel, unlike charges - attract.

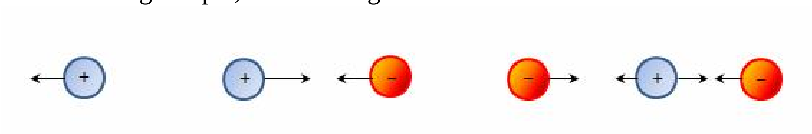


Figure 3.1

Electric charge is discrete, i.e charge of any body is an integer multiple of the elementary electric charge: $q = n\ e$, where n-positive integer, e - electron charge $e = -1.6{\cdot}10^{-19}$ C. Electron - an elementary carrier negative charge. Proton - nucleus of the hydrogen atom $_1H^1$ - container carrier elements of positive charge (Figure 3.2). In the structure of the hydrogen atom consists of one electron and one proton. Hydrogen atom, as the atoms of other substances is neutral, that is, net plus charge of atom is equal to the net subzero charge $Z_p = Z_e$. Atomistic charge is that the elementary negative and positive charges are part of an atom in an isolated atom and they are always the same number.

All bodies in nature can electrify i.e to get to (give) an electric charge. The electrification of bodies can be done in different ways, by contact (friction), electrostatic induction by placing the body in an external electric field, etc.

Every process electrification reduced charge separation in which one of the bodies (or body parts), there is an excess of positive charges, and the another (or other body parts) - an excess of negative charges. The total number of charges of both signs contained in the bodies does not change, only the charges are redistributed between the bodies.

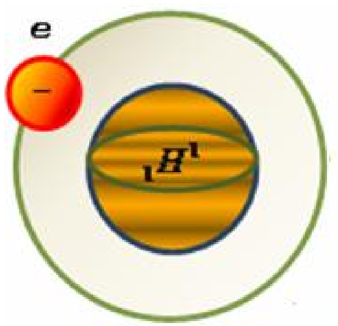


Figure 3.2

Electrically **closed** terms the system which is not exchanging charges with exterior bodies.

Charge conservation:

Algebraic sum of the electrical charges of any closed system remains constant, no matter what the process would not occur within the system

$$\sum_{i=1}^{N} q_i = const, \tag{3.1}$$

$$q_1 + q_2 + ... + q_N = q_1^` + q_2^` + ... + q_N^` = const, \tag{3.2}$$

$$q_1 + q_2 + ... + q_N = q_1^` + q_2^` + ... + q_N^` = const. \tag{3.3}$$

By the ability to pass an electrical current (ie charge transfer), all substances are divided into conductors, semiconductors and insulators.

Conductors body in which an electric charge can be moved over the entire volume of the conductor. Lead resistance is small, and the conductivity is high.

The conductors are divided into two groups:

I kind of conductors - metals - the charge carriers are electrons. Flow of electrical charge does not lead to chemical changes of matter.

Conductors II kind - solutions of acids, salts and melts - the charge carriers - electrons and ions, they transfer charges leads to chemical changes (electrolysis).

Dielectric (insulator) matter not conducting electrical current (glass, air, plastic, etc.). There are no free charges in the conductors, all charges related to the molecules of the dielectric. High resistance, conductivity is low.

Semiconductors - under certain conditions (high temperatures and electric fields) are able to conduct electricity (germanium, silicon, gallium arsenide).

A unit of electrical charge - coulomb - l electric charge passing through a cross-section at a current of 1 amp for the time of 1 *s*.

$$[q] = C, \tag{3.4}$$

$$q = I \cdot t, \tag{3.5}$$

$$[q] = [I][t], \tag{3.6}$$

$$C = 1A \cdot 1s. \tag{3.7}$$

3.1.2 Coulomb's law. The electrostatic field. Intensity of electrostatic field

Point charges are called charged bodies, the size of which can be neglected in comparison with the distance between them.

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 \varepsilon r^2}, \tag{3.8}$$

Coulomb's law: Two fixed point charges interact with the force F is directly proportional to the magnitude of the charges and inversely proportional to the square of the distance between them.

Coulomb force directed along the line joining the interacting charges, ie is central. $F< 0$ for opposite charges (charges attract); $F> 0$ for the same charges (charges repel).

Coulomb's law in vector form:

$$\overrightarrow{F_{12}} = \frac{q_1 q_2 \overrightarrow{r_{12}}}{4\pi\varepsilon_0 \varepsilon r_{12}^3}, \tag{3.9}$$

where $\overrightarrow{F_{12}}$ - the force on the 1st of the charge of the 2nd, $\overrightarrow{r_{12}}$ - radius-vector between charges 1 and 2; ε_0 - electric constant; $\varepsilon_0 = 8.85\ 10^{-12}$ F/m, F - farad - unit capacity; ε - dielectric constant of the medium, shows how many times the force between two point charges in the medium is less than the force of interaction in a vacuum, if the distance between the charges is not changed

$$\varepsilon = \frac{F_{vacuum}}{F_{medium}}, \tag{3.10}$$

ε - indicates weakening Coulomb force (and the electrostatic field) in the medium compared to vacuum $[\varepsilon] = 1$.

According to Newton's third law

$$\overrightarrow{F_{12}} = -\overrightarrow{F_{21}}\ . \tag{3.11}$$

Electric charges create an electric field around it. Field - a form of existence of matter. Field to explore, to describe its power, energy, and other properties. The field produced by stationary electric charges, called electrostatic. To investigate an electrostatic field use the test point positive charge - a charge that does not distort the investigated field (does not cause redistribution of the charges). If in the field produced by the charge q, put a test charge q_1 on it will be a force F_1, and the magnitude of this force depends on the charge placed in the given point of the field. If put into the same point charge q_2, then the Coulomb force $F_2 \sim q_2$, etc.

However, the ratio of the Coulomb force to the magnitude of the test charge is constant for a given point in space

$$\frac{F_1}{q_1} = \frac{F_2}{q_2} = ... = \frac{F_N}{q_N} = \frac{q}{4\pi\varepsilon_0 r^2}, \tag{3.12}$$

and characterizes the electric field at the point where the test charge.

This value is called the intensity $\vec{E}$ and is a power characteristic of the electrostatic field. The field intensity is a vector quantity which is numerically equal to the force acting on a unit positive point charge is placed at a given point of the field

$$\vec{E} = \frac{\vec{F}}{q}, \tag{3.13}$$

$$[E] = \frac{N}{C} = \frac{V}{m}. \tag{3.14}$$

The direction of the intensity vector coincides with the direction of the force.

We define the field strength generated by a point charge q at a distance r from it in a vacuum (Figure 3.3)

$$\vec{F} = \frac{qq_1}{4\pi\varepsilon_0\varepsilon r^2}\cdot\frac{\vec{r}}{r}, \tag{3.15}$$

$$\vec{E} = \frac{\vec{F}}{q_1} = \frac{q}{4\pi\varepsilon_0\varepsilon r^2}\cdot\frac{\vec{r}}{r}. \tag{3.16}$$

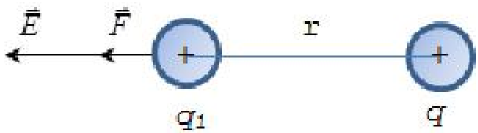


Figure 3.3

3.1.3 The principle of superposition of fields. The lines of force vector $\vec{E}$

Determine the value and direction of the field $\vec{E}$ produced by a system of fixed charges $q_1, q_2,..., q_n$. Net force $\vec{F}$, exerted by the field on the test charge q, is the vector sum of the forces $\vec{F_i}$, applied to it by each of the charges q_i (Figure 3.4)

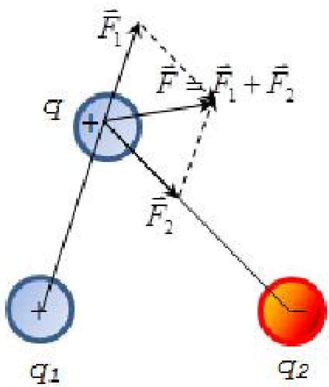


Figure 3.4

$$\vec{E} = \frac{\vec{F}}{q_1} = \frac{q}{4\pi\varepsilon_0\varepsilon r^2}\cdot\frac{\vec{r}}{r}, \tag{3.17}$$

$$\vec{F} = \sum_{i=1}^{n}\vec{F_i}. \tag{3.18}$$

Dividing by q, we obtain

$$\vec{E} = \sum_{i=1}^{n}\vec{E_i}. \tag{3.19}$$

Principle of superposition (overlay) fields:

Intensity $\vec{E}$ of resulting field created by a system of charges, equal to the geometric (vector) sum of the field intensities produced at this point each of the charges separately.

Electrostatic field can be represented very clearly with lines of Intensity or power lines of vector $\vec{E}$.

Field line intensity vector $\vec{E}$ is a curve whose tangent at every point in space coincides with the direction of the vector $\vec{E}$.

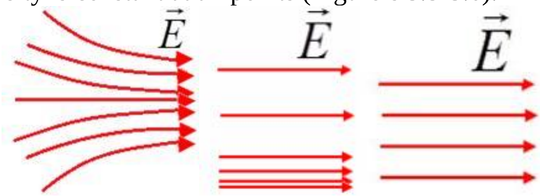
The principle of construction of power lines $\vec{E}$:

1. The lines of force vector $\vec{E}$ begin on positive charges and terminate at negative (ie directed from "+" to "-").

2. The lines of force vector $\vec{E}$ approach the surface charges at right angles.

3. For a quantitative description of the vector E field lines carried out with a certain density. The number of lines of intensity running through the unit area perpendicular to the lines of intensity must be equal to the modulus of a vector $\vec{E}$.

Homogeneous is a field that has a vector $\vec{E}$ in any point in space is constant in magnitude and direction, ie vector $\vec{E}$ parallel to the field lines and their density is constant at all points (Figure`s 3.5-3.6).



Inhomogeneous field **Homogeneous** field

Figure 3.5

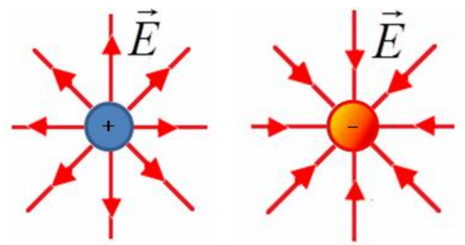


Figure 3.6

Painting lines isolated point charges.

3.1.4 Dipole. Dipole moment. Dipole field

Electric dipole is a system of two point-of opposite charges (+ and -) at a distance l.

Vector along the dipole axis (the line passing through the two charges) from the negative to the positive charge and is equal to the distance between them, called the ARM dipole $\vec{l}$. Vector $\overrightarrow{p_e} = q\vec{l}$.

coinciding with the direction of the dipole arm and the product of the charge q on the arm $\vec{l}$ is called an electric **dipole moment** $\overrightarrow{p_e}$ dipole moments (Figure 3.7).

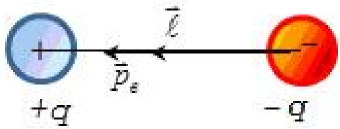


Figure 3.7

By the principle of superposition the field intensity E of dipole at any point

$$\overrightarrow{E_e} = \overrightarrow{E_+} + \overrightarrow{E_-}, \tag{3.20}$$

$\overrightarrow{E_+}$ - field generated by a positive charge, $\overrightarrow{E_-}$ - the field of negative charge. Field intensity on the extension of the axis of the dipole.

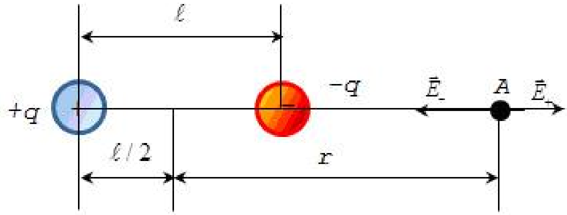


Figure 3.8

$$\overrightarrow{E_A} = \overrightarrow{E_+} + \overrightarrow{E_-}$$

$$E_A = E_+ - E_- = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\left(r-\frac{l}{2}\right)^2} - \frac{q}{\left(r+\frac{l}{2}\right)^2}\right) = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2ql}{r^3} \tag{3.21}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3},$$

because $\frac{l}{2} << r$ (Figure 3.8).

1. The field intensity at a perpendicular to the axis of the reconstructed from his middle

$$\overrightarrow{E_B} = \overrightarrow{E_+} + \overrightarrow{E_-}, \tag{3.22}$$

$$E_+ = E_- = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}. \tag{3.23}$$

Triangle $A'B'C$ 'similar to the triangle ABC, as equilateral and three angles are equal, so

$$\frac{E_B}{E_+} = \frac{l}{\sqrt{\frac{l^2}{2} + r^2}} \approx \frac{l}{r} \Rightarrow E_B = E_+ \frac{l}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2} \cdot \frac{l}{r} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{p}{r^3}.$$

The picture of the dipole field lines (Figure 3.9):

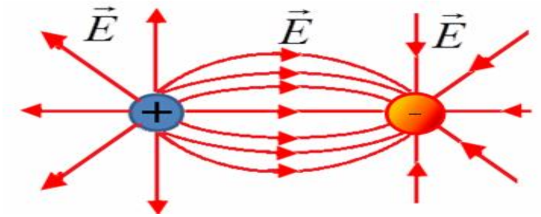


Figure 3.9

3.1.5 Flux of intensity

We define the flux of vector $\vec{E}$ through any surface dS, $\vec{n}$ - normal to the surface, α - the angle between the normal and the force line of vector $\vec{E}$. You can enter a area vector $d\vec{S} = dS\vec{n}$. **Flux of vector** $\vec{E}$ is called a scalar Φ_E equal to the scalar product of the intensity vector $\vec{E}$ on the area vector $d\vec{S}$.

For a homogeneous field

$$\Phi_E = \vec{E}\vec{S} = ES\cos\alpha. \tag{3.24}$$

For inhomogeneous field

$$d\Phi_E = \vec{E}d\vec{S} = \vec{E}dS\vec{n} = EdS\cos\alpha = E_n dS = EdS_E, \tag{3.25}$$

where E_n - the projection $\vec{E}$ on $\vec{n}$, dS_E - the projection $d\vec{S}$ on the $\vec{E}$.

$$[\Phi] = [E]\cdot[S] = \frac{V}{m}\cdot m^2 = V\cdot m. \tag{3.26}$$

In the case of a curved surface S it must be broken down into elementary surface dS, to calculate the flow $d\Phi_E$ through the unit area, and the total flux is equal to the amount or limit of integrals of elementary streams (Figure 3.10)

$$d\Phi_E = \oint_S \vec{E}d\vec{S} = \oint_S E_n dS, \tag{3.27}$$

where $\oint_S$ - the integral over a closed surface S (eg, sphere, cylinder, cube, etc.)

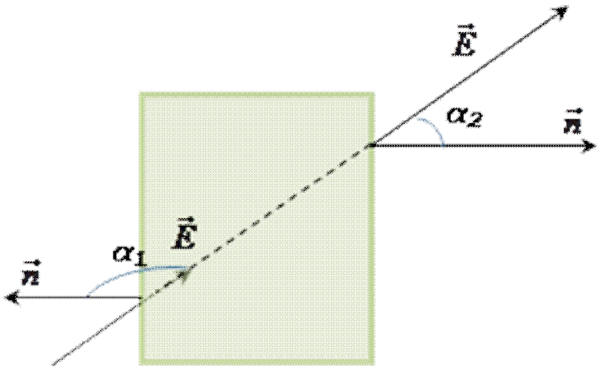


Figure 3.10

Flux Φ_E of $\vec{E}$ an algebraic value: not only depends on the configuration of the field $\vec{E}$, but the destinations of your choice $\vec{n}$. For closed surfaces, the positive direction of the normal is taken outside the normal, ie, normal outward field covered surface.

$$\Phi_{1E} = ES\cos\alpha_1 = ES\cos\left(180^0 - \alpha_2\right) = -ES\cos\alpha_2, \tag{3.28}$$

$$\Phi_{2E} = ES\cos\alpha_2, \tag{3.29}$$

$$\Phi_{1E} + \Phi_{2E} = 0. \tag{3.30}$$

For a uniform field flow through a closed surface is equal to zero. In the case of inhomogeneous field

$$\Phi_E \neq 0. \tag{3.31}$$

Gauss's theorem and its application to the calculation of the electrostatic field

I. Consider the electrostatic field generated by a single positive charge. We include it in the sphere of radius *R*. We define the flow of intensity $\vec{E}$ through a spherical surface of radius *R*.

We divide the surface of the sphere *S* at the elementary area *dS* (Figure 3.11). The normal to the area *dS* is directed through the sphere of radius and coincides with the direction $\vec{E}$: $\vec{n}$ parallel $\vec{E}$ so

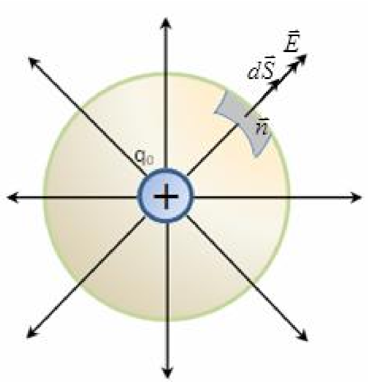


Figure 3.11

$$\overrightarrow{E_n} = \vec{E} \tag{3.32}$$

$$d\vec{S} = dS\vec{n};\ \cos\alpha = \cos 0^0 = 1 \Rightarrow \overrightarrow{E_n} d\vec{S} = EdS \,. \tag{3.33}$$

Then the flow of the vector $\vec{E}$ through the surface *S* is equal to the sum of flows through an area element *dS* and vanish *dS* we can write

$$\Phi_1 = \oint_S \vec{E} d\vec{S} = \oint_S E_n dS = \int_0^{4\pi R^2} EdS = 4\pi R^2 E. \tag{3.34}$$

Considering that the field of a point charge is

$$E = \frac{q}{4\pi\varepsilon_0 R^2}, \tag{3.35}$$

Get

$$\Phi_E = \frac{q}{\varepsilon_0}. \tag{3.36}$$

This result can be generalized to any surface. Given the principle of superposition can apply our results to any number of charge inside the surface.

Gauss' theorem: Flux of intensity $\vec{E}$ through an arbitrary closed surface is equal to the algebraic sum of the charges enclosed by this surface, divided by ε_0 (ε_0 -dielectric constant)

$$\oint_S \vec{E} d\vec{S} = \frac{\sum_{i=1}^{N} q_i}{\varepsilon_0}. \tag{3.37}$$

II. Application of the Gauss theorem.

1. The field intensity created by an infinitely extended plane uniformly charged with surface charge density σ.

Surface charge density indicates which charge per unit area $\sigma = \frac{q}{S}$.

Lines of intensity $\vec{E}$ are perpendicular to the surface and sent from it on both sides. Construct a cylinder with a base *S*, whose elements of cylinde are parallel lines to intensity $\vec{E}$.

Since elements of cylinde is parallel to the lines of $\vec{E}$ $\cos\alpha = \cos 0^0 = 1$, the flow through the base of *S* (Figure 3.12) is

$$\Phi_{E_{ocn}} = \oint_S \vec{E} d\vec{S} = \frac{\sum_{i=1}^{N} q_i}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0}. \tag{3.38}$$

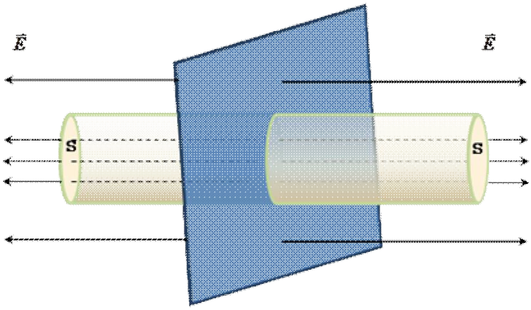


Figure 3.12

The flow through the lateral surface of the cylinder is equal to zero, because $\vec{E}$ perpendicular $S\cos\alpha = \cos 90^0 = 0$, so

$\Phi_{E_{po\ln}} = 2\Phi_{E_{ocn}}$;

$$\oint_S \vec{E}d\vec{S} = 2ES = \frac{\sigma S}{\varepsilon_0};\Rightarrow E = \frac{\sigma}{2\varepsilon_0}. \qquad (3.39)$$

2. The field intensity created by two parallel infinitely extended plates with surface charge density + σ and –σ (Figure 3.13). We find the field *E*, using the principle superposition of the fields. In the region between the planes

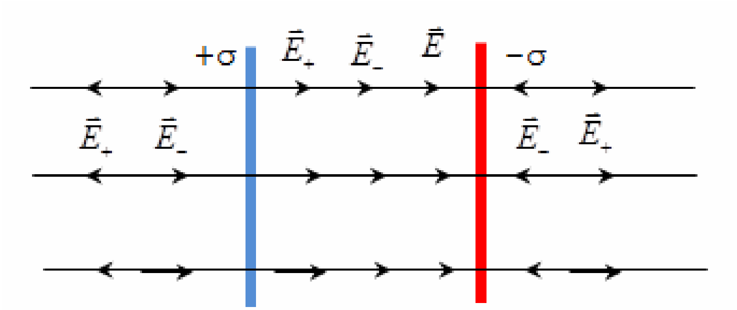


Figure 3.13

$$E = E_1 + E_2 = 2E_1 = 2\frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}, \qquad (3.40)$$

$$E = \frac{\sigma}{\varepsilon_0}. \qquad (3.41)$$

Left and right of the plane fields deducted as lines of intensity are directed toward each other $E = E_1 - E_2$.

3. The field intensity created by an infinitely extended thread with linear charge density τ (Figure 3.14). Linear charge density indicates which charge per unit length of the conductor

$$\tau = \frac{q}{l} = \frac{dq}{dl}. \qquad (3.42)$$

Required to determine the field intensity at a distance r from the filament. For this we construct a cylinder of radius r and height h, the axis of which passes the thread.

$$\tau = \frac{q}{l} = \frac{q}{h}, \quad l = h \cdot \qquad (3.43)$$

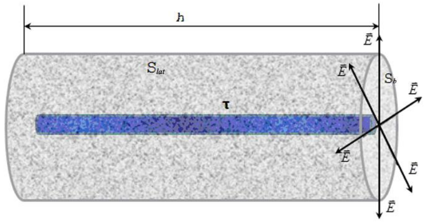


Figure 3.14

The flow through the base of the cylinder is considered zero, as perpendicular vector $\vec{E}$ and $\vec{S_b}$, therefore, the flow will be determined only flow through the lateral surface of the cylinder

$$\vec{S_b} = 2\pi r h, \qquad (3.44)$$

$$\oint_S \vec{E}d\vec{S} = \int_{S_{lat}} \vec{E}d\vec{S} + \int_{S_b} \vec{E}d\vec{S} = \int_{S_{lat}} \vec{E}d\vec{S} = ES_b = E \cdot 2\pi r h = \frac{\sum_{i=1}^{N} q_i}{\varepsilon_0} = \frac{\tau h}{\varepsilon_0}, \qquad (3.45)$$

$$E = \frac{\tau}{2\pi\varepsilon_0 r}. \qquad (3.46)$$

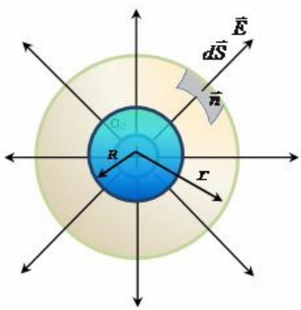


Figure 3.15

4. The field intensity generated by a spherical surface with a surface charge density σ. On a sphere of radius R is distributed charge q. The surface charge density

$$\sigma = \frac{q}{S} = \frac{q}{4\pi R^2}. \qquad (3.47)$$

Lines of intensity are directed radially, moving away from the surface of the sphere at right angles. The surrounding area of this sphere of radius r and determine the intensity $\vec{E}$ the flow through spherical surface of radius r.

When $r > R$ the entire charge q is inside the sphere r. Then, by the Gauss theorem

$$\oint_S \vec{E}d\vec{S} = \int_0^{4\pi r^2} E_n dS = E\cdot 4\pi r^2, \text{ because } E_n = E, \tag{3.48}$$

$$\oint_S \vec{E}d\vec{S} = \frac{\sum_{i=1}^{N} q_i}{\varepsilon_0} = \frac{\sigma\cdot 4\pi R^2}{\varepsilon_0}, \tag{3.49}$$

$$E\cdot 4\pi r^2 = \frac{\sigma\cdot 4\pi R^2}{\varepsilon_0} \Rightarrow E = \frac{\sigma\cdot R^2}{\varepsilon_0 r^2},\ r > R, \tag{3.50}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2},\ r > R. \tag{3.51}$$

When $r < R$, inside surface of radius r, and there are no charges so $E = 0$. Based on this screening - protection from external electric fields.

5. The field intensity of a body charged sphere with a volume charge density ρ (Figure 3.16). The bulk density of the charge indicates which charge per unit volume

$$\rho = \frac{q}{V}, \tag{3.52}$$

$$q = \rho V = \rho\frac{4}{3}\pi R^3. \tag{3.53}$$

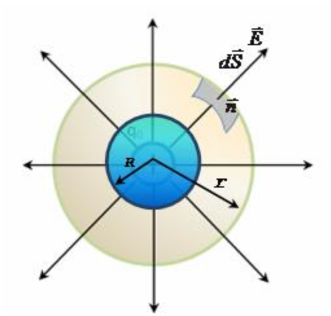


Figure 3.16

a) For $r > R$, under item 4 to determine the

$$E = \frac{q}{4\pi\varepsilon_0 r^2},\ r > R. \tag{3.54}$$

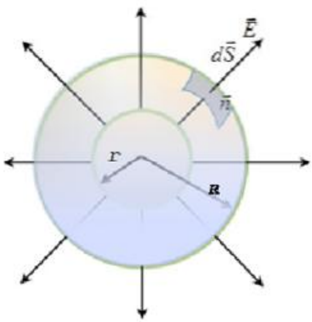


Figure 3.17

$$\oint_S \vec{E}d\vec{S} = \int_0^{4\pi r^2} E_n dS = E\cdot 4\pi r^2 = \frac{\rho\frac{4}{3}\pi R^3}{\varepsilon_0}, \Rightarrow$$

$$E = \frac{\rho R^3}{3\varepsilon_0 r^2},\ r > R. \tag{3.55}$$

b) For $r < R$

$$q` = \rho V` = \rho\frac{4}{3}\pi r`^3, \tag{3.56}$$

$$\rho = \frac{q}{\frac{4}{3}\pi R^3}, \tag{3.57}$$

$$S` = 4\pi r`^2, \tag{3.58}$$

$$4\pi r`^2 E = \frac{q`}{\varepsilon_0} = \frac{\rho \frac{4}{3}\pi r`^3}{\varepsilon_0}, \tag{3.59}$$

$$E = \frac{r`}{\varepsilon_0}\rho = \frac{r`}{\varepsilon_0}\frac{q}{\frac{4}{3}\pi R^3}, \quad r` < R. \tag{3.60}$$

3.1.6 Work of the electrostatic field intensity at moving charge. Potential nature of the field forces. Circulation of intensity

Consider the electrostatic field produced by the charge q. Let it move a test charge q_0. At any point of the field on the charge q_0 a force

$$\vec{F} = \left|\vec{F}\right| \cdot \vec{e_r}, \tag{3.61}$$

where $\left|\vec{F}\right|$ - force module, $\vec{e_r}$ - the unit vector of the radius vector $\vec{r}$, which determines the position of the charge q_0 relative to the charge q (Figure 3.18).

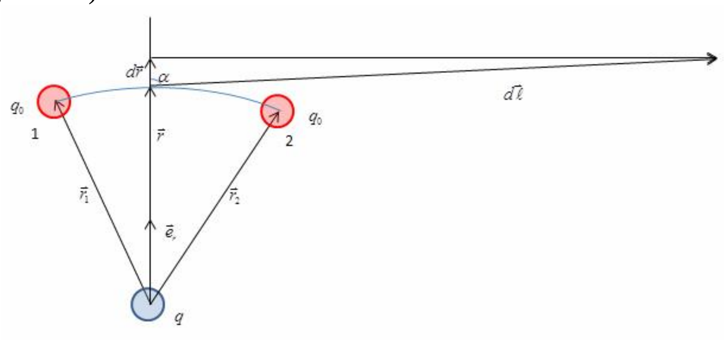


Figure 3.18

Since the force varies from point to point, the work force of the electrostatic field can be written as a variable work force:

$$A_{12} = \int_1^2 F\vec{e_r}d\vec{l}, \tag{3.62}$$

$$dr = dl\cos\alpha; \quad \left|\overrightarrow{e_r}\right| = 1; \quad \Rightarrow \tag{3.63}$$

$$\overrightarrow{e_r} \cdot d\vec{l} = 1 \cdot dl \cdot \cos\alpha = dr \Rightarrow \tag{3.64}$$

$$A_{12} = \int_1^2 F dr = \int_1^2 \frac{qq_0}{4\pi\varepsilon_0 r^2} dr = \frac{qq_0}{4\pi\varepsilon_0}\left(-\frac{1}{r}\right)\Big|_{r_1}^{r_2} = \frac{qq_0}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \tag{3.65}$$

Because the regarded charge transport from point 1 to point 2 along an arbitrary trajectory, it can be concluded that the work on the movement of a point charge in an electric field does not depend on the shape of the path but is determined only the initial and final positions of the charge. This indicates that the electrostatic field is potential, and the strength of Coulomb - conservative force. Work on moving charge in a field along a closed path is always zero.

$$\vec{F} = q\vec{E} \tag{3.66}$$

$$dA = \vec{F} \cdot d\vec{l} = q_0 \vec{E} \cdot d\vec{l}\,, \tag{3.67}$$

$$A = \int_{r_1}^{r_2} q_0 \vec{E} \cdot d\vec{l} = \int_{r_1}^{r_2} q_0 \overrightarrow{E_l} \cdot dl \cdot \tag{3.68}$$

E_l - projection, $\vec{E}$ - on the direction of the contour ℓ. We take into account that the work on a closed path is zero

$$\oint_l q_0 E_l \cdot dl = 0, \tag{3.69}$$

$$\oint_l E_l \cdot dl = 0. \tag{3.70}$$

$\oint_l E_l \cdot dl$ - circulation of intensity. Circulation of the electrostatic field, taken by an arbitrary closed path is always zero.

3.1.7 Potential. The link between the intensity and potential. Potencial Gradient. Equipotential surfaces

Since the electrostatic field is a potential job of moving the charge in such a field can be represented as a difference of the potential energy of a charge in the start and end points of the path. (The work is equal to the reduction of the potential energy, or the change in the potential energy take with minus sign.)

$$A = -\Delta W_{pot} = W_1 - W_2 = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r_1} - \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r_2} \Rightarrow \tag{3.71}$$

$$\Delta W_{pot} = \frac{qq_0}{4\pi\varepsilon_0}\frac{1}{r} + const. \tag{3.72}$$

Constant determined from the condition that the removal of the charge q_0 to infinity the potential energy must be equal to zero.

$$W_{pot} = \frac{q_0 q}{4\pi\varepsilon_0}\frac{1}{r}. \tag{3.73}$$

Various test charges q_{0i}, placed at a given point of the field will have at this point various potential energies:

$$W_{pot}^1 = \frac{q_{01}q}{4\pi\varepsilon_0}\frac{1}{r},\ W_{pot}^2 = \frac{q_{02}q}{4\pi\varepsilon_0}\frac{1}{r}\ \cdots W_{pot}^n = \frac{q_{0n}q}{4\pi\varepsilon_0}\frac{1}{r}. \tag{3.74}$$

The ratio W_{pot} i to the value of the test charge q_{0i}, placed at a given point of the field is constant for a given point of the field for all test charges. This ratio is called the potential.

Potential - energy characteristic of the electric field. **Potential** numerically equal to the potential energy, which has at a given point of the field unit positive charge.

$$\varphi = \frac{W_{pot}}{q_o} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r} \Rightarrow W_{pot} = \varphi \cdot q_0. \tag{3.75}$$

Работу по перемещению заряда можно представить в виде

$$A = -\Delta W_{pot} = W_1 - W_2 = q(\varphi_1 - \varphi_2), \tag{3.76}$$

$$A = q(\varphi_1 - \varphi_2)\ . \tag{3.77}$$

Potential is measured in volts

$$[\varphi] = \frac{J}{C} = V\ . \tag{3.78}$$

Equipotential surface is a surface of equal potential (φ = const). Work to move a charge along an equipotential surface is zero.

Relationship between the intensity $\vec{E}$ and the potential φ can be found, based on the fact that the job of moving a charge q at the elementary segment $d\ell$ can be written as

$$dA = q \cdot E_l \cdot dl\ . \tag{3.79}$$

On the other hand $dA = q(\varphi_1 - \varphi_2) = -q\dfrac{d\varphi}{dl}dl \Rightarrow$

$$E_l = -\frac{d\varphi}{dl}, \tag{3.80}$$

$$E_x = -\frac{\partial\varphi}{\partial x},\ E_y = -\frac{\partial\varphi}{\partial y},\ E_z = -\frac{\partial\varphi}{\partial z}. \tag{3.81}$$

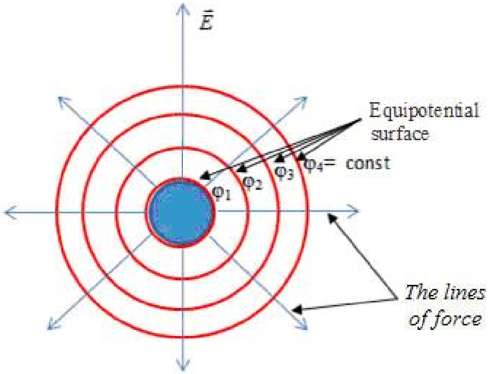


Figure 3.19

$$\vec{E} = \vec{i}E_x + \vec{j}E_y + \vec{k}E_z = -\left(\vec{i}\frac{\partial\varphi}{\partial x} + \vec{j}\frac{\partial\varphi}{\partial y} + \vec{k}\frac{\partial\varphi}{\partial z}\right) = -\frac{d\varphi}{dr},\ \frac{d\varphi}{dr}$$ - gradient of potential.

$$\vec{E} = -grad\varphi. \qquad (3.82)$$

Field intensity is equal to the potential gradient, to the negative (Figure 3.20).

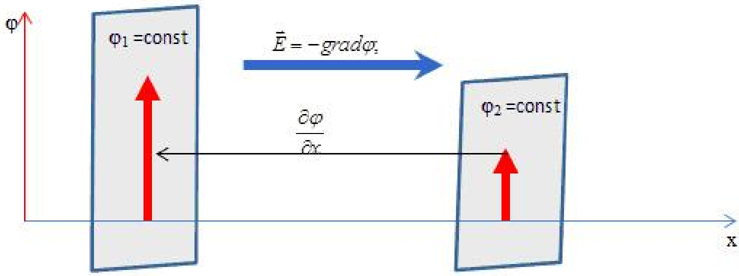


Figure 3.20

Potential gradient shows how change the capacity per unit length. Gradient perpendicular to the function and is directed towards the increase of the function. Therefore, the vector intensity perpendicular to the equipotential surface and is directed towards the decrease of the potential.

Consider the field created by a system N point charges q_1, q_2, … q_N. Distance from the charge to a given point of the field are r_1,r_2, … r_N. The work done by this field on the charge q_0, will be equal to the algebraic sum of the work force each charge separately.

$$A_{12} = \sum_{i=1}^{N} A_i , \tag{3.83}$$

where $A_i = \dfrac{q_i q_0}{4\pi\varepsilon_0}\left(\dfrac{1}{r_{i1}} - \dfrac{1}{r_{i2}}\right)$,

$$A_{12} = W_{pot1} - W_{pot_2} = \frac{1}{4\pi\varepsilon_0}\left(\sum_{i=1}^{N}\frac{q_i q_0}{r_{i1}} - \sum_{i=1}^{N}\frac{q_i q_0}{r_{i2}}\right), \tag{3.84}$$

$$\Rightarrow W_{pot} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i q_0}{r_i}, \qquad \Rightarrow \varphi = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{N}\frac{q_i}{r_i}, \tag{3.85}$$

$$\varphi = \sum_{i=1}^{N}\varphi_i . \tag{3.86}$$

The potential field generated by a system of charges is defined as the algebraic sum of the potentials produced at the same point each charge separately.

difference potentials of the plane, the two planes, spheres, ball, cylinder

Using the relation between φ and $\vec{E}$ odefine the potential difference between two arbitrary points (Figure 3.21).

$$\vec{E} = -\frac{d\varphi}{dr} = -grad\varphi, \ \Rightarrow \varphi = \int_{x_1}^{x_2} E_x dx . \tag{3.87}$$

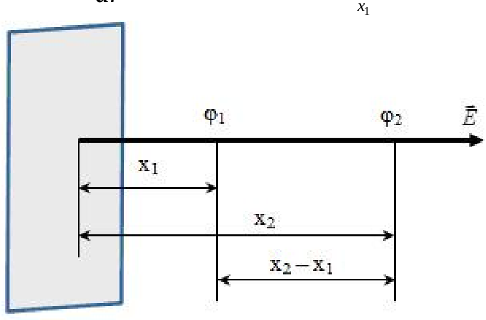


Figure 3.21

1. The potential difference of the field of a uniformly charged infinite plane with surface charge density σ .

$$\varphi_1 - \varphi_2 = \int_{x_1}^{x_2} Edx = \int_{x_1}^{x_2}\frac{\sigma}{2\varepsilon_0}dx = \frac{\sigma}{2\varepsilon_0}(x_2 - x_1). \tag{3.88}$$

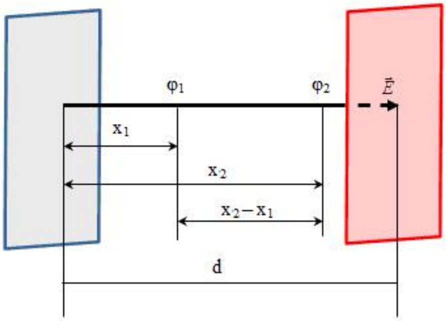


Figure 3.22

2. The potential difference of the field of two infinite parallel planes with an oppositely charged surface charge density σ (Figure 3.22).

$$\varphi_1 - \varphi_2 = \int_{x_1}^{x_2} E dx = \int_{x_1}^{x_2} \frac{\sigma}{\varepsilon_0} dx = \frac{\sigma}{\varepsilon_0}(x_2 - x_1). \tag{3.89}$$

If $x_1 = 0$; $x_2 = d$, then $\varphi_1 - \varphi_2 = \frac{\sigma}{\varepsilon_0} d$ or $U = Ed$.

3. The potential difference of the field of a uniformly charged spherical surface of radius R (Figure 3.23).

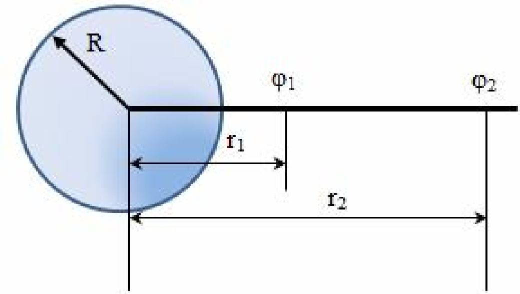


Figure 3.23

$$\varphi_1 - \varphi_2 = \int_{r_1}^{r_2} E dx = \int_{r_1}^{r_2} \frac{q}{4\pi\varepsilon_0 r^2} dr = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right). \tag{3.90}$$

If $r_1 = r$, $r_2 \rightarrow \infty$, the potential outside the spheres

$$\varphi_1 - \varphi_2 = \frac{q}{4\pi\varepsilon_0 r}. \tag{3.91}$$

Inside a spherical surface potential everywhere and is equal

$$\varphi = \frac{q}{4\pi\varepsilon_0 R}. \quad (3.92)$$

4. The potential difference of the field volume of a charged sphere of radius R and total charge q (Figure 3.24).

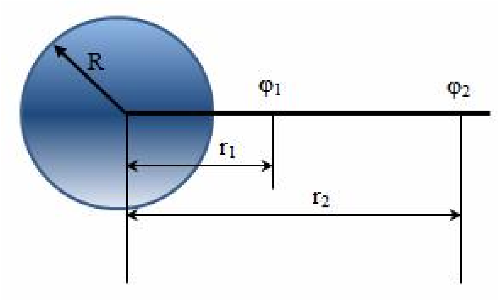


Figure 3.24

Outside the ball $\varphi_1 - \varphi_2 = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$. $r_1, r_2 > R$,

Inside the ball

$$\varphi_1 - \varphi_2 = \int_{r_1}^{r_2} Edr = \frac{q}{4\pi\varepsilon_0 R^3}\int_{r_1}^{r_2} rdr = \frac{q}{4\pi\varepsilon_0 R^3}\left(r_2^2 - r_1^2\right). \quad (3.93)$$

5. The potential difference of the field of a uniformly charged cylinder (or infinitely long thread). $r>R$:

$$\varphi_1 - \varphi_2 = \int_{r_1}^{r_2} Edr = \int_{r_1}^{r_2} \frac{\tau}{2\pi\varepsilon_0 r} dr = \frac{\tau}{2\pi\varepsilon_0} \ln\frac{r_2}{r_1}. \quad (3.94)$$

3.1.8 Conductors in electrostatic field. Of the charge distribution in the conductor. The connection between the field intensity at the surface of the conductor and the surface charge density

Free charges in the conductor can move under the influence of an arbitrarily small force. Therefore, for the balance of the charges in the conductor must meet the following conditions:

1. Field intensity inside the conductor must be zero $\vec{E} = 0$, since (Figure 3.25).

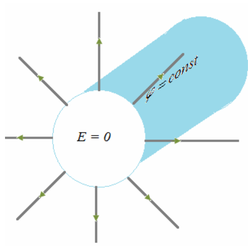


Figure 3.25

$$\varphi = \int Edx = const, \quad (3.95)$$

2. i.e potential within the conductor must be constant. Field intensity on surface of the conductor must be perpendicular to the surface

$$\vec{E} = \overrightarrow{E_n}. \quad (3.96)$$

Consequently, the surface charge of the conductor at equilibrium is an equipotential.

At equilibrium, the charges in any place inside the conductor can not be excess charges - they are distributed over the surface of a conductor with a density σ.

Consider a closed surface in the form of a cylinder whose generators are perpendicular to the surface of the conductor (Figure 3.26). On the surface of the conductor are free charges with surface density σ.

Because inside a conductor there are no charges, the flow $\vec{D}$ through the

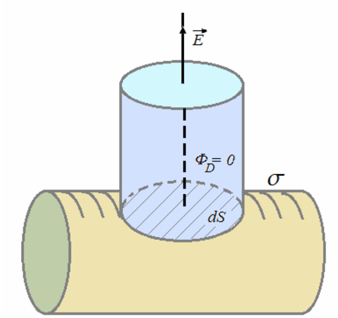


Figure 3.26

surface of the cylinder inside the conductor is zero. The flow through the top of the cylinder outside the a conductor on the Gauss theorem is equal

$$\oint_S \vec{D} d\vec{S} = q, \quad (3.97)$$

$$q = \sigma S = DS \Rightarrow D = \sigma. \quad (3.98)$$

I.e electric displacement vector equal to the surface density of free charges of a conductor or

$$E = \frac{\sigma}{\varepsilon_0 \varepsilon}. \quad (3.99)$$

II. In making an uncharged conductor to an external electrostatic field his free charges will move: positive - on the field, negative - against the field (Figure 3.27). Then on one side of the conductor will accumulate positive and the other negative charges. These charges are said to be induced. The redistribution of the charges will be as long as the intensity in the conductor becomes zero, and the line intensity outside of the conductor will be the perpendicular to the surface. Induced charges appear on the conductor due to the displacement, ie are surface density charges and shifted as $D = \sigma$, is therefore $\vec{D}$ called the electric displacement vector.

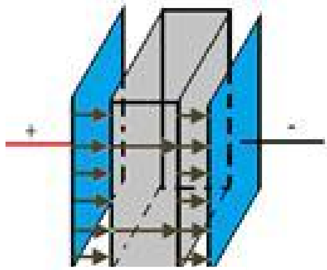


Figure 3.27

3.1.9 Capacitance conductors. Capacitors

I. Secluded called conductors, far from other conductors bodies charges. The potential of such a conductor is directly proportional to the charge on it

$$\varphi = \frac{q}{4\pi\varepsilon_0\varepsilon r} = k \cdot q. \qquad (3.100)$$

From experience, it follows that different conductors, being equally charged $q_1 = q_2$ takes various potentials $\varphi_1 \neq \varphi_2$ due to the different shapes, sizes, and surrounding environment conductor (ε). Therefore, for an isolated conductor have the formula

$$q = C \cdot \varphi, \qquad (3.101)$$

where $C = \frac{q}{\varphi}$ - **capacity of secluded conductor**. Capacity of the secluded conductor is equal to a charge q, give the conductor that changes its potential by 1 volt.

In the SI system capacity is measured in farads $[C] = F$.

Capacity of the ball

$$C = \frac{q}{\varphi} = 4\pi\varepsilon_0\varepsilon R. \qquad (3.102)$$

II. Capacity solitary conductors is very small. For practical purposes it is necessary to create such a device, which allows to store large charges at small sizes and capacities. Capacitor- device for charge storage and electrical power.

The simplest capacitor consists of two conductors separated by a gap of air or dielectric (air - is also a dielectric). The conductors are called the plates of the capacitor, and their location in relation to each other are selected such that the electric field is concentrated in the gap between them. Under the capacity of the capacitor understood the physical value of *C* equal to the ratio of the charge *q*, accumulated on the plates, to the difference of potentials $\varphi_1 - \varphi_2$ between the plates.

$$C = \frac{q}{\varphi_1 - \varphi_2}. \qquad (3.103)$$

Calculate the capacity of parallel-plate capacitor with plates of area *S*, the surface charge density σ, the dielectric constant ε of the dielectric between the plates, the distance between the plates is *d* (Figure 3.28). The field intensity is

$$E = \frac{\sigma}{\varepsilon_0\varepsilon}. \qquad (3.104)$$

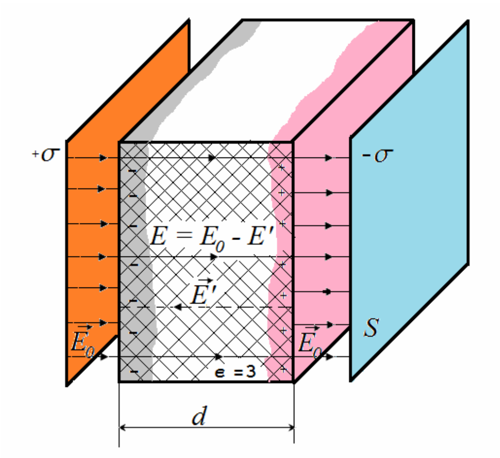


Figure 3.28

Using the relation $\Delta\varphi$ and E, we find

$$\varphi_1 - \varphi_2 = Ed = \frac{\sigma d}{\varepsilon_0 \varepsilon} = \frac{qd}{S\varepsilon_0 \varepsilon} \Rightarrow \tag{3.105}$$

$$\frac{q}{\varphi_1 \varphi_2} = \frac{\varepsilon_0 \varepsilon S}{d} = C\text{'} \tag{3.106}$$

$$C = \frac{\varepsilon_0 \varepsilon S}{d}. \tag{3.107}$$

$C = \dfrac{\varepsilon_0 \varepsilon S}{d}$ - capacity of plate capacitor. For a cylindrical capacitor:

$$C = \frac{2\pi\varepsilon_0 \varepsilon L}{\ln \dfrac{R_2}{R_1}}. \tag{3.108}$$

For a spherical capacitor:

$$C = 4\pi\varepsilon_0 \varepsilon \frac{R_2 R_1}{R_2 - R_1}. \tag{3.109}$$

Because for some values of the voltage in the dielectric breakdown occurs (electrical discharge through the dielectric layer), then there is a breakdown voltage capacitors. Breakdown voltage depends on the shape of facings, dielectric properties and its thickness.

III. Capacitance in parallel and series connection of capacitors parallel (Figure 3.29)

$$U_1 = U_2 = U_3 = U. \tag{3.110}$$

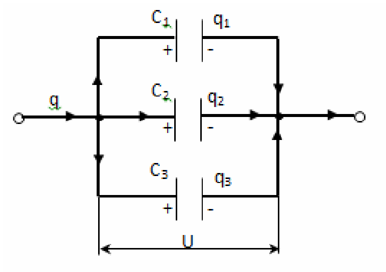


Figure 3.29

According to the law of conservation of charge $q = q_1 + q_2 + q_3$.

$$q = C \cdot U, \;\; q_1 = CU_1, \;\; q_2 = CU_2, \;\; q_3 = CU_3, \tag{3.111}$$

$$CU = C_1U + C_2U + C_3U, \tag{3.112}$$

$$C = C_1 + C_2 + C_3, \tag{3.113}$$

$$C = \sum_{i=1}^{N} C_i \cdot \tag{3.114}$$

c) serial connection (Figure 3.30).

$$U = U_1 + U_2 + U_3. \tag{3.115}$$

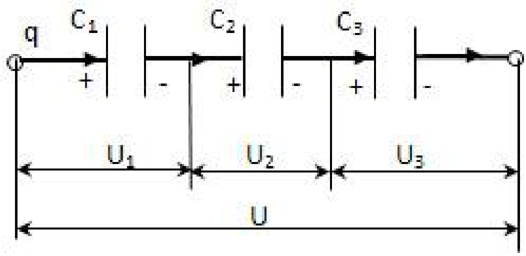


Figure 3.30

According to the law of conservation of charge $q = q_1 = q_2 = \ldots$

$$U = \frac{q}{C}, \Rightarrow U_1 = \frac{q_1}{C_1}, U_2 = \frac{q_2}{C_2}, U_3 = \frac{q_3}{C_3}, \tag{3.116}$$

$$\frac{q}{C} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}, \tag{3.117}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}, \tag{3.118}$$

$$\frac{1}{C} = \sum_{i=1}^{N} \frac{1}{C}. \tag{3.119}$$

3.1.10 The energy of the electrostatic field

1. Energy of the system of fixed point charges

The electrostatic field is potential. The forces acting between charges - conservative forces. System of fixed point charges should have potential energy (Figure 3.31). We find the potential energy of two fixed point charges q_1 and q_2, separated by a distance r from each other.

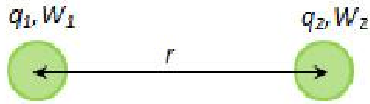


Figure 3.31

The potential energy of the charge q_2 in the field created charge q_1, equal to

$$W_2 = q_2\varphi_1 = q_2 \frac{q_1}{4\pi\varepsilon_0 \varepsilon r}. \qquad (3.120)$$

Similarly, the potential energy of the charge q_1 in the field created by a charge q_2, equal to

$$W_1 = q_1\varphi_2 = q_1 \frac{q_2}{4\pi\varepsilon_0 \varepsilon r}. \qquad (3.121)$$

It is seen that $W_1 = W_2$, then identify potential energy of the system of charges q_1 and q_2 in W, we can write

$$W = \frac{1}{2}\sum_{i=1}^{N} q_i\varphi_i, \qquad (3.122)$$

where φ_i - potential generated at the point where the charge q_i, all charges except the i-th.

2. The energy of secluded charged conductor

The energy of the electric field of a charged secluded conductor can be determined by considering the total work done on the movement of small amounts of charge dq from infinity to this conductor.

If the conductor has a charge q, capacitance C and potential φ, is to transfer the charge dq from infinity to conductor the work must be expended

$$dA = \varphi \cdot dq = \varphi \cdot C \cdot d\varphi\,. \qquad (3.123)$$

To charge conductor from ground potential to a potential φ must do the work

$$A = \int dA = \int_0^{\varphi} \varphi \cdot C \cdot d\varphi = \frac{C\varphi^2}{2}. \qquad (3.124)$$

The potential energy equal to the work that needs to be performed in order to charge the conductor

$$W_{pot} = A = \int dA = \int_0^{\varphi} \varphi \cdot C \cdot d\varphi = \frac{C\varphi^2}{2} = \frac{q\varphi}{2} = \frac{q^2}{2C}. \qquad (3.125)$$

3. The energy of a charged capacitor

We express the energy of the capacitor through the values characterizing the capacitor

$$C = \frac{\varepsilon_0 \varepsilon S}{d}, \tag{3.126}$$

$$U = E \cdot d, \tag{3.127}$$

$$V = S \cdot d, \tag{3.128}$$

$$W = \frac{CU^2}{2} = \frac{U^2}{2} \cdot \frac{\varepsilon_0 \varepsilon S}{d} = \frac{U^2 \varepsilon_0 \varepsilon S \cdot d}{2 \cdot d^2} = \frac{E^2 \varepsilon_0 \varepsilon}{2} \cdot V, \tag{3.129}$$

because field is uniform inside the capacitor, you can enter the volume energy density (bulk density - the energy per unit volume)

$$\varpi = \frac{W}{V}, \tag{3.130}$$

$$\varpi = \frac{\varepsilon_0 \varepsilon E^2}{2} = \frac{ED}{2} = \frac{D^2}{2\varepsilon_0 \varepsilon}. \tag{3.131}$$

3.2 LAWS of DIRECT CURRENT

3.2.1 The electric current. The power and current density. EMF and voltage

I. Any ordered (directed) movement of electrical charges called **electric current**. When an external electric field E in the conductor starts moving charges, i.e generates an electric current. With positive charges move across the field, and negative - against the field. Take over the direction of current direction of movement of the positive charges. For the origin and the existence of an electric current requires two conditions:

1) the presence of free charge carriers (ie the substance must be conductors or semiconductors at high temperatures),

2) The presence of an external electric field.

For a quantitative description of the electric current is introduced - current intensity - scalar physical quantity equal amount of electrical charge transferable per unit time through a cross-section S.

$$I = \frac{q}{t} \tag{3.132}$$

- for direct current, and

$$I = \frac{dq}{dt} \tag{3.133}$$

- for alternating-current.

Current, which intensity and direction do not change with time, is called permanent.

The current density $\vec{j}$ - vector physical quantity that is numerically equal to the force of current flowing through a unit area perpendicular to the current.

$$\vec{j} = \frac{I}{S}\vec{n} \qquad (3.134)$$

- for direct current, and

$$\vec{j} = \frac{dI}{dS}\vec{n} \qquad (3.135)$$

- for alternating-current.

II. To a portion of the conductor under consideration is a current *I*, is necessary to maintain a constant potential difference between these points of the conductor.

III. In order to maintain a constant potential difference across the ends of the conductor must be connected to a power source.

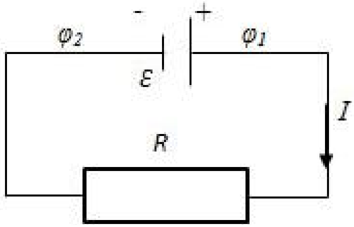


Figure 3.32

The current source does work on moving electrical charges along the chain. This work is done by external forces - forces no electrostatic origin, acting on charges of side of the power supply. The nature of external forces may be different (except fixed charges):

1) chemical reaction - in galvanic cells (batteries), rechargeable batteries,

2) Electromagnetic - in generators. The generator can use a) mechanical energy - hydro, b) nuclear - nuclear reactor) heat - TPS, z) of the tides - PES, D) Wind - Wind Farm, etc.

3) use of the photoelectric effect - photovoltage in calculators and solar power;

4) piezoelectric – pezo EDS, such as piezolighter,

5) contact potential - thermopower in thermocouples etc.

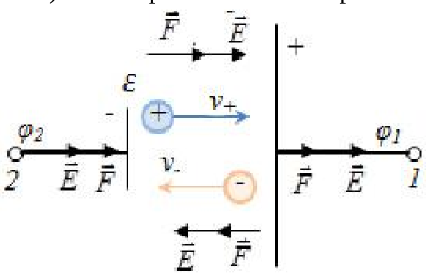


Figure 3.33

The field of external forces, electric charges move inside the power supply against the forces of electrostatic field, whereby the terminal current source and is supported by the potential difference in the circuit current is flowing.

The current source is characterized by an electromotive force – EMF

$$\varepsilon = \frac{A}{q}, \tag{3.136}$$

$$[\varepsilon] = \frac{J}{C} = V \cdot \tag{3.137}$$

EMF determined by the work performed by external forces to move a unit of positive charge along the closed circuit.

Sided force is equal to:

$$\overrightarrow{F_S} = q \cdot \overrightarrow{E_S}, \tag{3.138}$$

where E_S - the field of external forces. The work of external forces on the movement of the charge q on a closed portion of the chain is:

$$A = \oint_l \overrightarrow{F_S} \cdot d\vec{l} = q\oint_l \overrightarrow{E_S} \cdot d\vec{l},\ q \ \Rightarrow \varepsilon = \oint_l \overrightarrow{E_S} \cdot d\vec{l}, \tag{3.139}$$

i.e EMF equal to the circulation of the intensity vector of external forces. At site 1 - 2 (see picture) except of external forces force acting the electrostatic field

$$\overrightarrow{F_v} = q \cdot \vec{E}, \tag{3.140}$$

i.e the resultant force on the section 1 - 2 equals

$$\vec{F} = \overrightarrow{F_S} + \overrightarrow{F_v} = q\left(\overrightarrow{E_S} + \overrightarrow{E_v}\right), \tag{3.141}$$

then

$$A_{12} = q\int_1^2 \overrightarrow{E_{st}} d\vec{l} + q\int_1^2 \overrightarrow{E_v} d\vec{l} = q\vec{E} + q(\varphi_1 - \varphi_2), \tag{3.142}$$

For a closed circuit

$$A_{Electrfield} = 0 \Rightarrow A_{12} = q\varepsilon \,. \tag{3.143}$$

Voltage U on the site 1 -2 called physical quantity determined by the work done by the total field of electrostatic (Coulomb) and external forces when moving a unit positive charge on this part of the chain

$$U_{12} = \frac{A_{12}}{q} = \varphi_1 - \varphi_2 + \varepsilon. \tag{3.144}$$

$$U_{12} = \varphi_1 - \varphi_2 \quad \text{at } \varepsilon = 0. \tag{3.145}$$

3.2.2 Ohm's Law

1. Ohm's law for a homogeneous region of the chain. Called homogeneous area free of EMF.

The current in the homogeneous chain site is directly proportional to the voltage and inversely proportional to the resistance of chain (Figure 3.34).

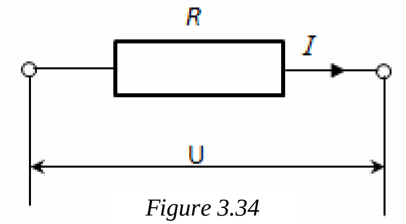


Figure 3.34

$$I = \frac{U}{R}. \quad (3.146)$$

$[R] = \Omega (Ohm)$.

1 *Ohm* - the resistance of the conductor, which at a voltage of 1 *V* 1 *A* current flows.

$$G = \frac{1}{R}, \quad (3.147)$$

G - electrical conductivity. $[G] = S$ (Siemens).

The resistance R of the conductor depends on its size and shape, as well as the conductor material (Figure 3.35).

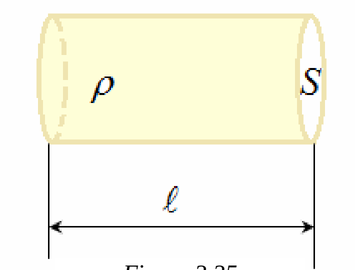


Figure 3.35

$$R = \rho \frac{l}{S}, \quad (3.148)$$

where ρ - resistivity of the conductor - the resistance per unit length of the conductor.

$$[\rho] = \Omega \cdot m, \quad (3.149)$$

ℓ - the length of wire, S - cross-sectional area of the conductor.

2. Ohm's law for the inhomogeneous region of the chain **Inhomogeneous** called chain section containing EMF.

$$U_{12} = \frac{A_{12}}{q} = \varphi_1 - \varphi_2 + \varepsilon, \quad (3.149,a)$$

$$I = \frac{U_{12}}{R} = \frac{\varphi_1 - \varphi_2 + \varepsilon}{R}. \quad (3.150)$$

Ohm's Law for the inhomogeneous region of the chain in the integrated form.

3. Ohm's law for a closed circuit (full circuit) (Figure 3.36).

$$I = \frac{U_{12}}{R`} = \frac{\varphi_1 - \varphi_2 + \varepsilon}{R`} = \frac{\varepsilon}{R`}, \quad (3.151)$$

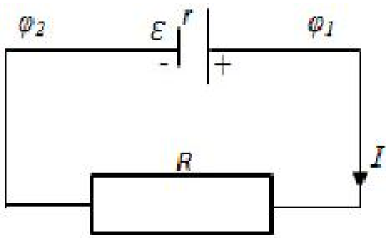


Figure 3.36

where $R` = R + r$, where R - the resistance of the external circuit, r - the source *EMF* impedance, then

$I = \dfrac{\varepsilon}{R + r}$ - Ohm's law for the complete chain.

4. Ohm's law in differential form

$$I = \frac{U}{R}, \qquad (3.152)$$

$$R = \rho \frac{l}{S}, \qquad (3.153)$$

$$I = \frac{U \cdot S}{\rho \cdot l}, \qquad (3.154)$$

$$j = \frac{I}{S}, \qquad (3.155)$$

$$\sigma = \frac{1}{\rho}, \qquad (3.156)$$

$$E = \frac{U}{d}, \qquad (3.157)$$

σ - electrical conductivity; $[\sigma] = \frac{S}{m}$.

$\vec{j} = \sigma \vec{E}$ - Ohm's law in differential form.

The current density $\vec{j}$ is directly proportional to the electric field E. The coefficient of proportionality σ - electrical conductivity.

3.2.3 Dependence of the resistance the conductor temperature. Superconductors

With increasing temperature, the resistance of the conductor increases linearly (Figure 3.37).

$$R = R_0(1 + \alpha t), \qquad (3.158)$$

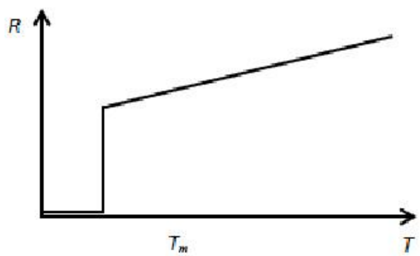


Figure 3.37

where R_0 - resistance at t = 0 C; *R*-resistance at temperature *t*, α-coefficient of thermal resistance, shows how changing resistance of a conductor temperature changes by 1 degree. For pure metals at not too low temperatures $\alpha \approx \frac{1}{273} K^{-1}$, i.e, we can write $R = \alpha R_0 T$.

At a certain temperature (0,14-20 *K*), called the "critical" conductor resistance sharply reduced to 0 and the metal becomes superconducting. For the first time in 1911, it discovered Kamerlingh Onnes for mercury.

In 1987 the designed ceramics, passing into the superconducting state at temperatures above 100 K, the so-called high-temperature superconductors - HTS.

3.2.4 The elementary classical theory of electrical conductivity of metals

Carriers in metals are free electrons, ie weakly bound electrons with ions of the crystal lattice of the metal. The presence of free electrons explained by that the formation of the crystal lattice of the metal during the approach of isolated atoms, valence electrons are weakly coupled with atomic nuclei, break away from the metal atom to become "free" socialized belonging not to an individual atom and the whole matter, and can move on throughout. In the classical electron theory, these electrons are treated as an electron gas with the properties of a monatomic ideal gas (Figure 3.38).

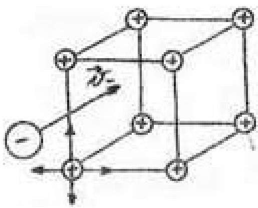


Figure 3.38

Conduction electrons in absence of an electric field inside the metal randomly move and collide with the ions of the crystal lattice of the metal. Thermal motion of the electrons, being chaotic, can not give rise to current. Average thermal velocity of the electrons

$$\bar{u} = \sqrt{\frac{8kT}{\pi m}} = 1{,}1 \cdot 10^{5}\ \frac{m}{s} \text{ at T = 300 K.}$$

The electric current in the metal arises under influence of an external electric field, which causes the orderly movement of electrons. Express the current intensity and the current density of the velocity *v* of the ordered motion of electrons in a conductor.

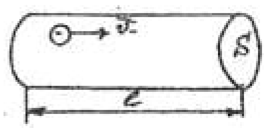


Figure 3.39

During the time d*t* through the cross section *S* of the conductor will be *N* electrons (Figure 3.39).

$$N = n \cdot dV = n \cdot S \cdot dl = n \cdot S \cdot \bar{\upsilon}.dt\,, \qquad (3.159)$$

$$dq = N \cdot e = n \cdot e \cdot S \cdot \bar{\upsilon} \cdot dt \Rightarrow I = \frac{dq}{dt} = n \cdot e \cdot S \cdot \bar{\upsilon}\,, \qquad (3.160)$$

$$j = n \cdot e \cdot \bar{\upsilon}\,, \qquad (3.161)$$

$$j_{cu} = 10^7 \frac{A}{m},\ n = 8 \cdot 10^{28} \frac{1}{m^3}, \Rightarrow$$

$$\overline{\upsilon} = 7{,}84 \cdot 10^{-4} \frac{m}{s},\ \Rightarrow \overline{\upsilon} << \overline{u} \cdot$$

The refore, even at very high current densities, the average velocity of the ordered motion of electrons $\overline{\upsilon}$, causes the electric current is much smaller than the speed of thermal motion $\overline{u}$.

The electric current in the circuit is set in a time $t = \frac{L}{c}$, where the L - chain length, c = $3 \cdot 10^8$ m/s - the speed of light in vacuum. The electric current in the circuit disappear almost simultaneously with its closure.

The mean free path of the electrons of the order of λ must be equal to the period of the crystal lattice of the met $\lambda = 10^{-10}$ м.

With increasing temperature, increasing the amplitude of oscillation of the crystal lattice of ions and electron bowl facing fluctuating ions, so it decreases the mean free path, and the resistance of the metal increases.

Shortcomings of the classical theory of electrical conductivity of metals:

$$\rho = \frac{2m\overline{u}}{ne^2\overline{\lambda}}, \tag{3.162}$$

because $\overline{u} \sim \sqrt{T}$, n and $\lambda \neq f(T) \Rightarrow \rho \sim \sqrt{T}$, i.e from the classical theory of electrical conductivity, the specific resistance is proportional to the square root of the temperature, and from experience that it is linearly dependent on the temperature, $\rho \sim T$.

Gives an incorrect value of the molar heat capacity of metals. According to the law of Dulong and Petit $C_\mu = 3R$,and the classical theory of C = 9/ $2R = C_{\mu \text{ ionic lattice}} = 3R + C_{\mu \text{ monatomic electron gas}} = 3/2R$.

The mean free path of electrons from (3.162) by substituting the experimental value ρ and the theoretical value of $\overline{u}$ gives 10^{-8}, which is two orders of magnitude larger than the mean free path assumed in the theory (10^{-10}).

3.2.5 Work and power supply. Joule-Lenz's law

Because charge is transferred to the conductor under the influence of the electrostatic field, his work is

$$dA = (\varphi_1 - \varphi_2)dq = Udq = UIdt = I^2Rdt = \frac{U^2}{R}dt\,. \tag{3.163}$$

Power - the work done per unit of time

$$P = \frac{A}{t} = \frac{dA}{dt} = IU = I^2 R = \frac{U^2}{R}. \quad (3.164)$$

[P] = *W* (watts).

If the current is a stationary conductor, the entire current work goes into heating the metal conductor, and the law of conservation of energy

$$dQ = UIdt = I^2 Rdt = \frac{U^2}{R} dt \quad (3.165)$$

- **Joule-Lenz`s law**.

Specific power current is the amount of heat per unit volume emitted a conductor per unit time.

$$R = \rho \frac{l}{S}, \quad (3.166)$$

$$I = jdS, \quad (3.167)$$

$$dQ = j^2 dS^2 \rho \frac{dl}{dS} dt = j^2 \rho \cdot dl \cdot dS \cdot dt = j^2 \rho \cdot Vdt, \quad (3.168)$$

$$\varpi = \frac{dq}{dVdt} = \rho j^2 = \sigma E^2 = jE \quad (3.169)$$

- Joule--Lenz's law in differential form.

3.2.6 Kirchhoff's rules for the branched chain

Any point of the branched chain, which converges at least three conductors a current, is called a node. In this talk, part of the node is positive, and going out – no.

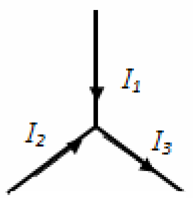


Figure 3.40

The first rule of Kirchhoff: algebraic sum of the currents that converge at a node is equal to zero (Figure 3.40).

$$\sum_i I_i = 0 \quad (I_1 + I_2 - I_3 = 0). \quad (3.170)$$

Kirchhoff's first rule follows from the law of conservation of charge (the charge, who entered to the node is the withdrawing charge).

The second rule of Kirchhoff: in any closed loop randomly chosen in a branched circuit, the algebraic sum of the products of forces of current I_i to resistance R_i - relevant sections of this circuit is equal to the algebraic sum of the EMF occurring in the circuit.

$$\sum_i I_i R_i = \sum_k \varepsilon_k \quad (3.171)$$

In the calculation of complex direct current circuits using Kirchhoff's rules should:

1. Choose an arbitrary direction of the currents at all stages of the chain, the actual direction of the currents is determined to solve the problem, if desired current is turned positive, the direction is right, if negative, the opposite of its true direction chosen.

2. Choose the direction of the circuit. Product IR is positive when the current at the site coincides with the direction of passage, and vice versa. EMF positive if they produce a current directed towards the contour - against negative.

3. Recorded the first rule to N -1 node.

4. Write the second Kirchhoff's rules for closed loops that can be allocated in the chain. Each considered circuit must contain at least one element that is not contained in the previous circuits.

The number of independent equations, but in accordance with the first and second rule of Kirchhoff, is equal to the number of different currents in the branched chain. Therefore, given the EMF and resistance to all areas of unbranched, it can be calculated all the currents

3.3 ELECTROMAGNETISM

3.3.1 The magnetic field in the vacuum. A magnetic field and its characteristics

Permanent magnets have been known 2000 years ago, but only in 1820, H. Oersted (Danish physicist) found that around a conductor with a current creates a magnetic field, which affects the magnetic needle. Later, it was found that the magnetic field is produced by moving bodies, or any charges. The magnetic field, like the electric, is a type of matter. The magnetic field has energy. By means of the magnetic field the interaction between electric currents moving charges.

Experience has shown that the effects of the magnetic field on the current varies depending on the shape of the conductor, in which the current flows, the location of the conductor and the direction of the current. Therefore, in order to characterize the magnetic field, it is necessary to consider the effect on a certain current.

For the study of the electric field using a test point charge. Similarly, for the study of the magnetic field using a current loop, whose dimensions are small compared with the distance to the currents that form a magnetic field. The orientation of the contour (with a current loop) in space is characterized by the normal to the contour.

The positive direction of the normal is determined by the right-hand rule: the four fingers of his right hand in the direction of the current position in the loop, deflected at right angles to the thumb indicates the direction of the normal. The magnetic field exerts on the loop with current orienting effect. The current loop is installed in a magnetic field so that it coincides with the normal direction of the magnetic field lines.

Magnetic moment $\overrightarrow{p_m}$ of current loop is a vector equal to the product of the current flowing through the loop on the vector square $\vec{S} = S \cdot \vec{n}$.

Direction $\overrightarrow{p_m}$ coincides with the direction $\vec{n}$. Direction $\overrightarrow{p_m}$ determined by the right-hand rule (Figure 3.41).

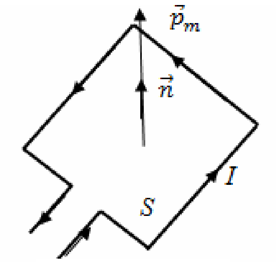


Figure 3.41

Because current loop experiences orienting action of the field, then it in a magnetic field exerts a force couple. Rotating moment forces depends on the properties of the field at a given point and the properties of current loop

$$\overrightarrow{M_{rot}} \sim \vec{B}, \tag{3.171}$$

$$\overrightarrow{M_{rot}} \sim \overrightarrow{p_m}, \tag{3.172}$$

$$\overrightarrow{M_{rot}} = \left[\overrightarrow{p_m} \times \vec{B}\right], \tag{3.173}$$

$$M_{rot} = \left|\vec{p}_m\right|\left|\vec{B}\right| \sin\left(\vec{p}_m \vec{B}\right), \tag{3.174}$$

$$M_{rot\,\max} = \left|\vec{p}_m\right|\left|\vec{B}\right|, \tag{3.175}$$

$\vec{B}$ - magnetic induction vector is a quantitative measure of force of the magnetic field. The unit of measurement of magnetic induction – Tesla.

$[B] = T\ (Tesla)$.

If at a given point of the magnetic field to make a variety of current loop with the magnetic moments of p_1, p_2, ... p_n, then the torque will be different for each current loop M_1, M_2, ...M_n, but the ratio

$$\frac{M_{rot\,\max}}{p_m} = B, \tag{3.176}$$

for all current loops is the same and can serve to characterize the magnetic field.

Magnetic induction $\vec{B}$ at a given point of a uniform magnetic field is numerically equal to the maximum torque $M_{rot\,\max}$, acting on the current loop with the magnetic moment of one, when the normal to the to the current loop is perpendicular to direction of the field ($\vec{B}$ also determined by the Lorentz force or Ampere force).

The direction of vector $\vec{B}$ coincides with the direction of $\overrightarrow{p_m}$ in the case when the current loop is in equilibrium and $M_{rot} = 0$.

A magnetic field conveniently represented with lines of force of vector $\vec{B}$. Force line of vector $\vec{B}$ called a line whose tangent at any point coincides with the direction of $\vec{B}$ at this point. The direction of lines of force of vector $\vec{B}$ determined by the right-hand rule. For linear conductor: thumb in the direction of the current, bent four fingers indicate the direction of the field line. For a circular coil with a current: four fingers - on the current direction, the thumb indicates the direction of the field line in the center of the coil.

Lines of magnetic induction $\vec{B}$, unlike force lines of vector $\vec{E}$, of the electric field is always closed and covered conductor. (The lines of force of vector $\vec{E}$ begin on positive charges and end on negative, approach perpendicular to the surface charge density of the lines of force characterizes the field.)

In some cases, along with the vector $\vec{B}$ applied vector of intensity of magnetic field $\vec{E}$,which is associated with the vector $\vec{B}$ by ratio

$$\vec{H} = \frac{\vec{B}}{\mu\mu_0}, \tag{3.177}$$

$$\vec{B} = \mu\mu_0\vec{H}, \tag{3.178}$$

where μ_0 - magnetic constant; $\mu_0 = 4\pi \cdot 10^{-7}\frac{N}{m}$, μ - magnetic permeability of the medium - shows how many times the magnetic field in the medium more (or less) of the magnetic field in the vacuum.

$$\mu = \frac{B}{B_0}, \tag{3.179}$$

where B - the magnetic field in the material, B_0 - external magnetizing field.

From a comparison of the characteristics of the electric field vector (vector $\vec{E}$ and a vector $\vec{D}$) and magnetic field (vector $\vec{B}$ and $\vec{H}$) it follows that intensity vector $\vec{E}$ of electric field is similar to the magnetic induction $\vec{B}$. Both determine the effect of the force fields and depend on the properties of the medium in which the fields are created.

Analogue of the electric displacement $\vec{D}$ is the vector of intensity of magnetic field $\vec{H}$. Vector $\vec{H}$ which describes the magnetic field macrocurrents macrocurrents - currents flowing through a conductor), so do not depend on the properties of the medium

$$[B]=\frac{M_{rot}}{p_m}=\frac{N\cdot m}{A\cdot m^2}=\frac{N}{A\cdot m}=T \quad (Tesla), \tag{3.180}$$

$$H=\frac{N}{l}I \Rightarrow [H]=\frac{A}{m}. \tag{3.181}$$

3.3.2 Biot-Savart-Laplace's Law

Bio and P. Savard in experimental studies of the magnetic fields produced by a current-carrying conductor, allowed theorist Pierre Simon de Laplace in 1820 to formulate the law of Biot-Savart-Laplace. This law determines the value $\vec{B}$ of at any point relative conductor.Magnetic induction $d\vec{B}$ field, created by the conductor element $d\vec{l}$, in which the current I flows, at some point A, whose position relative to the $d\vec{l}$ determined by the radius vector $\vec{r}$, determined by the Biot-Savart-Laplace law (Figure 3.42)

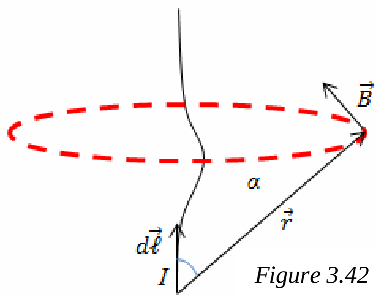


Figure 3.42

$d\vec{B}=\frac{\mu_0\mu}{4\pi}\cdot\frac{I[d\vec{l}\times\vec{r}]}{r^3}$ - the Biot-Savart-Laplace law (in vector form).

Because in the Biot-Savart-Laplace law there is a vector product $[d\vec{l}\times\vec{r}]$, then vector $d\vec{B}$ must be perpendicular to the plane of the vectors $d\vec{l}$ and $\vec{r}$. The direction of vector $d\vec{B}$ determined on the right-hand rule.

Modulus (magnitude) of the vector $d\vec{B}$ is equal to $d\vec{B}=\frac{\mu_0\mu}{4\pi}\cdot\frac{I\cdot dl\cdot r\cdot\sin\alpha}{r^2}$ - the Biot-Savart-Laplace law (in scalar form)

$$\left(\frac{[d\vec{l}\times\vec{r}]}{r^3}=\frac{|d\vec{l}||\vec{r}|\sin\alpha}{r^2 r}=\frac{dl\cdot r\cdot\sin\alpha}{r^2}\right), \tag{3.182}$$

where α –the angle between the $d\vec{l}$ and $d\vec{r}$.

The principle of superposition of fields:

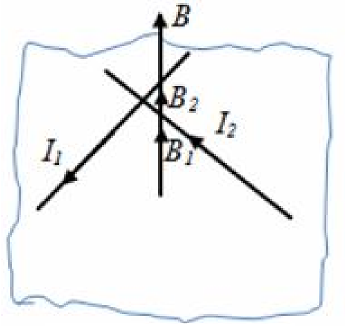


Figure 3.43

Magnetic induction of the resulting field, multi-currents (or moving charges), equal to the geometric (vector) sum of the magnetic induction generated by each current separately (Figure 3.43).

$$\vec{B} = \sum_{i=1}^{N} \vec{B}_i \,. \qquad (3.183)$$

Application of the Biot-Savart-Laplace's law to the calculation of magnetic fields.

a) A magnetic field of the direct current

$$dB = \frac{\mu_0 \mu}{4\pi} \cdot \frac{I \cdot dl \cdot \sin\alpha}{r^2}, \qquad (3.184)$$

$$r = \frac{r_0}{\sin\alpha}, \; dr = r \cdot d\alpha \,, \; dl = \frac{dr}{\sin\alpha} = \frac{r \cdot d\alpha}{\sin\alpha} = \frac{r_0 \cdot d\alpha}{\sin^2\alpha}, \qquad (3.185)$$

$$dB = \frac{\mu_0 \mu}{4\pi} \cdot \frac{I r_0 \cdot d\alpha}{\sin^2\alpha} \cdot \frac{\sin\alpha \cdot \sin^2\alpha}{r_0^2} = \frac{\mu_0 \mu}{4\pi} \cdot \frac{I \sin\alpha}{r_0} d\alpha \cdot \qquad (3.186)$$

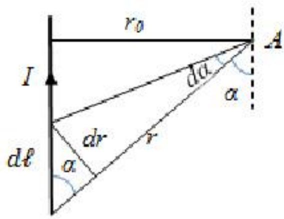


Figure 3.44

Since the induction created by different elementary sections, which we have broken conductor at this point have the same direction, we can sum the geometric vectors $d\vec{B}$ replace the scalar summation

$$B = \sum_{i=1}^{N} dB_i = \lim_{dl \to 0} \sum_{i=1}^{N} dB_i = \int_{\alpha_1}^{\alpha_2} \frac{\mu_0 \mu}{4\pi} \cdot \frac{I \sin\alpha}{r_0} d\alpha =$$

$$= \frac{\mu_0 \mu}{4\pi} \frac{I}{r_0} \int_{\alpha_1}^{\alpha_2} \sin\alpha d\alpha = \frac{\mu_0 \mu}{4\pi} \frac{I}{r_0} (\cos\alpha_1 - \cos\alpha_2), \qquad (3.187)$$

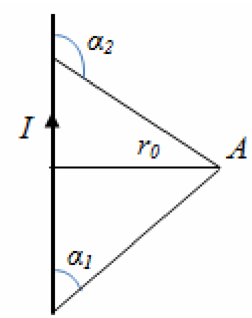


Figure 3.45

$B = \dfrac{\mu_0 \mu}{4\pi} \dfrac{I}{r_0} (\cos\alpha_1 - \cos\alpha_2)$ - magnetic induction linear conductor of finite length.

$H = \dfrac{1}{4\pi} \dfrac{I}{r_0} (\cos\alpha_1 - \cos\alpha_2)$ - intensity of the magnetic field of a conductor of finite length.

In the case of an infinitely long conductor $\alpha_1 = 0; \alpha_2 = \pi$ (Figure 3.45).

$$B=\frac{\mu_0\mu}{4\pi}\frac{I}{r_0},\quad H=\frac{1}{2\pi}\frac{I}{r_0}. \tag{3.188}$$

b) The magnetic field at the center of a circular current-carrying conductor (Figure 3.22).

$$dB=\frac{\mu_0\mu}{4\pi}\cdot\frac{Idl\sin\alpha}{R^2}, \tag{3.189}$$

$\alpha=90^0;\ \sin\alpha=1.$

$$dB=\frac{\mu_0\mu}{4\pi}\cdot\frac{I\cdot dl}{R^2}$$

$$B=\int_0^{2\pi R}\frac{\mu_0\mu}{4\pi}\cdot\frac{I\cdot dl}{R^2}=\frac{\mu_0\mu}{4\pi}\cdot\frac{2\pi RI}{R^2}=\frac{\mu_0\mu I}{2R}, \tag{3.190}$$

$$H=\frac{I}{2R}. \tag{3.191}$$

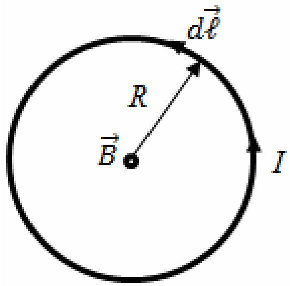


Figure 3.46

3.3.3 The Law of the total current. Vortex nature of the magnetic field

Circulation of vector $\vec{B}$ (or $\vec{H}$) in a closed loop is the integral over a closed contour L scalar product of vectors $\vec{B}$ (or $\vec{H}$) and $d\vec{l}$,where $d\vec{l}$ - vectors of the unit length of the contour.

$$\oint_L \vec{B}d\vec{l}=\oint_L B_L dl=\oint_L Bdl_B, \tag{3.192}$$

$$\oint_L \vec{H}d\vec{l}=\oint_L H_L dl=\oint_L Hdl_H, \tag{3.193}$$

where B_L – projection of vector $\vec{B}$ on vector $d\vec{l}$.

$$\left(d\vec{l},H\right)=\alpha, \tag{3.194}$$

$$\vec{H}d\vec{l}=Hdl\cos\alpha=H_L dl=Hdl_H, \tag{3.195}$$

$$\oint_L H_L dl=\oint_L Hdl_H=\int_0^{2\pi}HRd\alpha=\int_0^{2\pi}\frac{I}{2\pi R}Rd\alpha=,$$
$$=\frac{I}{2\pi}\int_0^{2\pi}d\alpha=I \tag{3.196}$$

Law of the total current:

Circulation of vector $\vec{H}$ an arbitrary closed loop is the sum of current covered by the circuit

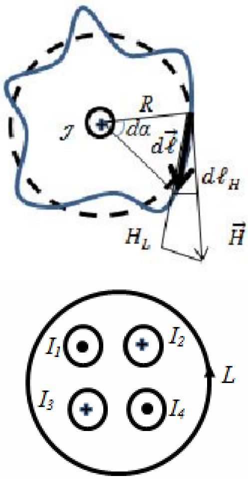


Figure 3.47

$$\oint_L \vec{H}d\vec{l} = \sum_{k=1}^{N} I_K , \tag{3.197}$$

$$\oint_L \vec{B}d\vec{l} = \mu_0 \sum_{k=1}^{N} I_K , \tag{3.198}$$

Positive are those currents, the direction of which to the direction of passage of obeys the right hand rule. Currents, whose direction is opposite to bypass, taken with the minus sign (Figure 3.47).

$$\oint_L \vec{H}d\vec{l} = I_1 - I_2 + I_3 - I_4 . \tag{3.199}$$

In contrast to the electric field, for which the circulation of the vector $\vec{E}$ equal zero $\oint_L \vec{E}d\vec{l} = 0$ and the electrostatic field is potential, the circulation of the magnetic field is not zero $\oint_L \vec{B}d\vec{l} = \mu_0 \sum_{k=1}^{N} I_K$, if a path on which we consider the circulation covers currents. Field, the circulation of which is non-zero, is called a vortex or solenoidal. Consequently, the magnetic field is a vortex. In vortex field force lines are closed, therefore, there is no magnetic charges.

3.3.4 A magnetic field of the solenoid and toroid

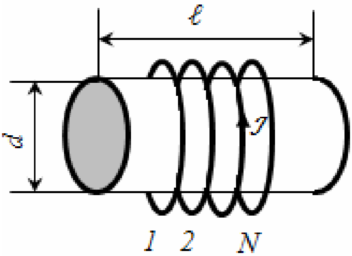


Figure 3.48

Solenoid is cylindrical shell, which are wound windings of wire. Consider an infinitely long solenoid, ie solenoid which ℓ >> *d*, where ℓ - length, *d* –diameter of the coil. Inside such a solenoid magnetic field is uniform. Uniform is a field, the field lines are parallel and their density is constant (Figure 3.48).

Apply the law of the total current to calculate the magnetic field of the solenoid. Represent the contour *L*, which is considered by the circulation of the vector $\vec{H}$, consisting of four related areas 1-2; 2-3, 3-4, 4-1. Then the circulation of the vector $\vec{H}$ the chosen us contour *L* is equal to

$$\oint_L H_L dl = \int_1^2 H_L dl + \int_2^3 H_L dl + \int_3^4 H_L dl + \int_4^1 H_L dl + = \int_0^{2\pi} HRd\alpha , \tag{3.200}$$

$$\int_1^2 H_L dl = Hl , \tag{3.201}$$

$$\int_2^3 H_L dl = 0, \text{ because } \vec{H} \perp d\vec{l} \text{ and therefore, } H_{L\,2-3} = 0, \tag{3.202}$$

$$\int_3^4 H_L dl = 0, \tag{3.203}$$

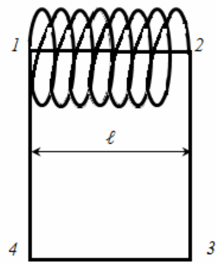


Figure 3.49

because we have chosen the area 3 - 4 far enough from solenoid and one can assume that the field far from the solenoid is zero,

$\int_4^1 H_L dl = 0$, because $\vec{H} \perp d\vec{l}$ and therefore, $H_{L\,4-1} = 0$.

Circuit L includes N currents, where N - number of turns the solenoid, then the law of the total current (Figure 3.49)

$$Hl = NI, \tag{3.204}$$

$H = \dfrac{N}{l} I = nI$ - The magnetic field of an infinitely long solenoid n - winding density - the number of turns per unit length

$$n = \frac{N}{l}. \tag{3.205}$$

Figure 3.50

The field intensity inside the solenoid is equal to the number of turns per unit length of the solenoid, multiplied by the current.

Toroid - torus, with coils wound on a wire. Unlike a solenoid, which has a magnetic field, both inside and outside, fully toroidal magnetic field is concentrated inside the coils, i.e there is no dissipation of magnetic field energy (Figure 3.50).

$$\oint_L \vec{H} d\vec{l} = H 2\pi r = NI \Rightarrow H \frac{NI}{2\pi r} = \frac{2\pi R n I}{2\pi r} = nI \frac{R}{r}, \tag{3.206}$$

where $N = n \cdot 2\pi R$.

$H = nI \dfrac{R}{r}$ - magnetic field of toroid. If $R >> R_{turn}$, then $R \approx r$ and $N = nI$.

3.3.5 Ampere force

Ampere studied the effect of magnetic field on the current-carrying conductors and found that the force $d\vec{F}$, with which the magnetic field acts on the element wire $d\vec{l}$ with current *I*, in a magnetic field $\vec{B}$, directly proportional to the current *I* and the vector product $d\vec{l}$ on the magnetic induction $\vec{B}$.

$$d\vec{F} = I[d\vec{l} \times \vec{B}] \tag{3.207}$$

– the Ampere force (or Ampere's law).

The direction of the Ampere force is situated by the rule of the vector product - on left-hand rule: four elongated fingers of his left hand placed on the direction of the current, vector $\vec{B}$ included in the palm, deflected at right angles to the thumb will show the direction of force acting on a current-carrying conductor. (You can also determine the direction of $\vec{F_A}$ with his right hand: turn the four fingers of the right hand of the first factor $d\vec{l}$ to second $\vec{B}$, thumb indicates the direction of $\vec{F_A}$).

Module of Ampere force

$$dF = I \cdot dl \cdot B \cdot \sin\alpha\ , \tag{3.208}$$

where α - the angle between the vectors $d\vec{l}$ and $\vec{B}$, $\alpha = \left(d\vec{l}, \vec{B}\right)$.

If the field is uniform, and the current-carrying conductor of finite size, the

$$\vec{F} = I[\vec{l} \times \vec{B}], \tag{3.209}$$

$$F = IlB \cdot \sin\alpha\ . \tag{3.210}$$

At $\vec{B}$ perpendicular $\vec{l}$

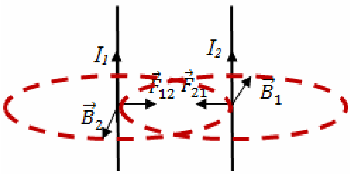
$$F = IlB\ . \tag{3.211}$$

3.3.6 Definition of the unit of measurement of the current

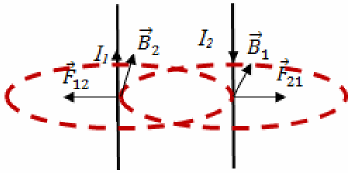
Any current-carrying conductor generates a magnetic field around itself. If you put it in the field of the other current-carrying conductor, the conductor between the forces of interaction. In this case, co-directional parallel currents attract each the opposite direction - are repelled.

Consider two infinitely long parallel conductors with currents I_1 and I_2, in a vacuum at a distance *d* (for vacuum μ = 1) (Figure 3.51). According to Ampere's law

$$F_{21} = I_2 l B_1\ , \tag{3.212}$$



a)



b)

Figure 3.51

A magnetic field of direct current is

$$B_1 = \frac{\mu_0\mu}{2\pi}\cdot\frac{I_1}{d}, \tag{3.213}$$

Then

$$F_{21} = \frac{\mu_0\mu}{2\pi}\cdot\frac{I_1 I_2 l}{d}, \tag{3.214}$$

the force per unit length of the conductor

$$\frac{F}{l} = \frac{\mu_0\mu}{2\pi}\cdot\frac{I_1 I_2}{d}. \tag{3.215}$$

The force per unit length of the conductor between two infinitely long conductor with a current directly proportional to the current in each conductor and inversely proportional to the distance between them.

Definition of the unit of measurement of the current - Ampere:
Per unit of current in the SI in place a DC current which is flowing in two infinitely long parallel conductors infinitesimal cross section, located in vacuum at a distance of 1 m from each other, is the force exerted per unit length of the conductor is equal to $2\cdot10^{-7}$ N.
$\mu = 1$; $I_1 = I_2 = 1$ *A*; $d = 1$ *m*; $\mu_0 = 4\pi\cdot10^{-7}$ *H/m* –magnetic constant.

$$\frac{F}{l} = \frac{\mu_0 1}{2\pi}\cdot\frac{1\cdot1}{1} = 2\cdot10^{-7}\,\frac{N}{m}. \tag{3.216}$$

3.3.7 The Lorentz force

Under the Ampere's law, force acting on the current element $Id\vec{l}$, determined by the formula

$$dF = IdlB. \tag{3.217}$$

Consider that the elementary current is none other than the directional movement of electric charges

$$I = jS,\ Idl = jSdl = jV, \tag{3.218}$$

$$j = en\bar{\upsilon}. \tag{3.219}$$

where V –volume, n - the carrier density, j - current density, S - cross-sectional area of the conductor, e - electron charge ($e = 1{,}6\cdot10^{-19}$ C), dl -the element length of the conductor, $\overline{\upsilon}$ - velocity of the electron motion.

$$Idl = edN\overline{\upsilon}, \qquad (3.220)$$

$$dN = ndV, \qquad (3.221)$$

$$d\vec{F} = edN[\overline{\upsilon} \times \vec{B}]. \qquad (3.222)$$

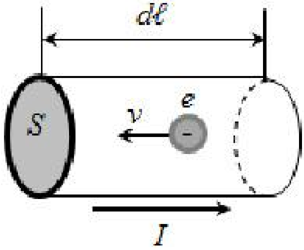


Figure 3.52

Ampere force acting on the elementary current $Id\vec{l}$ can be seen as the resultant force of the all forces exerted by the magnetic field on each charge separately (Figure 3.52). Then, the force acting on a moving charge in a magnetic field, we find by dividing the number of Ampere charge in this volume element of the conductor

$$\overrightarrow{F_L} = \frac{d\overrightarrow{F_A}}{dN} = e[\vec{\upsilon} \times \vec{B}], \qquad (3.223)$$

This force is called the Lorentz force:

$$\overrightarrow{F_L} = q[\vec{\upsilon} \times \vec{B}], \qquad (3.224)$$

$F_L = q\upsilon B \sin\alpha$ - module of the Lorentz force.

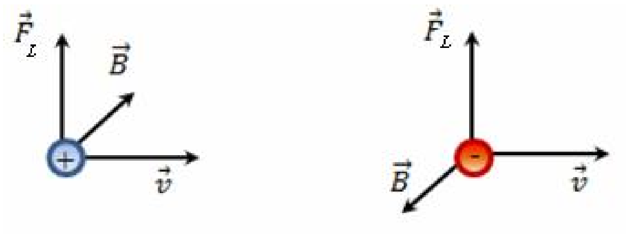


Figure 3.53

The direction of the Lorentz force is determined by the left-hand rule: four fingers of his left hand - the speed, the vector $\vec{B}$ enters in the palm, deflected at right angles to the direction of the thumb shows the Lorentz force to the positive charge. For a negative charge - four fingers against the speed, then the same as for the positive charge (Figure 3.53).

3.3.8 Dia-and paramagnetic

Diamagnetic are those substances which have a magnetic moment of the atom in the absence of an external magnetic field is zero.

$$\vec{p}_{m\,aton} = 0 \text{ at } \vec{B}_0 = 0, \qquad (3.224,a)$$

When an external magnetic field is placed a substance, all the atoms of the material are in a magnetic field that changes the motion of electrons in the atom, so that an additional current, similar to the induction current. If the vector $\vec{p}_m$ and $\vec{B}$ form an angle α, then in a magnetic field $\vec{B}$ electron orbit will rotate around the direction $\vec{B}$ with some angular velocity ($\varpi = \dfrac{eB}{2m}$ - Larmor precession frequency) (Figure 3.54). Such movement mechanics called **precession**.

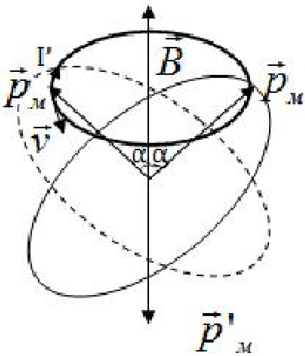


Figure 3.54

The precession of the electron orbit is equivalent to an additional electron motion around the magnetic field in addition to rotation around its axis of rotation and orbit. This extra electron motion in the magnetic field leads to a vicious induced current, which has a magnetic moment, which is always directed against the field. Thus, the cause of the additional magnetic moments - the precession of the orbit of the electron.

Since diamagnetics magnetized opposite to the magnetic field, their magnetization is negative.

For diamagnetic include metals *Bi*, *Ag*, *Au*, *Cu*; water, glass, inert gases, etc.

Diamagnetism is common to all substances, but a number of substances diamagnetic effect is blocked as stronger effects.

Paramagnetic called substances in which the atoms in the absence of an external magnetic field have some permanent magnetic moment

$$\vec{p}_{m\,aton} \neq 0 \text{ at } \vec{B}_0 = 0. \qquad (3.225)$$

However, due to thermal motion of the magnetic moments are oriented randomly, so $\sum\limits_{i=1} \vec{p}_{mi} = 0$.

When a magnetic field, forces that guide the magnetic moments of each atom. The magnetic moments of trying to line up on the field. Thus, the paramagnetic magnetized, creating its own magnetic field collinear with the external field and reinforcing it.

The process of the magnetic moments of the atoms in a magnetic field is called the paramagnetic effect.

Arrayed in paramagnetic forces are relatively small compared with the forces of thermal motion, to break down the ordering. Therefore, with decreasing temperature paramagnetic susceptibility usually increases.

Paramagnetic to include rare earth metals *Pt*, *Al*, *Mg*, *Cr*, O_2 etc.

3.3.9 A magnetic field in the materials. Magnetic susceptibility and permeability

The current flowing through a conductor is called macrocurrent. The magnetic field generated by these currents, called the field of macrocurrent and denote macrocurrents $\vec{B}_0$.

If an object is placed in this field $\vec{B}_0$, then the magnetic moments of the atoms of matter will be oriented against the field in the diamagnetic and paramagnetic in the field. Ie microcurrents substances create an internal field $\vec{B}$, opposite directions in the diamagnetic and paramagnetic in collinear. Then the magnetic induction vector of the resultant magnetic field in matter is equal to the vector sum of the magnetic induction of the external field $\vec{B}_0$ and microcurrents field $\vec{B}`$

$$\vec{B} = \vec{B_0} + \vec{B}`, \tag{3.226}$$

where $\vec{B}_0 = \mu_0 \vec{H}$.

If we consider the matter in any section of a cylinder perpendicular to its axis, the substance inside the molecular currents of neighboring atoms are directed towards each other and cancel each other out. Not be compensated only molecular currents on the side of the cylinder. The current through the cylinder surface is similar to the current in the solenoid, and creates within it the field

$$B` = \mu_0 \frac{I`N}{l} = \mu_0 \frac{I`}{l}, \;\; N = 1, \mu = 1, \tag{3.227}$$

$$p_m = I`S\frac{l}{l} = \frac{I`V}{l}, \tag{3.228}$$

$$\frac{p_m}{V} = j \Rightarrow j = \frac{I`}{l}, \tag{3.229}$$

$$\vec{B}` = \mu_0 \vec{j}, \tag{3.230}$$

$$\vec{B} = \vec{B_0} + \vec{B}` = \mu_0 \vec{H} + \mu_0 \vec{j}, \tag{3.231}$$

$$\frac{\vec{B}}{\mu_0} = \vec{H} + \vec{j}. \tag{3.232}$$

Experience shows that in weak magnetic fields, the magnetization is proportional to the field intensity $\vec{H}$, causing the magnetization

$$\vec{j} = \chi \vec{H}. \qquad (3.233)$$

[χ] = 1,

where χ - a dimensionless quantity called the magnetic susceptibility shows how the substance reacts (magnetized) to an external field.

$$\vec{B} = \mu_0 (1+\chi)\vec{H}, \qquad (3.234)$$

$$\mu = 1+\chi. \qquad (3.235)$$

- link permeability μ and susceptibility χ.

$$\vec{B} = \mu_0 \mu \vec{H} \Rightarrow \mu = \frac{\vec{B}}{\mu_0 \vec{H}} = \frac{\vec{B}}{\vec{B}_0}. \qquad (3.236)$$

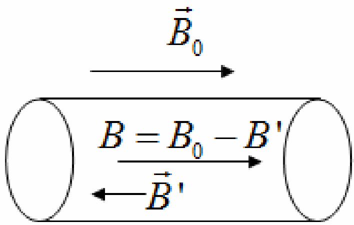


Figure 3.55

Magnetic permeability shows how many times the resulting magnetic field in a substance greater than the external magnetizing field macrocurrents $\vec{B}_0$ (Figure 3.55).

For diamagnetic:

$\chi < 0$, $\mu < 1$, $\vec{B} = \vec{B}_0 + \vec{B}`$,

$B = B_0 - B`$, $\chi \sim 10^{-5} \div 10^{-7}$. (3.237)

For paramagnetic:

$\chi > 0$; $\mu > 1$; $\vec{B} = \vec{B}_0 + \vec{B}`$; $B = B_0 + B`$; $\chi \sim 10^{-3} \div 10^{-5}$. (3.237,a)

3.3.10 Electromagnetic induction. The phenomenon of electromagnetic induction. Faraday's law

Faraday's experiments

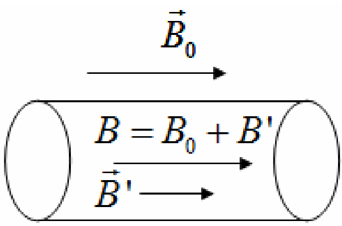


Figure 3.56

a) The solenoid, closed the galvanometer, pushed into and put forward the permanent magnet. The galvanometer deflection will, and it will be longer than the faster the pushed into and put forward. When you change the direction of the poles of the magnet deflection change.

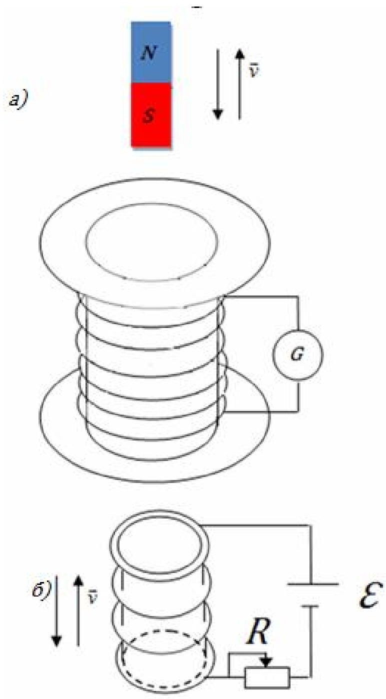


Figure 3.57

b) solenoid, closed the galvanometer is inserted coil (another solenoid) through which current is passed. When you turn on and off (ie, any change in the current) is deflection of the galvanometer. Direction of the deviation varies with On - Off, decrease - the current increases, vdviganii - sliding out the coils.

The phenomenon of electromagnetic induction is that in a closed conducting circuit when the flow of magnetic induction covered by this contour, an induction (induced) electric current.

Occurrence of the induced current means that the circuit operates electromotive force E_i- induced emf.

EMF induction, which occurs in the conducting circuit, equal to the rate of change of magnetic flux through the area bounded by this contour - Faraday's law.

$$\varepsilon_i = -\frac{d\Phi}{dt} \tag{3.238}$$

In 1834, E.H. Lenz established the law, allowing to determine the direction of induced current.

Lenz rule: induction loop current always has a direction such that the magnetic field created by it prevents the change in magnetic flux that caused this induced current.

The minus sign in Faraday's Law is a mathematical expression of the rule of Lenz.

If the circuit in which the induced emf is not composed of a single turn, and of N turns (eg, solenoid), if windings are connected in series, E_i will be equal to the emf induced in each of the coils individually (Figure 3.56):

$$\vec{B} = \vec{B_0} + \vec{B`}, \tag{3.239}$$

$$\varepsilon_i = -\sum_i \frac{d\Phi_i}{dt} = -\frac{d}{dt}\sum_i d\Phi_i = -\frac{d\psi}{dt}, \tag{3.240}$$

$\psi = \sum_i d\Phi_i$ - flux linkage or the total magnetic flux.

$[\psi] = Wb$.

If $\Phi_1 = \Phi_2 = \ldots = \Phi_n$, then

$$\psi = N\Phi . \qquad (3.241)$$

Because $\Phi_B = BS\cos\alpha$, then in order to change the magnetic flux *F* can be changed: 1) magnetic field $\vec{B}$; 2) the area *S*; 3) the angle α.

Spin current loop in a magnetic field

The phenomenon of electromagnetic induction is used to convert mechanical energy and the energy of the electric current in the generator current loop area *S* rotates in a uniform magnetic field ($\vec{B} = const$) evenly with a constant angular velocity ω.

$\alpha = \omega t$.

Then

$$\Phi = BS\cos\alpha = BS\cos\varpi t . \qquad (3.242)$$

$$\varepsilon_i = \frac{d\Phi}{dt} = BS\varpi \sin\varpi t \cdot \qquad (3.243)$$

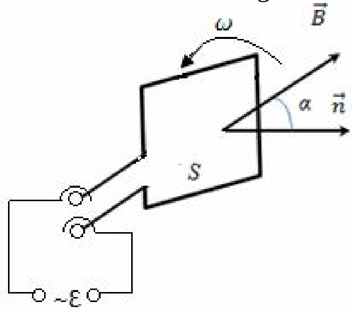


Figure 3.58

At $\sin\omega t = 1$

$$\varepsilon_{i\max} = BS\varpi , \qquad (3.244)$$

and

$$\varepsilon_i = \varepsilon_{i\max} \sin\varpi t. \qquad (3.245)$$

Because network frequency $\nu = \frac{\omega}{2\pi} = 50\ Hz = const$, then for increase $\varepsilon_{i\max}$ need to increase *B* and *S* (Figure 3.58). As can be increased by using powerful permanent magnets or electromagnets to pass large currents. Core with a large electromagnet choose μ. To increase the use of *S* multiturn coil.

If a current loop placed in a magnetic field, an electric current, then it will be acted torque

$$M_{rot} = Bp_m = BIS . \qquad (3.246)$$

and the current loop starts to rotate. Based on this principle of the motor, designed to transform electrical energy into mechanical energy.

3.3.11 Foucault currents. Of the circuit inductance. Self-induction

Induced currents can be excited in continuous bulk conductors. In this case they are called Foucault currents or eddy currents. The electrical resistance of a bulk conductor is small, so eddy currents can reach a very large force.

Eddy currents, as well as induced currents in linear conductors are subject Lenz's law: their magnetic field is directed so to counteract changes in the magnetic field that induces eddy currents.

Therefore, moving in a strong magnetic field are good conductors under heavy braking due to the interaction of the eddy currents and the magnetic field. It is used for damping (steadying) of moving parts galvanometers, seismographs, etc. Thermal effect of the eddy currents is used in induction melting furnaces.

To reduce eddy currents transformer cores made of individual plates and the plates are perpendicular to the currents of Foucault.

S due to eddy currents rapidly current unevenly distributed over the cross section of wire - it pushed to the surface of a conductor - the skin effect. Therefore, at high frequencies using hollow wire.

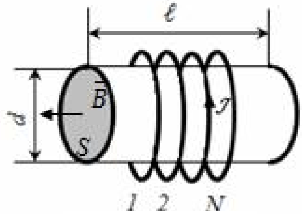


Figure 3.59

In any case where the contour of the electric current creates a magnetic field. In this case, there is always a magnetic flux *F* passing through the surface bounded by the circuit under consideration.

Any change in the current in the circuit changes the magnetic field, coupled with the circuit, and this in turn causes the induced current. This phenomenon is called self-induction: the emergence of the emf induced in a conductor when the current in it (Figure 3.59).

Of the Biot-Savart-Laplace should

$$dB = \frac{\mu\mu_0}{4\pi} \cdot \frac{dl \sin\alpha}{r^2} \cdot I = cI\,, \tag{3.247}$$

ie magnetic flux linked with the circuit is proportional to the current *I* in the circuit

$$\Phi = LI\ . \tag{3.248}$$

$[L] = H$ (henry).

1 *H*- inductance of the loop, the magnetic flux is self-induced by a current of 1 A is 1 Wb.

Calculate the inductance *L* of the solenoid

$$\Phi_B = BS\cos\alpha = BS\,, \tag{3.249}$$

magnetic induction of the solenoid

$$\Phi = BS\cos\alpha = BS\cos\varpi\ , \tag{3.250}$$

$$\psi = \Phi_1 N = \mu_0\mu\frac{N}{l}ISN = \mu_0\mu\frac{N^2}{l}IS = LI\,, \tag{3.251}$$

$$L = \mu_0\mu\frac{N^2}{l}S\,. \tag{3.252}$$

ie inductance depends on the geometry of the solenoid (l, S), number of turns and the magnetic permeability of the core solenoid. Therefore we can say that the inductance L capacitance C analogue isolated conductor, which also depends on the geometry, the shape and the dielectric constant of the medium.

Applying the self-induction Faraday's law, we find that self-induced emf

$$\varepsilon_{is} = -\frac{d\Phi}{dt} = -\frac{d}{dt}(LI) = -\left(L\frac{dI}{dt} + I\frac{dL}{dt}\right). \tag{3.253}$$

If $L = const$

$$\varepsilon_{is} = -L\frac{dI}{dt}, \tag{3.254}$$

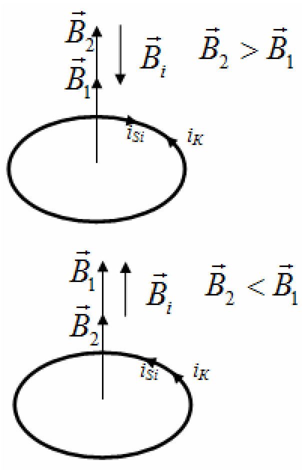


Figure 3.60

where the minus sign, due to Lenz's law, shows that the presence of inductance in the circuit slows current change in him.

If the current increases with time, then $\frac{dI}{dt} > 0$, and $\varepsilon_{is} < 0$, (Figure 3.60) i.e current is directed towards the self-inductance of the current, due to the external source and inhibits its growth.

If the current time is decreasing, then $\frac{dI}{dt} < 0$ and $\varepsilon_{is} > 0$, i.e induced current has the same direction as the decreasing current in the circuit, and slows its decay. Consequently, the circuit having inductance is electrically inert, consists in the fact that any change in the current is inhibited, the stronger, the more inductance.

3.3.12 Ferromagnetic Materials

Ferromagnetic materials behave just like paramagnetic materials but the effect is much more intense. Thus they are attracted by a magnet much more strongly. They always settle down in the direction of the magnetic field and their magnetization is positive and very much greater.

They comprise iron, nickel, cobalt , gadolinium and certain alloys. The value of μr though high, is not constant but varies with B. For cast iron, which is not a very good magnetic material, μr has a maximum value of about 350. A silicon steel, stalloy, which is widely used in a.c. generator and transformers, has a maximum μr of about 6000. Some nickel-iron alloys have values of μr, up to 100.000, but they require careful heat treatment and are susceptible to mechanical strians.

In the atoms of ferromagentic materials, there are vacancies in the inner electron shells. The electrons in these shells are, therefore, not paired off with equal and opposite orbital magnetic moments and anti-parallel spins. In the case of iron, for example, as many as 5 out of 6 electrons in the n = 3, l = 2 sub-shell have parallel spins. The atoms of these elements, therefore, possess appreciable magnetic moments (Figure 3.61).

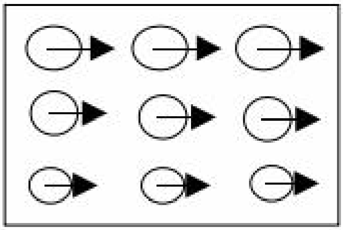
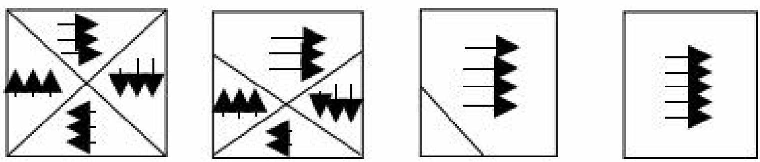


Figure 3.61

Another important property of these materials, is that an unpaired electron in atom internets strongly with the unpaired electron in the atom adjustment to it.Hence the magnetic moment get all aligned in the same direction as shown in figure 3.61. This is known as exchange interaction.

The atoms in all these elements (which are crystalline in nature) seem to group themselves togather in small and separate assemblies, called domains, each about 5 x 10^{-5} m across. The magnetic moments in one domain are parallel to each other but not necessarily in the same direcion as those in a neighbouring domain.

In the unmagnetised state, the domains are oriented randomly, as it is in figure 3.62 (a). Therefore, the material shows no sign of magnetization. When a magnetic field is applied to the material, alignment may occur in one or two ways: either (i) the magnetic moment in all the domains line up in the same direction as the field or (ii) if the material be pure and homogenous, the domain in which the magnetic moments are in the direction of the field continually expands at the expense of the others, as shown in figure 3.62 (b), (c) and (d). The first procedure requires a much stronger magnetic field than the second.



a) *b)* *c)* *d)*

Figure 3.62

Ferromagnetic materials retain their magnetization even after the magnetising field is removed. This is why permanent magnets are made of such materials. With temperature rise, the atomic alignment within the domains gets disturbed and at temperature near about 750^0C, called the curie temperature, the material is reduced to a paramagnetic one.

You should note that ferromagnetism is not an atomic property but just s special arrangement of groups of atoms into magnetic domain.

An unmagnetised ferromagnetic material, placed in a magnetic field, becomes magnetised and thereby makes substantial alteration in the magnetic field that would otherwise be present, typically increasing the field by a factor of a thousand at points within or near the material. Permanent magnets retain the alignment of their different domains. Other 'softer' ferromagnetic material tends to revert to random domain alignment when a magnetising field is removed.

Figure 3.63 show the rather complicated relationship between B and H in ferromagnetic material. If the sample is initially unmagnetised and H is steadily increased from zero, a B-H graph called the magnetization curve is obtained. This is the graph from state 0 to sate 1 in Figure 3.63. The permeability, μ = B/H, is not constant. A typical ferromagnetic material, annealed ion, has a relative permeability with an initial value of $3x10^2$, a maximum value of 5×10^3, and a limiting value of 1 (the value for a vacuum) as H approaches infinity.

With the sample magnetised state 1, reduction in H does not result in B values lying on the magnetization curve. As H decreases B decreases, but along a different curve. When H has been decreased to zero (state 2), a magnetic field still remains. This magnetic field Br is called the remanence. Continuing with changes in the same direction, the field H is now established in the reverse direction, and b continues to decrease reaching the value zero at state 3. The corresponding magnetic intensity Hc is called the coercive force. With further change in the same direction we reach state 4, where B and H both have directions opposite to their directions in state 1. If we now reverse the direction of change of H we trace out the lower curve back to the original state 1.

This closed B-H curved is called a Hysteris loop (Figure 3.63) and the phenomenon that the magnetization is not retraced is referred to as hysteresis (or lagging of the magnetic effect behind the magnetic field).

The cause of hysteresis has been traced to the fact that domain boundaries, instead of shifting freely when H and M change tend to become stuck at crystal imperfections.

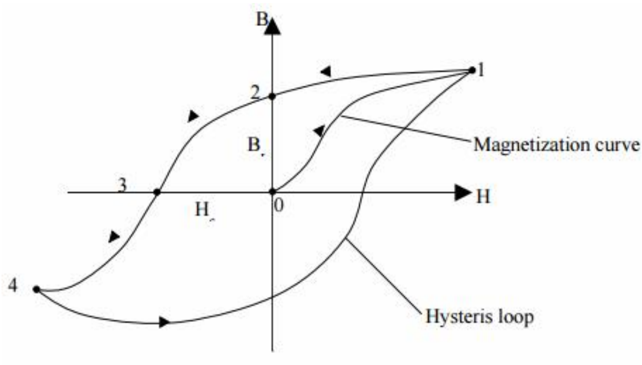


Figure 3.63

Hysteresis makes possible the existence of permanent magnetism. A good material for a permanent magnet should have both a large remanence Br so that the magnet will be strong and a large coercive force Hc so that the field will not be greatly reduced by modest values of reverse magnetic intensity.

Because of hysteresis, the B-H relationship in a ferromagnetic material is always dependent on the history of the material. Such material has a memory, a fact that is exploited in magnetic types.

Chapter 4 OSCILLATIONS

4.1. THE HARMONIOUS OSCILLATIVE MOTION

4.1.1 Kinematics of harmonious oscillations

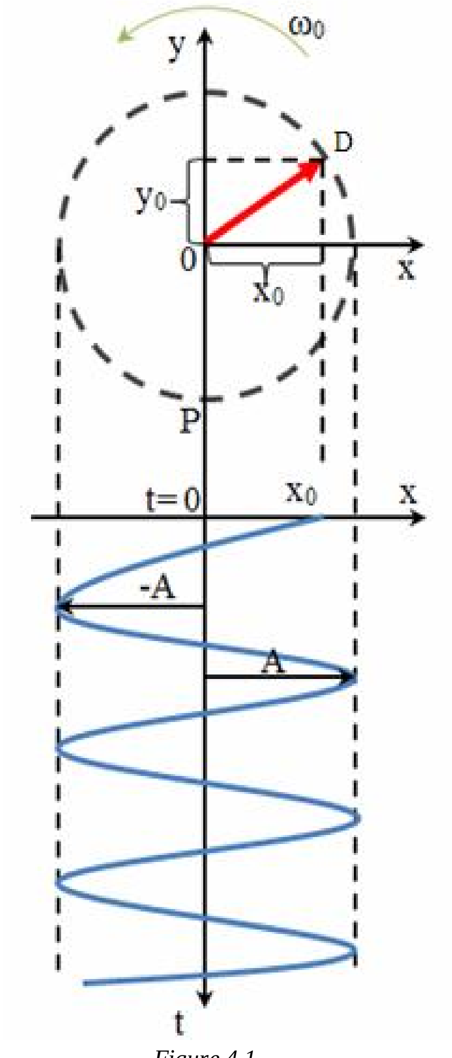


Figure 4.1

Processes, repeated over time are called vibrations.

Depending on the nature of vibrational process and the excitation mechanism are: mechanical oscillations (oscillations of pendulums, strings, buildings, a ground surface etc.); electromagnetic oscillations (alternating current oscillations, oscillations of vectors and $\vec{E}$ and $\vec{B}$ in an electromagnetic wave etc.); electromechanical oscillations (oscillations of a membrane of phone, a loudspeaker diaphragm, etc.); vibrations of the nuclei and molecules as a result of a thermal motion in atoms.

Consider the segment [OD] (the radius vector), perform a rotational movement around the point 0. Length | AP | = A. Rotation occurs at a constant angular velocity ω_0. Then the angle φ between the radius vector and the axis of x varies with time as

$$\varphi = \omega_0 t + \varphi_0\,, \qquad (4.1)$$

where φ_0 - angle between [OD] and the x-axis at time t =0. Projection of [OD] on the x-axis at time t = 0 (Figure 4.1).

$$x_0 = A\cos\varphi_0\,, \quad (t=0)\,, \qquad (4.2)$$

and at any given time

$$x = A\cos\varphi = A\cos(\omega_0 t + \varphi_0) \quad (\forall t) \;. \tag{4.3}$$

Thus, the projection of [OD] on the x-axis oscillates occurring along the x axis, and these fluctuations are described by cosine law (Eq. (4.2)).

Vibrations, which are described by the cosine

$$x = A\cos(\omega_0 t + \varphi_0)\,, \tag{4.4}$$

or a sine

$$y = A\cos(\omega_0 t + \varphi_0)\,. \tag{4.5}$$

called **harmonic**.

Harmonic oscillations are periodic, as value of $x(y)$ is repeated at regular intervals.

If [OD] s is the lowest position in the picture, ie Point D is the point P, then its projection on the$x$-axis is zero. We call this state of [OD] equilibrium position. Then we can say that the value of x describes the displacement of the vibrating point of equilibrium. The maximum displacement from the equilibrium position is called the **amplitude** fluctuations

$$x_{\max} = \pm A\,. \tag{4.6}$$

Quantity

$$\varphi = \omega_0 t + \varphi_0\,, \tag{4.7}$$

which is under the sign of the cosine of the phase is called. Phase determines the displacement from equilibrium at an arbitrary time t. Phase at the initial time $t = 0$, equal to φ_0 is called the initial phase (Figure 4.2).

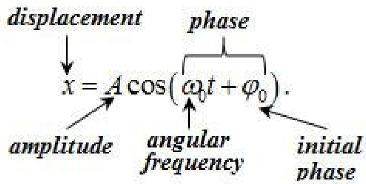


Figure 4.2

The length of time for which is made one complete oscillation is called the oscillation period T. The number of oscillations per unit of time is called the oscillation frequency ν

$$T = \frac{1}{\nu}\,. \tag{4.8}$$

After a time interval equal to the period T, i.e, an increase in the argument of the cosine $\omega_0 T$, movement is repeated, and the cosine takes the old value

$$\cos(\omega_0 t+\varphi_0)=\cos(\omega_0(t+T)+\varphi_0)\,. \qquad (4.9)$$

Since the period of cosine is equal 2π, hence, $\omega_0 T = 2\pi$ (Figure 4.3)

$$\omega_0=\frac{2\pi}{T}=2\pi\nu\,, \qquad (4.10)$$

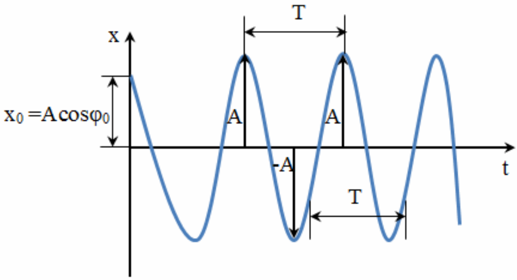


Figure 4.3 - Figure harmonic oscillation

A - amplitude, *T* - period, *x* - displacement, *t* - time.
thus, ω_0 - is the number of oscillations of the body for the 2π seconds. ω_0 - circular or **angular frequency**.

Speed oscillating point is found by differentiating equation displacement $x(t)$ in time

$$\upsilon=\frac{dx}{dt}=\frac{d(A\cos(\omega_0 t+\varphi_0))}{dt}=-A\omega_0\sin(\omega_0 t+\varphi_0)=A\omega_0\cos\left(\omega_0 t+\varphi_0+\frac{\pi}{2}\right) \qquad (4.11)$$

$$\upsilon_{\max}=A\omega_0,$$

i.e speed υ is different in phase from the displacement *x* on π/2.

Acceleration - the first derivative of the velocity (the second derivative of the displacement) over time

$$a=\frac{d\upsilon}{dt}=\frac{d(-A\omega_0\sin(\omega_0 t+\varphi_0))}{dt}=-A\omega_0^2\cos(\omega_0 t+\varphi_0)=A\omega_0^2\cos(\omega_0 t+\varphi_0+\pi) \qquad (4.12)$$

$$a_{\max}=A\omega_0^2,$$

i.e. acceleration a different from displacement in phase on π (Figure 4.4).

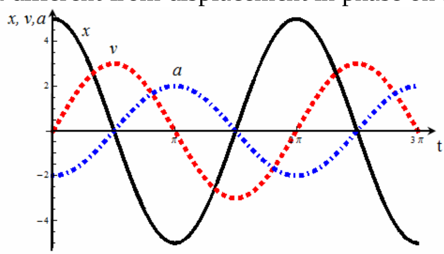


Figure 4.4

Plot $x(t)$, $\upsilon(t)$ and $a(t)$ in the same coordinate system (for simplicity we take $\varphi_0 = 0$ and $\omega_0 = 1$).

Free or **own** called vibrations that occur in the system to its own after it was removed from the equilibrium position.

Spring pendulum. Elastic and quasi-elastic forces. The equation of a vibrating spring

Consider a body of mass *m*, mounted on a spring with spring constant *k* (spring mass is neglected). Stretch the spring by *x*. Then Hooke's law on the body will act elastic force F_{el} (Figure 4.5).

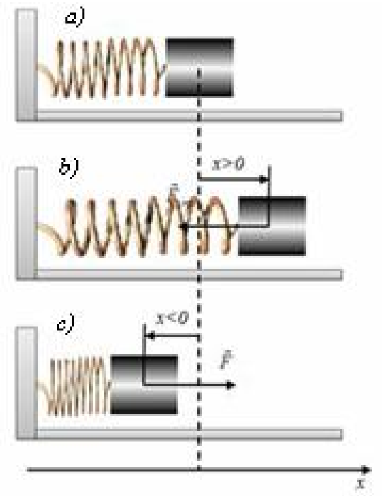


Figure 4.5

1. amount of force is proportional to the deviation of the system from equilibrium

$$F \sim x. \tag{4.13}$$

2. direction opposite to the direction of the force displacement, ie force is always directed towards the equilibrium position (at $x > 0$, $F_{el} < 0$, при $x < 0$, $F_{el} > 0$).

3) In equilibrium $x = 0$ и $F_{el} = 0$.

On a Hooke's law

$$F_{el} = -kx. \tag{4.14}$$

System consisting of a material point of mass *m* and absolutely elastic spring with spring constant *k*, which may be free oscillations of the pendulum is called.

We write Newton's second law for Figure 4.5. b

$$F_{bl} = ma, \tag{4.15}$$

$$F_{el} = -kx, \tag{4.16}$$

$$ma = -kx, \tag{4.17}$$

$$a = -A\omega_0^2 \cos(\omega_0 t + \varphi_0) = -\omega_0^2 x, \tag{4.18}$$

$$-m\omega_0^2 x = -kx \Rightarrow m\omega_0^2 = k, \tag{4.19}$$

i.e

$$\omega_0 = \sqrt{\frac{k}{m}}, \tag{4.20}$$

then

$$\nu = \frac{1}{2\pi}\omega_0 = \frac{1}{2\pi}\sqrt{\frac{k}{m}}, \tag{4.21}$$

and

$$T = \frac{1}{\nu} = 2\pi\sqrt{\frac{m}{k}} \cdot \qquad (4.22)$$

If power is not in their nature elastic, but subject to the law $F = -kx$, it is called quasi-elastic force.

Obtain the equation of the pendulum. We take into account in the record of the second law of Newton, that

$$a = \frac{d^2x}{dt^2}, \qquad (4.23)$$

then

$$ma = -kx, \qquad (4.24)$$

$$m\frac{d^2x}{dt^2} + kx = 0, \qquad (4.25)$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0, \qquad (4.26)$$

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \cdot \qquad (4.27)$$

- differential equation point oscillates (differential equation of the pendulum).

Solution of differential equations:

$$x = A\cos(\omega_0 t + \varphi_0) \qquad (4.28)$$

-oscillating point equation (the equation of a vibrating spring).

$$\omega_0 = \sqrt{\frac{k}{m}} \qquad (4.29)$$

- natural frequency of oscillations.

4.1.2 Mathematical and physical pendulums

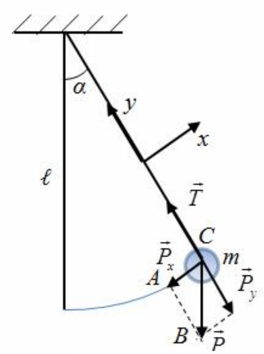


Figure 4.6

Periods of oscillation of mathematical and physical pendulum

Mathematical pendulum - point mass suspended on a weightless inextensible thread, and oscillate in a vertical plane under the influence of gravity (Figure 4.6).

Material point - a body whose mass is concentrated in the center of mass and size in terms of this problem can be neglected.

Mathematical pendulum at fluctuations is moved along a circular arc radius l.His movement is subject to the laws of rotational motion.

The basic equation of rotational motion can be written as

$$\overrightarrow{M} = I\vec{\varepsilon} \tag{4.30}$$

M - the moment of forces, I – a moment of inertia, ε – a angular acceleration.

$$\overrightarrow{M} = (\vec{T} + \vec{P})l\,, \tag{4.31}$$

$$I = ml^2\,, \tag{4.32}$$

$$\varepsilon = \frac{d^2a}{dt^2}\cdot \tag{4.33}$$

Resultant force $\vec{T}$ and $\vec{P}$ is equal to $\overrightarrow{P_x}$.

Of the triangle *ABC*

$$P_x = -P\sin\alpha = -mg\sin\alpha\,, \tag{4.34}$$

i.e

$$P_x = -k\sin\alpha\,, \tag{4.35}$$

Thus, the oscillations of a mathematical pendulum occur under the quasi-elastic force - gravity.

Then (4.30) can be written as

$$-mgl\sin\alpha = ml^2\frac{d^2a}{dt^2} \tag{4.36}$$

The minus sign takes into account that the vectors $\overrightarrow{M}$ and $\vec{a}$ have opposite directions (the angle of rotation can be regarded as a pseudovector of angular displacement $\vec{a}$, the vector direction is defined by the rule of the right propeller, because of the minus sign is guided $\vec{a}$ in the opposite side).

Having reduced in (4.36) on m and l we will gain

$$\frac{d^2a}{dt^2} + \frac{g}{l}\sin\alpha = 0\cdot \tag{4.37}$$

At small angles of oscillations α = 5 ÷6°, $\sin\alpha \approx \alpha$, we will gain

$$\frac{d^2a}{dt^2} + \frac{g}{l}\sin\alpha = 0\cdot \tag{4.38}$$

Label input

$$\omega_0^2 = \frac{g}{l}\,, \tag{4.39}$$

obtain the differential equation for the oscillations of a mathematical pendulum

$$\frac{d^2a}{dt^2} + \omega_0^2\alpha = 0\cdot \tag{4.40}$$

Its solution:

$$\alpha = \alpha_0 \cos(\omega_0 t + \varphi_0) \tag{4.41}$$

-the equation of a mathematical pendulum.

One can see that the angle α varies as the cosine. α_0 - amplitude, ω_0 - cyclic frequency, φ_0 - an initial phase.

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{4.42}$$

- the period of oscillationmof a mathematical pendulum.

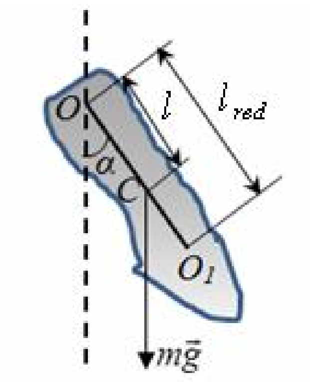


Figure 4.7

Physical pendulum - solid, vibrating by gravity around a fixed horizontal axis that does not pass through the center of gravity of the body, called the axis of oscillation of the pendulum (Figure 4.7).

The basic equation for the rotational motion of a physical pendulum is written as

$$I\frac{d^2a}{dt^2} = -mgl\sin\alpha \cdot \tag{4.43}$$

At small angles of oscillation $\sin\alpha \approx \alpha$ and the equation motion has the form

$$\frac{d^2a}{dt^2} + \frac{mgl}{I}\alpha = 0 \cdot \tag{4.44}$$

Then we put

$$\omega_0^2 = \frac{mgl}{I} \tag{4.45}$$

get

$$\frac{d^2a}{dt^2} + \omega_0^2\alpha = 0 \tag{4.46}$$

-the differential equation of a physical pendulum.

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{mgl}} \tag{4.47}$$

-the period of oscillation of a physical pendulum

Equating $T_{Phys} = T_{mat}$:

$$2\pi\sqrt{\frac{I}{mgl}} = 2\pi\sqrt{\frac{l}{g}}, \tag{4.48}$$

therefore, a mathematical pendulum with length

$$l_{red} = \frac{I}{ml}, \tag{4.49}$$

has the same period of oscillation, and this physical pendulum.

l_{red} - the reduced length of a physical pendulum - is the length of the a mathematical pendulum, the oscillation period coincides with the period of the physical.

4.1.3 The energy of the harmonic oscillations

By definition, the kinetic energy of a body of mass *m*, moving with speed $\vec{\upsilon}$ equal

$$E_K = \frac{m\upsilon^2}{2} = \frac{m\omega_0^2 A^2}{2}\sin^2\omega_0 t = \frac{m\omega_0^2}{2}\cos^2\left(\omega_0 t + \frac{\pi}{2}\right), \quad (4.50)$$

$$\cos^2\varphi = \frac{1}{2}(1+\cos 2\varphi), \quad (4.51)$$

$$E_K = \frac{m\omega_0^2 A^2}{2}\left(1+\cos 2\left(\omega_0 t + \frac{\pi}{2}\right)\right) = \frac{m\omega_0^2 A^2}{4}\left(1+\cos(2\omega_0 t + \pi)\right). \quad (4.52)$$

The potential energy is equal to

$$U = -\int_0^x F_q dx = \int_0^x kx dx = \frac{kx^2}{2} = \frac{kA^2}{2}\cos^2\omega_0 t = \frac{m\omega_0^2 A^2}{2}\cos^2\omega_0 t = \frac{m\omega_0^2 A^2}{2}(1+\cos 2\omega_0 t). \quad (4.53)$$

Total energy (Figure 4.8) is equal

$$W = E_k + U = \frac{m\omega_0^2 A^2}{2}(\sin^2\omega_0 t + \cos^2\omega_0 t) = \frac{m\omega_0^2 A^2}{2} = \frac{kA^2}{2}. \quad (4.54)$$

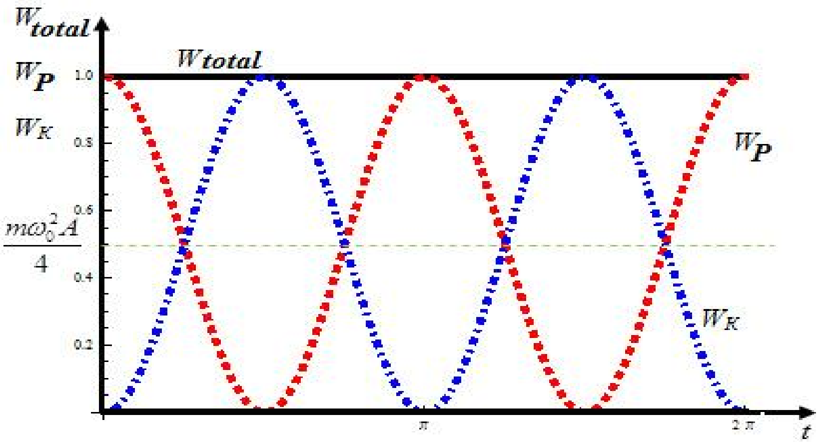


Figure 4.8

Quasi-elastic force is conservative, so the total energy of the harmonic motion is constant. In the process of oscillations is turning kinetic energy into potential energy and vice versa.

Fluctuations E_K and U have a frequency $2\omega_0$, ie twice the frequency of harmonic oscillations.

4.1.4 Addition of harmonic oscillations. Image fluctuations in vector diagram

1. Let oscillations are described by the equation

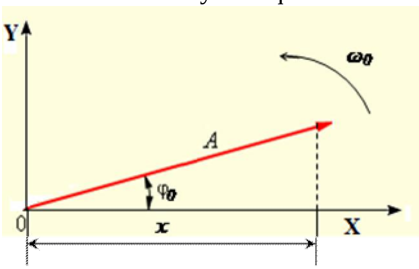


Figure 4.9

$$x = A\cos(\omega_0 t + \varphi_0) \tag{4.55}$$

Laid from point A on the vector length, making an angle φ_0 with Ox. If this vector to begin rotating with angular velocity ω_0, then the projection of the end of the vector will change with time as the cosine (4.55), i.e, harmonic motion can be described by a vector whose length is equal to the amplitude of the oscillations A, and the direction of the vector form the x-axis angle of the initial phase φ_0 (Figure 4.9).

Addition of two harmonic oscillations of the same direction and the same frequency (Figure 4.10).

$$x_1 = A_1\cos(\omega_0 t + \varphi_{10}), \tag{4.56}$$

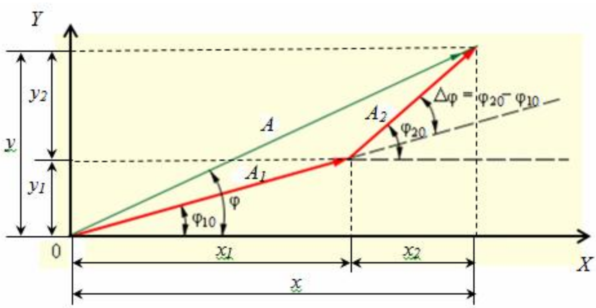


Figure 4.10

$$x_2 = A_2 \cos(\omega_0 t + \varphi_{20}), \tag{4.57}$$

$$x = x_1 + x_2 = A_1 \cos(\omega_0 t + \varphi_{10}) + A_2 \cos(\omega_0 t + \varphi_{20}). \tag{4.58}$$

The resulting vector $\vec{A}$ is equal to

$$\vec{A} = \vec{A_1} + \vec{A_2}. \tag{4.59}$$

And find on the parallelogram, its projection on the X axis equal to

$$X = X_1 + X_2. \tag{4.60}$$

Length of the resulting vector, or the amplitude of the resulting oscillation is on the law of cosines and equal

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_{20} - \varphi_{10})}. \tag{4.61}$$

The initial phase of the resulting oscillation is determined by the condition

$$tg\varphi_0 = \frac{Y_{10} + Y_{20}}{X_{10} + X_{20}} = \frac{A_1 \sin\varphi_{10} + A_2 \sin\varphi_{20}}{A_1 \cos\varphi_{10} + A_2 \cos\varphi_{20}}. \tag{4.62}$$

The addition of two harmonic oscillations with the same frequency and the same direction, the resulting motion is also a harmonic oscillation with the same period and an amplitude A, which lies within

$$|A_2 - A_1| \le A \le A_1 + A_2. \tag{4.63}$$

Fluctuations that have $\varphi_{10} = \varphi_{20}$, $A = A_1 + A_2$ are called in-phase.

Fluctuations that have $\varphi_{10} - \varphi_{20} = \pi$, $A = |A_2 - A_1|$ called antiphase.

If $A_1 = A_2$, when $\varphi_{10} = \varphi_{20}$, $A = 2A_1$, at $\varphi_{10} - \varphi_{20} = \pi$, $A = |A_2 - A_1| = 0$.

4.1.5 Beats

Beats - addition of oscillations with close frequencies $\omega_1 \approx \omega_2$.

With the addition of harmonic oscillations differ slightly in frequency resulting motion is a harmonic oscillation with pulsing amplitude. Such vibrations are called beats.

For simplicity, assume

$$A = A_1 = A_2,\ \ \varphi_{10} = \varphi_{20} = 0. \tag{4.64}$$

Then

$$\begin{cases} x_1 = A\cos\varphi_1 = A\cos\omega_1 t \\ x_2 = A\cos\varphi_2 = A\cos\omega_2 t \end{cases}, \tag{4.65}$$

where $\omega_1 - \omega_2 << \omega_1, \omega_2$

$$x = A\cos\omega_1 t + A\cos\omega_2 t = 2A\cos\left(\frac{\omega_1 + \omega_2}{2}t\right)\cos\left(\frac{\omega_1 - \omega_2}{2}t\right) \tag{4.66}$$

The resulting expression is the product of two oscillations.

Factor $\cos\left(\frac{\omega_1+\omega_2}{2}t\right)$ has a frequency an average of two terms of vibrations. ie close to their frequencies ω_1 and ω_2. The second factor $\cos\left(\frac{\omega_1-\omega_2}{2}t\right)$ has in virtue of proximity ω_1 and ω_2 low frequency, i.e large period. This allows us to consider the resulting motion as nearly harmonic oscillation with an average angular frequency $\omega=\frac{\omega_1+\omega_2}{2}$ and slowly varying amplitude $2A\cos\left(\frac{\omega_1-\omega_2}{2}t\right)$.

When $\varphi_1\approx\varphi_2$, $A_r\approx 2A$. After a interval $\frac{\pi}{\omega_1-\omega_2}$, one of the vibrations behind the other in phase by π and $A_r\to 0$. This gradual increase and decrease the amplitude of the resulting oscillation is called a beat (Figure 4.11).

If ω_1 and ω_2 are comparable, i.e can be found two numbers n_1 and n_2, that $\frac{T_2}{T_1}=\frac{\omega_1}{\omega_2}=\frac{n_1}{n_2}$, then after that interval of time $\tau=(n_1-n_2)\frac{2\pi}{\omega_1-\omega_2}=(n_1+n_2)\frac{2\pi}{\omega_1+\omega_2}$.

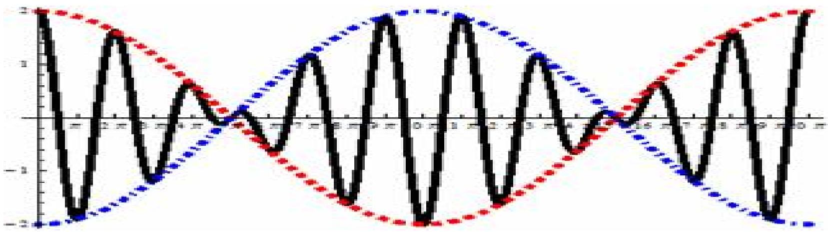


Figure 4.11

1, 2 – graph the slowly varying amplitude.

3 – graph of the resulting oscillation.

The arguments of both factors in (4.67) to change the whole (though different) number of times 2π, their product will take the same value as in the beginning of period τ. The value of τ is the time period of the resulting oscillation. If the frequency is not comparable, the resulting oscillation will non-periodic.

4.1.6 Addition perpendicular vibrations

Consider the result of the addition of two harmonic oscillations of the same frequency $\omega_1 = \omega_2 = \omega$, occurring in mutually perpendicular directions along the *x* and *y* axes (Figure 4.12).

$$\begin{cases} x = A\cos(\omega t + \varphi_{10}) \\ y = B\cos(\omega t + \varphi_{20}) \end{cases}. \qquad (4.68)$$

a) Let $\varphi_{10} = \varphi_{20}$. (4.69)

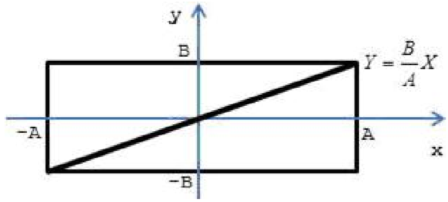


Figure 4.12

Then $\frac{x}{y} = \frac{A}{B}$ and $y = \frac{B}{A}x$ - trajectory the diagonal of a rectangle with sides 2A (*x*-axis) and 2B (*y*-axis) (Figure 4.13).

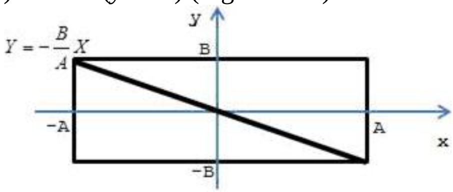


Figure 4.13

b) Let $\varphi_{10} = \varphi_{20} + \pi$ (Figure 4.14). Then $y = -\frac{B}{A}x$. (4.70)

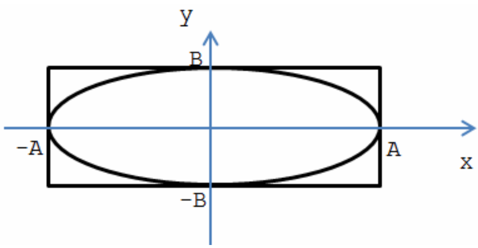


Figure 4.14

c) Let $\varphi_{10} = \varphi_{20} + \pi/2$, $\Delta\varphi = \frac{\pi}{2}$, (4.71)

$$\frac{x}{A} = \cos(\omega t + \varphi_{10}), \tag{4.72}$$

$$\frac{x}{B} = \cos\left(\omega t + \varphi_{10} + \frac{\pi}{2}\right) = \sin(\omega t + \varphi_{10}), \tag{4.73}$$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = \cos^2(\omega t + \varphi_{10}) + \sin^2(\omega t + \varphi_{10}), \tag{4.74}$$

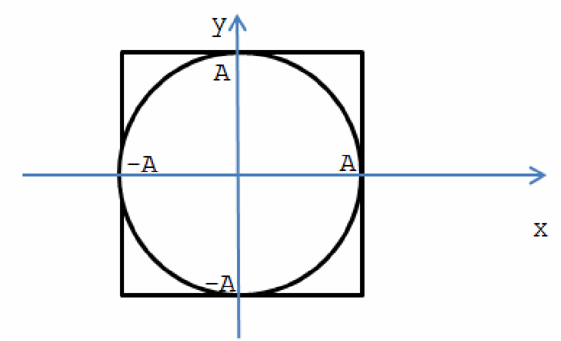


Figure 4.15

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \text{ - ellipse.} \tag{4.75}$$

If $A = B$ - circle.

d) $\varphi_{10} = \varphi_{20} - \pi/2$ - ellipse, but changes the direction of the circuit.

e) Arbitrary φ_{10} and φ_{20} - also an ellipse with equation

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos(\varphi_{20} - \varphi_{10}) = \sin^2(\varphi_{20} - \varphi_{10}). \tag{4.76}$$

In the general case

1. $\varphi_{20} - \varphi_{10} = 2k\pi$, (4.77)

$$\frac{x}{A} - \frac{y}{B} = 0, \tag{4.78}$$

$$y = \frac{B}{A}x. \tag{4.79}$$

2. $\Delta\varphi = (2k+1)\pi$, (4.80)

$$\frac{x}{A} + \frac{y}{B} = 0, \tag{4.81}$$

$$y = -\frac{B}{A}x. \qquad (4.82)$$

3. $\Delta\varphi = \pm\pi / 2k$, (4.83)

$$\cos\alpha = 0, \quad \sin\alpha = 1, \qquad (4.84)$$

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1. \qquad (4.85)$$

f) **Lissajous figures** (Figure 4.16)

In the case where the frequency of oscillation perpendicular, in which both involved the point under consideration are as integers, the trajectory is a complex curves, known as Lissajous figures. The shape of the curve depends on the ratio of amplitude, frequency and phase difference summed vibrations.

Ratio of frequency foldable vibrations is the ratio of the number of intersections of the Lissajous figures with lines parallel to the axes. By type of Lissajous figures can be determined from the known unknown frequency, or to determine the frequency ratio ω_1 and ω_2.

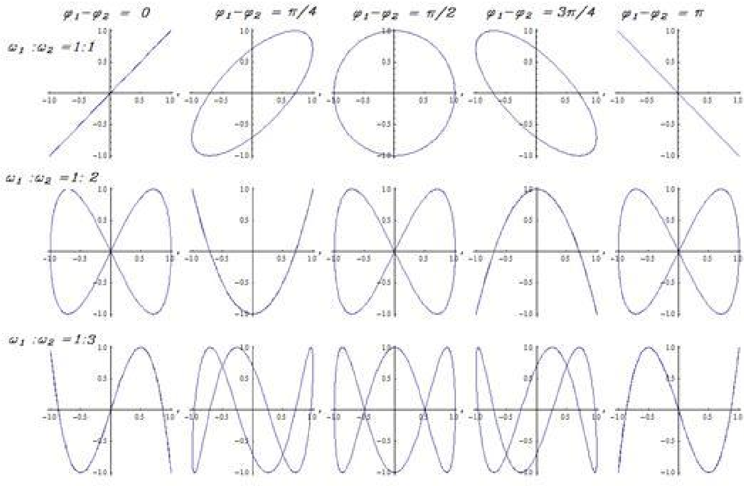


Figure 4.16

4.2 DAMPED OSCILLATORS

4.2.1 Damped oscillations. Damping rate. Logarithmic decrement. Quality

Free vibrations of engineering systems in the real world takes place when they are forces of resistance. The effect of these forces leads to a decrease in the amplitude of the fluctuating value.

Fluctuations with amplitude due to energy losses of real oscillating system decreases over time are called damped.

The most common cases where the resistance force is proportional to the velocity of the motion

$$F_r \sim \upsilon\,,\; F_r = -r\upsilon\,, \tag{4.86}$$

where r - coefficient of resistance of the medium. The minus sign indicates that the F_r is directed in the direction opposite the velocity.

We write the equation of vibrations at the point oscillating in a medium whose resistance coefficient r. According to Newton's second law

$$ma = -kx - r\upsilon, \tag{4.87}$$

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{k}{m}x = 0, \tag{4.88}$$

$$\omega_0^2 = \frac{k}{m}, \tag{4.89}$$

$$\beta = \frac{r}{2m}, \tag{4.90}$$

where β - coefficient of damping. This ratio describes the rate of damping. In the presence of the forces of resistance the energy of the oscillating system will gradually decrease, the oscillations are damped.

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = 0 \tag{4.91}$$

- Differential equation of damped oscillations.

$$x = Ae^{-\beta t}\cos(\omega t + \varphi_0) \tag{4.92}$$

-The equation of damped oscillations.

ω - the frequency of the damped oscillations:

$$\omega = \sqrt{\omega_0^2 - \beta^2}\,. \tag{4.93}$$

The period of the damped oscillations

$$T = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}\,. \tag{4.94}$$

Damped oscillations in the rigorous treatment are not periodic. Therefore, the period of damped oscillations can say when β is small. If you are weak damping $\beta \to 0$, then

$$T = T_0 = \frac{2\pi}{\omega_0}\,. \tag{4.95}$$

Damped oscillations can be considered as harmonic oscillations whose amplitude varies exponentially

$$A = A_0 e^{-\beta t}. \tag{4.96}$$

In equation (4.66) A_0 and φ_0 - arbitrary constants that depend on the choice of the point in time at which we consider vibrations.

$$t_0 = 0,\ x = A_0 \cos\varphi_0. \tag{4.97}$$

Consider the oscillations for at some time τ, for which the amplitude is reduced by a factor *e*

$$A(t) = A_0 e^{-\beta t}, \tag{4.98}$$

$$A(t+\tau) = A_0 e^{-\beta(t+\tau)}, \tag{4.99}$$

$$\frac{A(t)}{A(t+\tau)} = \frac{A_0 e^{-\beta t}}{A_0 e^{-\beta(t+\tau)}} = e^{\beta\tau}, \tag{4.100}$$

$$\frac{A(t)}{A(t+\tau)} = e = e^{\beta\tau} \Rightarrow \beta\tau = 1, \tag{4.101}$$

$\beta = \dfrac{1}{\tau}$, where τ - relaxation time.

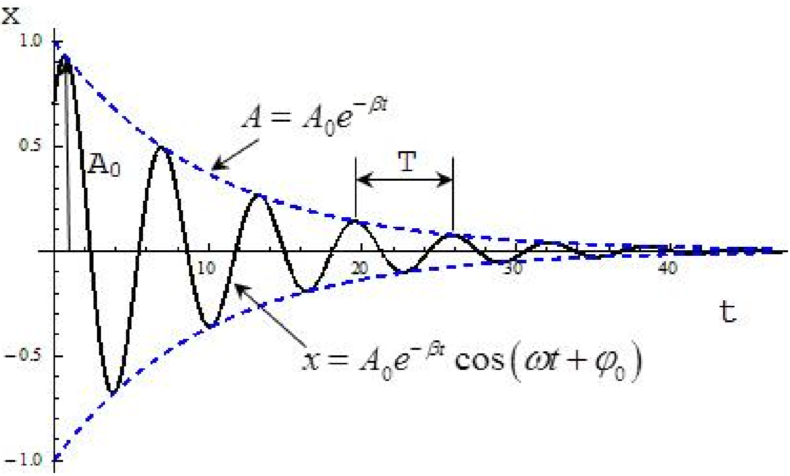


Figure 4.17

The damping coefficient β is inversely proportional time, during which the amplitude is reduced by a factor e. However, the damping factor is not enough to describe the damping vibrations. It is therefore necessary to introduce this feature for vibration damping, which includes the time of one vibrations (Figure

4.15). This characteristic is the **decrement** damping *D*, which is the ratio of the amplitudes, which are separated in time by a period:

$$D = \frac{A(t)}{A(t+T)} = \frac{A_0 e^{-\beta t}}{A_0 e^{-\beta(t+T)}} = e^{\beta T}. \tag{4.102}$$

Logarithmic decrement is equal to the logarithm of *D*:

$$\lambda = \ln D = \beta T = \ln \frac{A(t)}{A(t+T)}, \tag{4.103}$$

$$\lambda = \beta T = \frac{T}{\tau} = \frac{1}{N_e}. \tag{4.104}$$

Damping constant is inversely proportional to the number of vibrations that result in decreased amplitude of e. Damping constant - constant for a given system magnitude. Another feature of the system is the vibrational quality factor *Q*.

$$Q = \frac{\pi}{\lambda} = \pi N_e = \frac{\pi}{\beta T}. \tag{4.105}$$

Quality factor is proportional to the number of vibrations committed system during the relaxation time τ. Quality factor *Q* vibrating system is a measure of relative dissipation of energy.

Quality factor *Q* oscillating system is a number indicating how many times the force of elasticity greater resistance forces.

$$Q = \frac{F_{biast}}{F_{resist}} = \frac{kx_{\max}}{r\upsilon_{\max}} = \frac{m\omega_0^2 A}{r\omega_0 A} = \frac{m\omega_0}{r} = \frac{\sqrt{mk}}{r}. \tag{4.106}$$

The higher the quality factor, the slower the damping, the damped oscillations close to free harmonic.

4.2.2 Forced oscillations. Resonance

In many cases there is a need for systems that commit sustained oscillations. Get undamped oscillations in the system can compensate for the loss of energy when, acting on a system of periodically changing force.

Let

$$F_{ariv} = F_0 \cos \omega t. \tag{4.107}$$

We write the expression for the equation of motion of a particle undergoing harmonic oscillatory motion by the driving force. According to Newton's second law:

$$ma = -kx - r\upsilon + F_0 \cos \omega t, \tag{4.108}$$

$$\omega_0^2 = \frac{k}{m}, \tag{4.108,a}$$

$$\beta = \frac{r}{2m}, \tag{4.109}$$

$$\frac{F_0}{m} = f_0, \tag{4.110}$$

$$\frac{d^2x}{dt^2} + 2\beta\frac{dx}{dt} + \omega_0^2 x = f_0\cos\omega t, \tag{4.111}$$

Differential equation of forced oscillations. This differential equation is linear inhomogeneous. His solution is the total solution of the homogeneous equation and a particular solution of the inhomogeneous equation:

$$X = X_{tot.n} + X_{part.\inf}, \tag{4.112}$$

$$X_{tot.n} = A_0 e^{-\beta t}\cos\omega t. \tag{4.113}$$

We find a particular solution of the inhomogeneous equation. To do this, we rewrite (4.111) as follows:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_0 e^{i\omega t}. \tag{4.114}$$

The particular solution of this equation sought in the form:

$$x = Ae^{i\gamma t}. \tag{4.115}$$

Then

$$\dot{x} = i\gamma Ae^{i\gamma t}, \tag{4.116}$$

$$\ddot{x} = -\gamma^2 Ae^{i\gamma t}. \tag{4.117}$$

In (4.114):

$$Ae^{i\gamma t}(-\lambda^2 + 2i\beta\gamma + \omega_0^2) = f_0 e^{i\omega t}, \tag{4.118}$$

because holds for any *t*, we must have $\gamma = \omega$, hence

$$A = \frac{f_0}{-\omega^2 + 2i\beta\omega + \omega_0^2} \,|\, \frac{\cdot\left(\omega_0^2 - \omega^2 - 2i\beta\omega\right)}{\cdot\left(\omega_0^2 - \omega^2 - 2i\beta\omega\right)} = f_0\frac{(\omega_0^2 - \omega^2) - 2i\beta\omega}{(\omega_0^2 - \omega^2) + 4i\beta^2\omega^2}, \tag{4.119}$$

This complex number is conveniently written as

$$x = Ae^{-i\varphi}, \tag{4.120}$$

where *A* is defined by (3 below), and φ - by the formula (4.123), therefore, the solution (4.114) in the complex form is

$x = Ae^{-i(\omega t+\varphi)}$.

Its real part is the solution of equation (4.111) is:

$$X_{partic} = Ae^{-\beta t}\cos(\omega t + \varphi), \tag{4.121}$$

where

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - \omega^2) + 4i\beta^2\omega^2}}, \tag{4.122}$$

$$tg\varphi = \frac{2\beta\omega}{\omega_0^2 - \omega^2}, \tag{4.123}$$

X_{th} term plays a significant role only in the initial stage of the establishment of of oscillations as long as the amplitude of the forced oscillations reaches the value defined by equation (4.123). In steady state forced oscillations occur with a frequency ω and are harmonic. The amplitude (4.123) and phase (4.122) induced oscillations depend on the frequency of the driving force.

At a certain frequency of the driving force amplitude can reach very high values. The sharp increase in the amplitude of the forced oscillations in the frequency of the driving force to the natural frequency of the mechanical system is **resonance**.

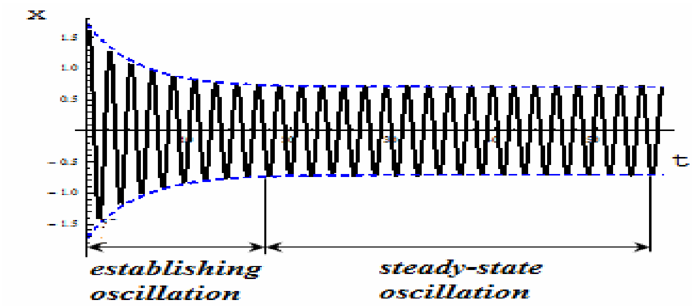


Figure 4.18

Frequency ω of the driving force at which a response is called resonance. In order to find the value ω_{res} need to find the condition of the maximum amplitude. To do this, you need to determine the minimum condition of the denominator in (4.122) (i.e the study (4.122) on the extreme).

$$\frac{\partial}{\partial\omega}\left((\omega_0^2 - \omega^2)^2 - 4\beta^2\omega^2\right) = 0, \tag{4.124}$$

$$-4\omega_0^2\omega + 4\omega^3 + 8\beta^2\omega = 0, \tag{4.125}$$

$$\omega_{res} = \sqrt{\omega_0^2 - 2\beta^2}. \tag{4.126}$$

The amplitude of the fluctuating value of the frequency of the driving force is called the **resonance curve**. Resonance curve will be higher, the lower the damping factor β and decreasing β, the maximum resonance curves mixed right. If β = 0 (Figure 4.19), then $\omega_{res} = \omega_0$.

When $\omega \to 0$, all the curves come to value f_0/ω_0^2 - static deflection.

$\frac{f_0}{\omega_0^2} = \frac{F_0}{m\omega_0^2} = \frac{F_0}{k}$.

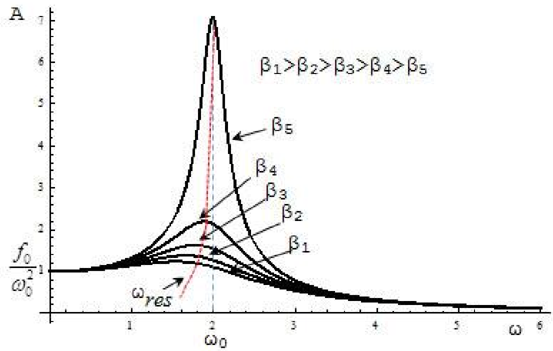


Figure 4.19

4.3 ELASTIC WAVES

4.3.1 The Waves in an elastic medium

If the vibrating body (tuning fork, strings, membranes, etc.) is in an elastic medium, it leads to the oscillatory motion of a particle in contact with the environment, resulting in the body adjacent to this element of the medium, periodic deformation (for example, compression and tension). In these strains in the environment appear elastic forces tend to return the elements of the environment to its original state of equilibrium, due to the interaction of neighboring elements of the environment, the elastic deformation will be transferred from one part of the environment to other, more remote from the oscillating body.

Thus, the periodic deformations caused in any place of the elastic medium, will be distributed in the environment with a rate that depends on its physical properties. The particles of the medium oscillates about equilibrium positions. From one part of the environment to another is transmitted only the state of deformation.

The propagation of the vibrational motion in the environment is called the wave process, or just **wave**. Depending on the nature of emerging with the elastic deformation distinguish longitudinal and transverse waves. In **longitudinal waves** (Figure 4.20), particles of the medium oscillate along the direction of oscillation. In **transverse waves (**Figure 4.21), particles of the medium oscillate perpendicular to the direction of propagation.

Liquid and gaseous media do not have elastic shear, so they are excited only longitudinal waves propagating in the form of alternating compressions and rarefactions. Waves excited on the surface, are transverse, they are due to the earth's gravity.

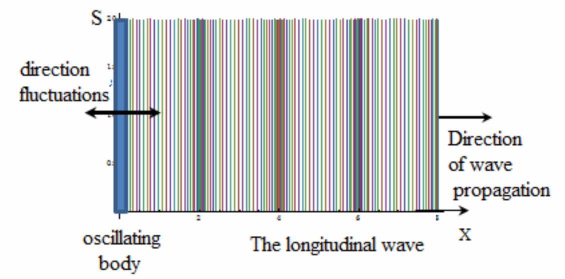


Figure 4.20

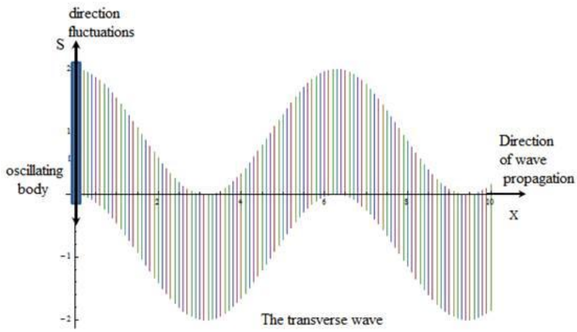


Figure 4.21

Suppose that a point source wave began excite vibrations in the environment at time t = 0, after a time t is the oscillation spread in different directions at a distance $r = \upsilon_i t$, where υ_i - wave velocity in this direction. The surface to which the vibration comes at some point in time is called the **wave front**. The shape of the wave front is defined by the configuration of the source of vibrations and the properties of the medium. In homogeneous media, the propagation velocity of the wave is the same everywhere. Medium called isotropic if the speed is the same in all directions. Wave front from a point source of oscillations in a homogeneous and isotropic medium has the form spheres, and such waves are called **spherical**.

In a heterogeneous and isotropic (anisotropic) environment, as well as from non-point sources of vibration wave front has a complex shape. If the wave front is a plane, and this form is saved as wave propagation in the medium, the wave is **flat**.

Surface waves, point of of which oscillate in the same phases, called **wave or phase surfaces**.

Graph showing the distribution in the medium fluctuating value at a given time is called the **waveform** (Figure 4.22).

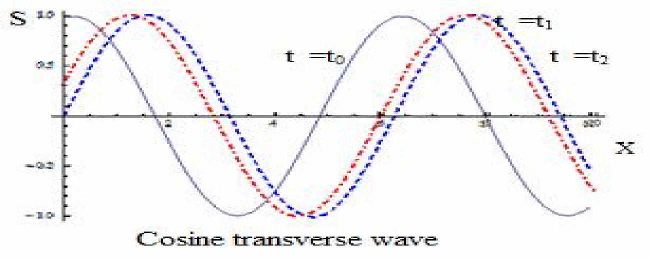


Figure 4.22

The equation of a plane wave

Wave equation allows us to find the displacement from equilibrium oscillating point (*x, y, z*) at time *t*.

$$S = S(x, y, z)\,. \qquad (4.127)$$

Let vibrations points lying in the plane $x = 0$, are due to the cosine law

$$S(0,t) = A\cos(\omega t + \varphi_0)\,. \qquad (4.128)$$

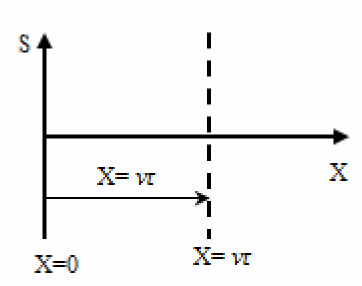


Figure 4.23

Find the form of vibrations of points in a plane corresponding to an arbitrary value of *x*. In order to go from $x = 0$ to the plane wave needs time $\tau = \dfrac{x}{\upsilon}$, υ – speed of wave propagation, therefore, the fluctuations of the particles lying in the *x*, will lag in time by τ of oscillations of the particles in the $x = 0$; i.e, have the form

$$S(x,t) = A\cos[\omega(t-\tau) + \varphi_0] = A\cos\left[\omega\left(t - \frac{x}{\upsilon}\right) + \varphi_0\right]. \qquad (4.129)$$

Equation of the incident, the traveling wave (equation of waves propagating in the direction of the axis *X*).

S-shift of the point of equilibrium in the plane at a distance x from the source of vibrations, *A* - wave amplitude, φ_0 - initial phase.

For a single wave can choose *S* and *t*, so that $\varphi_0 = 0$. For several waves can not. If the wave propagates in the direction of decreasing *x* coordinate, the oscillations in the *x* plane will begin earlier on $\tau = \frac{x}{\upsilon}$, than in the $x = 0$. Then the equation of the reflected wave can be written as

$$S(x,t) = A\cos\left[\omega\left(t + \frac{x}{\upsilon}\right) + \varphi_0\right]. \qquad (4.130)$$

4.3.2 The concept of the phase velocity. Relation between the phase and group velocities

1. Fix some value of the phase, which stands in the equation of a traveling wave

$$\omega\left(t - \frac{x}{\upsilon}\right) + \varphi_0 = const. \qquad (4.131)$$

It follows from it relation between the time t and the place's, in which the phase has a fixed value. Flows out of it value $\frac{dx}{dt}$ gives the rate at which the value is moving phase. Differentiating (4.131), we obtain

$$dt - \frac{1}{\upsilon}dx = 0, \qquad (4.132)$$

$$\frac{dx}{dt} = \upsilon, \qquad (4.133)$$

$$\frac{2\pi}{\lambda} = k, \qquad (4.134)$$

$$\lambda = \upsilon T, \qquad (4.135)$$

$$k = \frac{\omega}{\upsilon}, \qquad (4.136)$$

$$S = A\cos(\omega t - kx + \varphi_0), \qquad (4.137)$$

k - wave number, λ – wave length.

Thus, the velocity υ in the equation of the propagating wave is phase velocity, i.e, it shows how fast spread phase of the wave (**speed of the phase**).

In all real wave processes have to deal with more complex waves with non-sinusoidal character. Such a complex wave can be represented as the sum of cosine or sine waves, or a group of such waves. In reality, there is the movement of groups of waves, each of which differs from the other in frequency. At any given time you can find the point at which there is a maximum vibration resulting overlay of these waves. At this point, any phase of the waves is the same. This point is called the center of the waves.

The position of the center of the waves varies with time. This point corresponds to the maximum energy of the oscillating waves. The energy of the oscillating wave group moved at a speed equal to the speed of the center of a group of waves. This velocity is called the **group velocity**. It is denoted by *u*.

2. The link between the group and phase velocities.

To find this relationship, we use the fact that the center of the group phase of the waves all waves are the same. The group velocity is

$$u = \frac{\Delta\omega}{\Delta k}, \tag{4.138}$$

$$\omega = \upsilon k, \tag{4.139}$$

$$u = \frac{d(\upsilon k)}{dk} = \upsilon + k\frac{d\upsilon}{dk}, \tag{4.140}$$

$$\frac{d\upsilon}{dk} = \frac{d\upsilon}{d\lambda}\cdot\frac{d\lambda}{dk}, \tag{4.141}$$

$$\lambda = \frac{2\pi}{k} \Rightarrow \frac{d\lambda}{dk} = -\frac{2\pi}{k^2} = -\frac{\lambda}{k}, \tag{4.142}$$

$$\frac{d\upsilon}{dk} = -\left(\frac{d\upsilon}{d\lambda}\right)\frac{\lambda}{k}, \tag{4.143}$$

$$u = \upsilon - \lambda\frac{d\upsilon}{d\lambda}. \tag{4.144}$$

Depending on the sign of $\frac{d\upsilon}{d\lambda}$ the group velocity u may be less or greater than the phase velocity υ. In the absence of dispersion $\frac{d\upsilon}{d\lambda} = 0$ the group velocity coincides with the phase.

4.3.3 Electromagnetic Oscillations and Waves of the resonant circuit. The natural oscillations in resonant circuit. Thomson formula. Damped and forced oscillations

1. Free oscillations in r.c.

Oscillating circuit (R.C.) is a chain consisting of capacitors and inductors. Under certain conditions r.c. can cause electromagnetic oscillations of the charge, current, voltage and power (Figure 4.24).

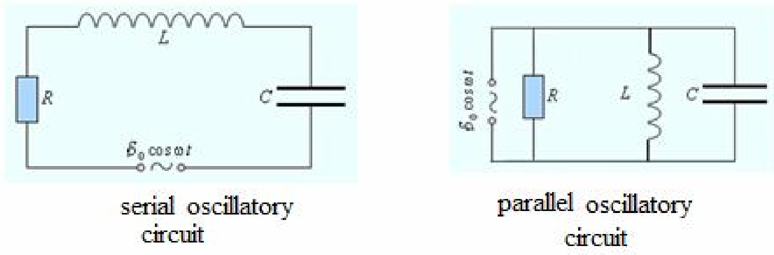


Figure 4.24

Consider the circuit shown in Figure 4.25. If you put the key in position 1, there will be a charge on the capacitor and its plates will charge Q and voltage UC. If you then put the key in position 2, the capacitor will discharge, a current will flow in the circuit, and the energy of the electric field between the plates of a capacitor, will be transformed into the energy of the magnetic field is concentrated in the inductor L. The presence of the inductor causes the current in the circuit increases, not immediately, but gradually due to the phenomenon of self-induction. As the discharge of the capacitor charge on its plates will be reduced, the current in the circuit to increase. Maximum loop current to reach at a charge equal to zero on the plates.

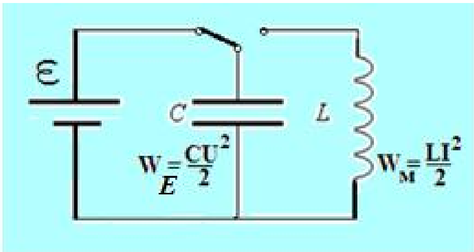


Figure 4.25

Since then loop current will begin to decrease, but, due to the phenomenon of self-induction, it will be supported by the magnetic field of the coil, i.e at full capacitor discharge magnetic energy stored in the inductor, the energy will go into the electric field. Because of the loop current will begin recharging the capacitor and its plates will accumulate a charge opposite to the original. Recharging of the capacitor will be long until the power of the magnetic field coil goes into the energy of the electric field condenser. The process is then repeated in the opposite direction, and, thus, in the chain of any electromagnetic vibrations.

We write the 2-nd Kirchoff's considered r.c.

$$U_C = \varepsilon_S, \tag{4.145}$$

$$\frac{q}{C} = -L\frac{dI}{dt}, \tag{4.146}$$

$$\frac{q}{C} = -L\frac{dI}{dt}, \tag{4.147}$$

$$\frac{q}{C} = -L\frac{dI}{dt}, \tag{4.148}$$

$$\omega_0^2 = \frac{1}{LC}, \tag{4.149}$$

$$\frac{d^2q}{dt^2} + \omega_0^2 q = 0 \cdot \tag{4.150}$$

- Differential equation *R.C.*

We obtain the differential equation for the oscillations of charge *r.c.* This equation is similar to the differential equation describing the motion of a body under the quasi-elastic force. Therefore, will likewise be recorded and solution of this equation

$$q = q_0 \cos(\omega_0 t + \alpha), \tag{4.151}$$

- The equation of charge oscillations *r.c.*

$$U_C = \frac{q_0}{C}\cos(\omega_0 t + \alpha), \tag{4.152}$$

- Equation of voltage fluctuations on the capacitor plates *r.c.*

$$\frac{q}{C} = -L\frac{dI}{dt} = -q_0\omega_0 \sin(\omega_0 t + \alpha), \tag{4.153}$$

- Equation of the current oscillations *r.c.*

2. Damped oscillations r.c.

Consider the *R.C.* containing capacitance, inductance and resistance. 2-nd Kirchhoff's law in this case is written as

$$IR + U_C = \varepsilon_S, \tag{4.154}$$

$$U_R = IR, \tag{4.155}$$

$$U_C = \frac{q}{C}, \tag{4.156}$$

$$\varepsilon_S = -L\frac{dI}{dt}, \tag{4.157}$$

$$\omega_0^2 = \frac{1}{LC}, \tag{4.158}$$

$$L\frac{dI}{dt} + IR + \frac{q}{C} = 0, \tag{4.159}$$

$\beta = \frac{R}{2L}$ - damping factor, ω_0^2 - own cyclic frequency.

$$I = \frac{dq}{dt} \quad , \tag{4.160}$$

$$\frac{dI}{dt} = \frac{d^2q}{dt^2}, \tag{4.161}$$

$$\frac{d^2q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = 0 \tag{4.162}$$

- differential equation of damped oscillations *r.c.*

$$q = q_0 e^{-\beta t}\cos(\omega_0 t + \alpha) \tag{4.163}$$

- equation of damped oscillations of the charge *r.c.*

$$q = q_0 e^{-\beta t} \tag{4.164}$$

- law of variation of the amplitude of the charge for the damped oscillations in the *R.C.:*

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}} = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}}, \tag{4.165}$$

-The period of the damped oscillations.

$$D = \frac{q(t)}{q(t+T)} = e^{\beta t} = e^{\frac{R}{2L}T} \tag{4.166}$$

- damping rate.

$$\lambda = \ln D = \ln\left(\frac{q(t)}{q(t+T)}\right) = \beta T \tag{4.167}$$

- logarithmic damping decrement.

$$Q = \frac{\pi}{\lambda} = \frac{\pi}{\beta T} = \frac{\pi\omega_0}{\frac{R}{2L}2\pi} = \frac{L}{R}\sqrt{\frac{1}{LC}} \tag{4.168}$$

- quality factor of the circuit.

If $\omega_0^2 >> \beta^2$ the damping is weak, then $T \approx T_0$.

$$Q = \frac{1}{R}\sqrt{\frac{1}{LC}}\,. \tag{4.169}$$

Investigate the change in the voltage on the capacitor plates.

$$U = \frac{q}{C} = \frac{q_0 e^{-\beta t}}{C}\cos(\omega t + \alpha), \tag{4.170}$$

$$I = \frac{dq}{dt} = \frac{q_0}{C} e^{-\beta t}(-\beta)\cos(\omega t+\alpha) - \omega q_0 e^{-\beta t}\sin(\omega t+\alpha) = \frac{q_0 e^{-\beta t}}{C}(-\beta\cos(\omega t+\alpha) -$$

$$-\omega\sin(\omega t+\alpha)) \mid \frac{\omega_0}{\omega_0} = \frac{q_0\omega_0 e^{-\beta t}}{C}\left(\frac{-\beta}{\omega_0}\cos(\omega t+\alpha) - \frac{\omega}{\omega_0}\sin(\omega t+\alpha)\right) = \quad (4.171)$$

$$= \frac{q_0\omega_0 e^{-\beta t}}{C}\cos(\omega t+\alpha+\varphi),$$

$$\frac{\beta}{\omega_0} = \cos\varphi, \quad (4.172)$$

$$\frac{\omega}{\omega_0} = \sin\varphi, \ \frac{\pi}{2} < \varphi < \pi. \quad (4.173)$$

Current change is different in phase φ of the voltage.

$$\omega = \sqrt{\omega_0^2 - \beta^2} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}, \quad (4.174)$$

at $\omega_0^2 > \beta^2$, - possible damped oscillations,

at $\omega_0^2 = \beta^2$ - emergency,

$$\frac{1}{LC} = \frac{R^2}{4L^2} \Rightarrow R_K = \sqrt{\frac{4L}{C}}. \quad (4.175)$$

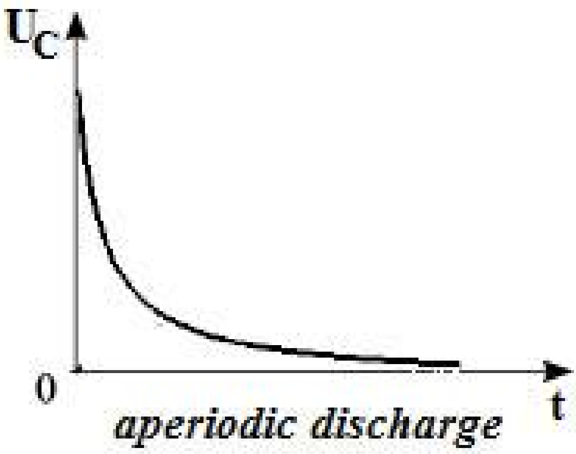


Figure 4.26

at $\omega_0^2 < \beta^2$, i.e R > R_K - fluctuations do not occur (aperiodic discharge capacitor).

3. forced oscillations

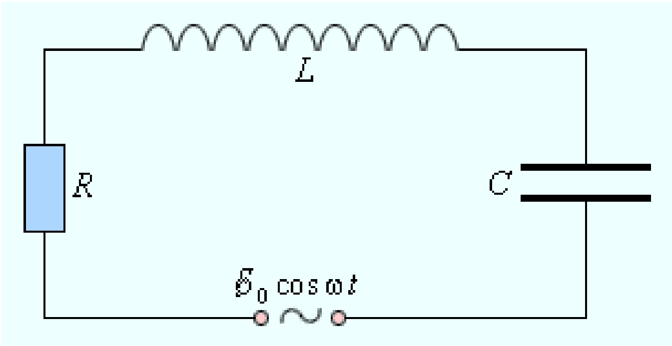


Figure 4.27

Undamped electromagnetic oscillations will occur in *r.c.* containing *R*, *L* and *C* in the case in this circuit to enter the *EMF* varies as a sine or cosine. In the circuit after the time of formation of forced oscillations occur undamped electromagnetic waves with frequency of the driving force.

Apply the 2-nd Kirchhoff's law to the subject *R.C.*

$$U_R + U_C = \varepsilon_{driv} + \varepsilon_S \,, \tag{4.176}$$

$$L\frac{dI}{dt} + IR + \frac{q}{C} = \varepsilon_0 \cos\omega t \,, \tag{4.177}$$

$$\frac{d^2q}{dt^2} + 2\beta\frac{dq}{dt} + \omega_0^2 q = \frac{\varepsilon_0}{L}\cos\omega t \,, \tag{4.178}$$

ω –the frequency of the driving force

$$q = q_{total.} + q_{runic.} \,, \tag{4.179}$$

$$q_{p.i.} = q_0 \cos(\omega t + \varphi) \,, \tag{4.180}$$

$$q_0 = \frac{\varepsilon_0}{L\sqrt{\left(\omega_0^2 - \omega^2\right) + 4\beta^2\omega^2}} \,, \tag{4.181}$$

$$tg\varphi = \frac{2\beta\omega}{\omega_0^2 - \omega^2} \,, \tag{4.182}$$

$$U = \frac{q}{C} = \frac{\varepsilon_0 \cos(\omega t + \varphi)}{LC\sqrt{\left(\omega_0^2 - \omega^2\right) + 4\beta^2\omega^2}} \,. \tag{4.183}$$

The law of change current

$$I = \frac{dq}{dt} = \frac{\varepsilon_0 \omega \cos\left(\omega t + \frac{\pi}{2} + \varphi\right)}{LC\sqrt{\left(\omega_0^2 - \omega^2\right) + 4\beta^2\omega^2}} \,, \tag{4.184}$$

$$q = \frac{\varepsilon_0}{L\sqrt{\left(\frac{1}{LC} - \omega^2\right)^2 + \frac{R^2}{L^2}\omega^2}} = \frac{\varepsilon_0}{\omega\sqrt{\left(\frac{1}{\omega C} - \omega L\right)^2 + R^2}}. \quad (4.185)$$

The law of change charge.

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}. \quad (4.186)$$

Impedance.

$X_L = \omega L$ - Inductance. $X_C = \frac{1}{\omega C}$ - Capacitance (Figure 4.28)

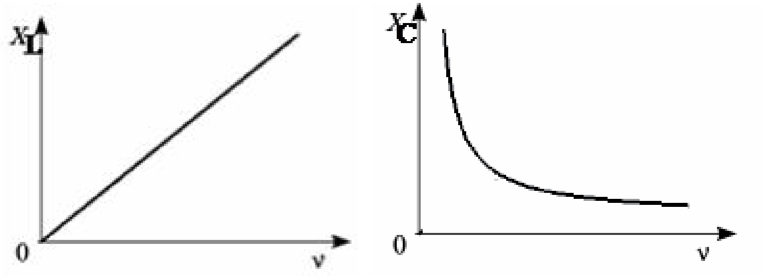


Figure 4.28

4.4 SOUND

This study of sound will concentrate on only a few main ideas - sound as an example of a longitunal wave exhibits all the properties that all waves exhibit including a speed that depends on the medium that carries the wave and both interference and diffraction. Sound level measurements (the decibel scale) are related to the energy density in the wave, and the apparent frequency of the sound one hears depends on both the speed of the source and the speed of the listener relative to the speed of sound in air (the Doppler effect). The most general study of sound would include discussions of how sound propagates through air as well as in liquids and solids, our perceptions of sound - which would require understanding the physiology of hearing, and would ultimately lead to the study of musical instruments and the complex study of acoustics.

Sound as a wave

The general principles studied in the discussion of wave motion apply equally well to sound. That includes, of course, the most general relationships between wave speed, wavelength, and frequency: That is,

$$\upsilon = \lambda f\ , \tag{4.187}$$

where λ represents the wavelength and f is the frequency. The wave speed v is determined by properties of the medium - which we will consider is air. The wave itself is longitudinal rather than transverse as are waves on strings and the surface waves on water. But the waves propagate in all directions as a speed determined by the compressibility and mass density of the air. The propagation of a disturbance in air is described by a differential equation of the same form as for transverse waves on a taut string. So the solutions to the equation are necessarily of the same form as well. That is, a disturbance in air is governed by a wave equation of the form

$$\frac{\partial^2 y(x,t)}{\partial t^2} = v^2 \frac{\partial^2 y(x,t)}{\partial x^2}\ , \tag{4.188}$$

which has solutions representing waves traveling a speed v that depends on the bulk modulus B (which in turn depends on pressure and temperature) and on the mass density ρ of the air. That is, the disturbance that propagates through the air that we call "sound" can be described with the same type of wave equation that was used to describe waves on strings. And the speed of those waves depends on properties of the medium through which they travel.

$$\upsilon = \sqrt{\frac{B}{\rho}}\ , \tag{4.189}$$

where *B* plays the same role as the tension in the string and ρ is the mass per unit volume of the air rather than the linear mass density of the string which supported a transverse wave.

In air, the wave speed can be related to the air temperature by

$$\upsilon = \sqrt{\frac{\gamma RT}{\mu}} = 331\, m/s \sqrt{\frac{T}{273}}\ . \tag{4.190}$$

Where γ is a constant for air, R is the Universal Gas Constant, μ is the average molecular mass of air and the temperature is the Absolute temperature on the Kelvin scale. The equation can be simplified in terms of the speed of sound in air at 273 K (i.e, ice point).

Sound Intensity

The intensity of a wave is simply the energy per unit time that is transferred per unit area of a surface that the wave impinges on. But energy per time is just the power that is delivered by the source. And that energy is distributed over an ever increasing area as the wave propagates away from the source. Assuming a point source of sound, with waves spreading outward in spherical wave fronts - and assuming no energy dissipation as the wave propagates through the air - the intensity decreases as the inverse square of the distance from the source as the energy is spread over an ever increasing spherical surface. So the intensity, or power per unit area, is simply given by

Intensity in *watts*/m^2

$$I = \frac{P_{avg}}{4\pi r^2}, \qquad (4.191)$$

where P_{avg} is the power emitted by the source and r is the distance from the source. The intensity I would be measured in watts per square meter.

In practice, the intensity of a sound is much more complicated, since the above expression assumes a point source of sound that spreads out uniformly in all directions. Ignored in this expression is the absorption of sound by the air itself and reflections from surfaces that the sound Graphing the intensity as a function of distance from the source shows how intensity diminishes as the energy of the sound waves is spread over an ever increasing area. As the distance from the source increases, the intensity decreases until it would be undetectable. The weakest sound intensity that most humans can hear is about 10-12 *w*/m^2.

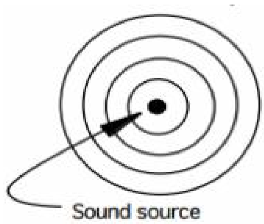


Figure 4.29

That level is called the threshold of hearing and is assigned the symbol I_0. All other sound intensities can be related to Io. That is, a sound intensity one hundred times the threshold of hearing would be written $I = 10^2 \cdot I_0$, etc. The loudest sounds that humans can endure over any prolonged time - although that level is physically uncomfortable - has an intensity of about one *watt*/m^2 and is called the threshold of pain. And we can detect much larger intensities even than that - although they can cause permanent hearing loss.

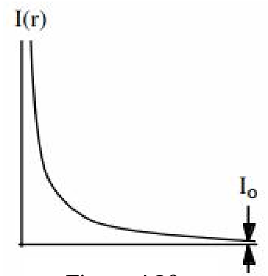


Figure 4.30

4.4.1 The Relationship between Intensity and Loudness - the Sound Level in Decibels

Because human hearing covers such an enormously large range in intensities the measure of sound levels makes use a logarithmic scale called the decibel scale which more accurately reflects our perception of the loudness of sound.

The sound level (sometimes called the intensity level - a term which I think is too easily confused with "intensity" which is measured in w/m^2) associated with any sound is determined by the intensity ratio relative to the threshold of hearing and is measured in decibels. That is, the sound level in decibels is defined by

Sound level in decibels

$$\beta = 10\log\left(\frac{I}{I_0}\right). \tag{4.192}$$

It is useful to notice that the sound level of the threshold of hearing would be given as 0 *Db*, since log(1) = 0. It is also useful to notice that the sound intensity that humans find physically uncomfortable (called the threshold of pain) is about 1 w/m^2 - which is 10^{12} times the threshold of hearing. The corresponding sound level is thus 10 log (10^{12}), or 120 *Db*.

When comparing two different sound intensities, say I_1 and I_2 the difference in sound levels would be given by $\Delta\beta = \beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_1}\right) - 10\log\left(\frac{I_1}{I_0}\right)$, which would reduce to

$$\Delta\beta = 10\log\left(\frac{I_2}{I_1}\right). \tag{4.193}$$

When the source of sound is a "point source" – i.e, the sound spreads out uniformly in all directions - the intensity is given by $I = \frac{P}{4\pi r^2}$. So the sound level difference between the two points at different distances from the same source is given by

$$\Delta\beta = 10\log\left(\frac{r_1^2}{r_2^2}\right) = 20\log\left(\frac{r_1}{r_2}\right). \tag{4.194}$$

4.4.2 Electromagnetic Waves

Consider a plane *electromagnetic wave* propagating through a vacuum in the z -direction. Incidentally, electromagnetic waves are the only commonly occurring waves that do not require a medium through which to propagate. Suppose that the wave is linearly polarized in the x -direction: that is, its electric component oscillates in the x -direction. It follows that the magnetic component of the wave oscillates in the y -direction (Fitzpatrick 2008). According to standard electromagnetic theory, the wave is described by the following pair of coupled partial differential equations:

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0}\frac{\partial H_y}{\partial z}, \tag{4.195}$$

$$\frac{\partial E_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z}, \qquad (4.196)$$

where $E_x(z,t)$ is the *electric field-strength*, and $H_y(z,t)$ is the *magnetic intensity* (i.e., the magnetic field-strength divided by μ_0). Observe that Equations (4.195) and (4.196), which govern the propagation of electromagnetic waves through a vacuum, which govern the propagation of electromagnetic signals down a transmission line. In particular, E_x has units of voltage over length, H_y has units of current over length, ε_0 has units of capacitance per unit length, and μ_0 has units of inductance per unit length.

Equations (4.195) and (4.196) can be combined to give

$$\frac{\partial^2 E_x}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 E_x}{\partial z^2}, \qquad (4.197)$$

$$\frac{\partial^2 H_y}{\partial t^2} = \frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 H_y}{\partial z^2}. \qquad (4.198)$$

It follows that the electric and the magnetic components of an electromagnetic wave propagating through a vacuum both separately satisfy a wave . Furthermore, the phase velocity of the wave is the velocity of light in vacuum,

$$c = \frac{1}{\sqrt{\varepsilon_0\mu_0}} = 2{,}998\cdot 10^8 m/s. \qquad (4.199)$$

Let us search for a traveling wave solution of Equations (4.195) and (4.196), propagating in the positive z - direction, whose electric component has the form

$$E_x(z,t) = E_0\cos(\omega t - kz - \varphi) \qquad (4.200)$$

This is a valid solution provided that

$$\omega = kc. \qquad (4.201)$$

According to Equation (4.196), the magnetic component of the wave is written

$$H_y(z,t) = \frac{1}{Z}E_0\cos(\omega t - kz - \varphi), \qquad (4.202)$$

Where

$$Z = Z_0 \equiv \sqrt{\frac{\mu_0}{\varepsilon_0}}, \qquad (4.203)$$

and Z_0 is the impedance of free space. Thus, the electric and magnetic components of an electromagnetic wave propagating through a vacuum are mutually perpendicular, and also perpendicular to the direction of propagation. Moreover, the two components oscillate in phase (i.e, they have simultaneous maxima and zeros), and the amplitude of the magnetic component is that of the electric component divided by the impedance of free space.

Multiplying Equation (4.195) by $\varepsilon_0 E_x$, Equation (4.196) by $\mu_0 H_y$, and adding the two resulting expressions, we obtain the energy conservation equation

$$\frac{\partial \varepsilon}{\partial t} = \frac{\partial \chi_z}{\partial z} = 0 \cdot \tag{4.204}$$

Where

$$\varepsilon = \frac{1}{2}\left(\varepsilon_0 E_x^2 + \mu_0 H_y^2\right), \tag{4.205}$$

is the energy density (i.e., energy per unit volume) of the wave (Fitzpatrick 2008), whereas

$$\chi_z = E_x H_y \ , \tag{4.206}$$

is the energy flux (i.e., power per unit area) in the positive z -direction. The mean energy flux associated with the z -directed electromagnetic wave specified in Equations (4.200) and (4.202) is thus

$$\langle \chi_Z \rangle = \frac{1}{2}\frac{E_0^2}{Z} \ . \tag{4.207}$$

For a similar wave propagating in the negative z -direction, it can be demonstrated that

$$E_x(z,t) = E_0 \cos(\omega t + kz - \varphi), \tag{4.208}$$

$$H_y(z,t) = \frac{1}{Z} E_0 \cos(\omega t + kz - \varphi), \tag{4.209}$$

And

$$\langle \chi_Z \rangle = -\frac{1}{2}\frac{E_0^2}{Z} \ . \tag{4.210}$$

Consider a plane electromagnetic wave, linearly polarized in the x -direction, that propagates in the z -direction through a transparent *dielectric medium*, such as glass or water. As is well-known (Fitzpatrick 2008), the electric component of the wave causes the neutral molecules making up the medium to polarize: that is, it causes a small separation to develop between the mean positions of the positively and negatively charged constituents of the molecules (i.e., the atomic nuclei and the orbiting electrons). [Incidentally, it can be shown that the magnetic component of the wave has a negligible influence on the molecules, provided the wave amplitude is sufficiently small that the wave electric field does not cause the electrons and nuclei to move with relativistic velocities (ibid.).]

If the mean position of the positively charged constituents of a given molecule, of net charge $+q$, develops a vector displacement d with respect to the mean position of the negatively charged constituents, of net charge $-q$, in response to a wave electric field E, then the associated *electric dipole moment* is $p = qd$, where d is generally parallel to E (ibid.). Furthermore, if there are N such molecules per unit volume then the *electric dipole moment per unit volume* is written $p = Nqd$. In a linear, isotropic, dielectric medium (ibid.),

$$p = \varepsilon_0(\varepsilon - 1)E, \tag{4.211}$$

where $\varepsilon > 1$ is a dimensionless quantity, known as the *relative dielectric constant*, that is a property of the medium in question. In the presence of a dielectric medium, Equations (4.208) and (4.209) generalize to give

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon_0}\left(\frac{\partial P_x}{\partial t} + \frac{\partial H_y}{\partial z}\right), \tag{4.212}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z}. \tag{4.213}$$

When combined with Equation (4.210), these expressions yield

$$\frac{\partial E_x}{\partial t} = -\frac{1}{\varepsilon\varepsilon_0}\frac{\partial H_y}{\partial z}, \tag{4.214}$$

$$\frac{\partial H_y}{\partial t} = -\frac{1}{\mu_0}\frac{\partial E_x}{\partial z}. \tag{4.215}$$

It can be seen that the previous equations are just like the corresponding vacuum equations, (4.195) and (4.196), except that ε_0 has been replaced by $\varepsilon\varepsilon_0$. It immediately follows that the phase velocity of an electromagnetic wave propagating through a dielectric medium is

$$\upsilon = \frac{1}{\sqrt{\varepsilon\varepsilon_0\mu_0}} = \frac{c}{n}, \tag{4.216}$$

where $c = \dfrac{1}{\sqrt{\varepsilon_0\mu_0}}$, is the velocity of light in vacuum, and the dimensionless quantity

$$n = \sqrt{\varepsilon}\ , \tag{4.217}$$

is known as the *refractive index* of the medium. Thus, an electromagnetic wave propagating through a transparent dielectric medium does so at a phase velocity that is less than the velocity of light in vacuum by a factor n (where $n > 1$). The dispersion relation of the wave is thus

$$\omega = k\upsilon = \frac{kc}{n}. \tag{4.218}$$

Furthermore, the impedance of a transparent dielectric medium becomes

$$Z = \sqrt{\frac{\mu_0}{\varepsilon\varepsilon_0}} = \frac{Z_0}{n}, \tag{4.219}$$

where Z_0 is the impedance of free space.

Incidentally, the signal that travels down a transmission line is a form of guided electromagnetic wave. It follows that if the space between the two conductors that constitute the line is filled with dielectric material of relative dielectric constant ε then the signal propagates down the line at the reduced phase velocity

$$\upsilon = \frac{c}{\sqrt{\varepsilon}}. \tag{4.220}$$

This occurs because the dielectric material increases the capacitance per unit length of the line by a factor ε, but leaves the inductance per unit length unchanged. For the same reason, the presence of the dielectric material decreases the impedance of the line by a factor $\sqrt{\varepsilon}$. Hence, the impedance of a dielectric filled co-axial cable is

$$Z = \frac{1}{2\pi\sqrt{\varepsilon}} \ln\left(\frac{b}{a}\right) Z_0. \tag{4.221}$$

Here, a and b are the radii of the inner and outer conductors, respectively.

Suppose that the plane $Z = 0$ forms the interface between two transparent dielectric media of refractive indices n_1 and n_2. Let the first medium occupy the region $Z < 0$, and the second the region $Z > 0$. Suppose that a plane electromagnetic wave, linearly polarized in the x -direction, and propagating in the positive Z -direction, is launched toward the interface from a wave source of angular frequency ω situated at $Z = -\infty$. We expect the wave incident on the interface to be partly reflected, and partly transmitted. The wave electric and magnetic fields in the region $Z < 0$ are written

$$E_x(z,t) = E_i \cos(\omega t + k_1 z) + E_r \cos(\omega t + k_1 z)\ , \tag{4.222}$$

$$H_y(z,t) = \frac{1}{Z} E_i \cos(\omega t - k_1 z) - \frac{1}{Z} E_r \cos(\omega t + k_1 z), \tag{4.223}$$

where E_i is the amplitude of (the electric component of) the incident wave, E_r the amplitude of the reflected wave, $k_1 = \frac{n_1\omega}{c}$, and $Z_1 = \frac{Z_0}{n_1}$. The wave electric and magnetic fields in the region $Z > 0$ take the form

$$E_x(z,t) = E_t \cos(\omega t - k_2 z), \tag{4.224}$$

$$H_y(z,t) = \frac{1}{Z} E_t \cos(\omega t - k_2 z), \tag{4.225}$$

where E_t is the amplitude of the transmitted wave $k_2 = \frac{n_2\omega}{c}$, and $Z_2 = \frac{Z_0}{n_2}$. According to standard electromagnetic theory, the appropriate matching conditions at the interface ($Z = 0$) are that E_x and H_y are both continuous. Thus, continuity of E_x yields

$$E_i + E_r = E_t, \tag{4.226}$$

whereas continuity of H_y gives

$$n_1(E_i - E_r) = n_2 E_t, \tag{4.227}$$

because $\frac{1}{Z} \infty n$. It follows that

$$E_r = \left(\frac{n_1 - n_2}{n_1 + n_2}\right) E_i, \tag{4.228}$$

$$E_t = \left(\frac{2n_1}{n_1 + n_2}\right) E_i \cdot \tag{4.229}$$

The coefficient of reflection, R, is defined as the ratio of the reflected to the incident energy flux, so that

$$R = \left(\frac{E_r}{E_i}\right)^2 = \left(\frac{n_1 - n_2}{n_1 + n_2}\right)^2. \tag{4.230}$$

Likewise, the coefficient of transmission, T, is the ratio of the transmitted to the incident energy flux, so that

$$T = \frac{Z_2^{-1}}{Z_1^{-1}} \left(\frac{E_t}{E_i}\right)^2 = \frac{n_2}{n_1} \left(\frac{E_t}{E_i}\right)^2 = \frac{4n_1 \cdot n_2}{(n_1 + n_2)^2} = 1 - R \tag{4.231}$$

It can be seen, first of all, that if $n_1 = n_2$ then $E_r = 0$ and $E_t = E_i$. In other words, if the two media have the same indices of refraction then there is no reflection at the interface between them, and the transmitted wave is consequently equal in amplitude to the incident wav**e**. On the other hand, if $n_1 \neq n_2$ then there is always some reflection at the interface. Indeed, the amplitude of the reflected wave is roughly proportional to the difference between n_1 and n_2. This has important practical consequences. We can only see a clean pane of glass in a window because some of the light incident on an air/glass interface is reflected, as a consequence of the different refractive indices of air and glass. As is well-known, it is a lot more difficult to see glass when it is submerged in water. This is because the refractive indices of glass and water are quite similar, and so there is very little reflection of light incident on a water/glass interface.

When $\frac{E_r}{E_i} < 0$ when $n_2 > n_1$. The negative sign indicates a π radian phase shift of the (electric component of the) reflected wave, with respect to the incident wave.

We conclude that there is a π radian phase shift of the reflected wave, relative to the incident wave, on reflection from an interface with a medium of greater refractive index. Conversely, there is zero phase shift on reflection from an interface with a medium of lesser refractive index.

4.4.3 Doppler Effect

Consider a sinusoidal wave of angular frequency ω and wavenumber k that is propagating in the $+x$ -direction. We can write represent the wave in terms of a *wave function* of the form

$$\psi(x,t) = \psi_0 \cos(\omega t - kx)\,. \tag{4.232}$$

The wavelength and frequency of the wave, as seen by a stationary observer, are $\lambda = \frac{2\pi}{k}$ and $f = \frac{\omega}{2\pi}$, respectively. Consider a second observer moving with uniform speed u_0 in the $+x$ -direction.

What are the wavelength and frequency of the wave seen by the latter observer? Assuming non-relativistic motion, the x -coordinate in the moving observer's frame of reference is given by the standard Gallilean transformation formula $x` = x - u_0 t$ (Rindler 1997). Both observers measure the same time. Hence, in the second observer's frame of reference, the wavefunction is written

$$\psi(x`,t) = \psi_0 \cos(\omega`t - kx`), \qquad (4.233)$$

Where

$$\omega` = \omega - ku_0 . \qquad (4.234)$$

Here, we have replaced x by $x` = x + u_0 t$ in Equation (4.232). Thus, the moving observer sees a wave possessing the same wavelength (i.e., the same k) but a different frequency (i.e., a different ω) to that seen by the stationary observer. This phenomenon is known as the *Doppler effect*.

If f is the wave frequency (in hertz) seen by the stationary observer then the wave frequency seen by the moving observer is

$$f` = \left(1 - \frac{u_0}{\upsilon}\right) f \ , \qquad (4.235)$$

where $\upsilon = \frac{\omega}{k}$ is the characteristic wave speed. Thus, an observer moving in the same direction as a wave sees a lower frequency than a stationary observer. On the other hand, an observer moving in the opposite direction to a wave sees a higher frequency than a stationary observer. Hence, the general Doppler shift formula (for a moving observer and a stationary wave source) is

$$f` = \left(1 \mp \frac{u_0}{\upsilon}\right) f \ , \qquad (4.236)$$

where the upper/lower signs correspond to the observer moving in the same/opposite direction to the wave.

Consider a stationary observer measuring a wave emitted by a source that is moving towards the observer with speed u_s. Let υ be the characteristic propagation speed of the wave. Consider two neighboring wave crests emitted by the source. Suppose that the first is emitted at time $t = 0$, and the second at time $t = T$, where $T = \frac{1}{f}$ is the wave period in the frame of reference of the source. At time t , the first wave crest has traveled a distance $d_1 = \upsilon t$ towards the observer, whereas the second wave crest has traveled a distance $d_2 = \upsilon(t - T) + u_s T$ (measured from the position of the source at $t = 0$). Here, we have taken into account the fact that the source is a distance $u_s T$ closer to the observer when the second wave crest is emitted. The effective wavelength, $\lambda`$, seen by the observer is the distance between neighboring wave crests. Hence,

$$\lambda` = d_1 - d_2 = (\upsilon - u_s)T \; . \qquad (4.237)$$

where f is the wave frequency in the frame of reference of the source. We conclude that if the source is moving towards the observer then the wave frequency is shifted upwards. Likewise, if the source is moving away from the observer then the frequency is shifted downwards.

This manifestation of the Doppler effect is familiar from everyday experience. When an ambulance passes us on the street, its siren has a higher pitch (i.e., a high frequency) when it is coming towards us than when it is moving away from us. In fact, the oscillation frequency of the siren never changes. It is the Doppler shift induced by the motion of the siren with respect to a stationary listener that causes the frequency change.

The general formula for the shift in wave frequency induced by relative motion of an observer and a source is

$$f` = \left(\frac{1 \mp {u_0}/{\upsilon}}{1 \mp {u_s}/{\upsilon}} \right) f \; , \qquad (4.238)$$

where u_0 is the speed of the observer, and u_s is the speed of the source (both measured relative to the wave medium). The upper/lower signs correspond to relative motion by which the observer and the source move apart/together. If the observer and source are not moving directly toward or directly away from one another then the quantities u_0 and u_s in the above formulae correspond to the components of the observer and source velocities, respectively, along the straight-line that instantaneously joins them.

An important proviso to the previous formula is that it is strictly classical, and only holds for non-relativistic motion (i.e., u_0, $u_s << c$, where c is the velocity of light in vacuum.) In fact, when applied to light propagation in a vacuum, the formula is only accurate up to first-order in u_0 / c and u_s / c (Rindler 1997). In other words, for light propagation the previous equation reduces to

$$f` = \left[1 - \frac{u}{c} + 0\left(\frac{u^2}{c^2} \right) \right] f , \qquad (4.239)$$

where u is the relative radial velocity of the source with respect to the observer (being positive when the source and observer are moving apart, and vice versa).

Probably the most well-known use of the Doppler effect in everyday life is in police speed traps. In such a trap, a policeman fires radar waves (i.e., electromagnetic waves of centimeter wavelength) of fixed frequency at an oncoming car. These waves reflect off the car, which effectively becomes a moving source. Hence, by measuring the frequency increase of the reflected waves, the policeman can determine the car's speed.

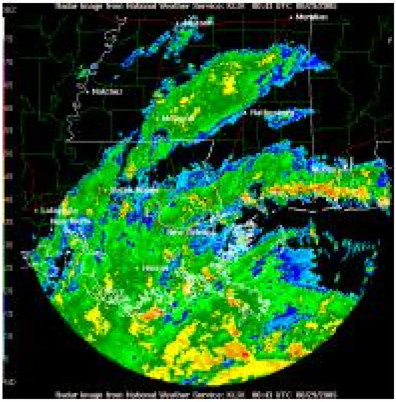


Figure 4.31

Doppler radar. The first use of radar was by Britain during World War II: antennas on the ground sent radio waves up into the sky, and detected the echoes when the waves were reflected from German planes. Later, air forces wanted to mount radar antennas on airplanes, but then there was a problem, because if an airplane wanted to detect another airplane at a lower altitude, it would have to aim its radio waves downward, and then it would get echoes from the ground. The solution was the invention of Doppler radar (Figure 4.31), in which echoes from the ground were differentiated from echoes from other aircraft according to their Doppler shifts. A similar technology is used by meteorologists to map out rainclouds without being swamped by reflections from the ground, trees, and buildings.

Chapter 5 WAVE OPTICS

5.1 INTERFERENCE OF LIGHT

5.1.1 The coherence and monochromatic light waves

Let the two waves of the same frequency, superimposed on each other, excited at some point in space vibrations of the same direction:

$$\xi_1 = A_1 \cos(\omega t - kS_1 + \varphi_{01}) = A_1 \cos\Phi_1, \tag{5.1}$$

$$\xi_2 = A_2 \cos(\omega t - kS_2 + \varphi_{02}) = A_2 \cos\Phi_2, \tag{5.2}$$

$$\omega_1 = \omega_2 = \omega. \tag{5.3}$$

The amplitude of the resulting oscillation at a given point is given by

$$A^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos[k(S_2 - S_1) + (\varphi_{20} - \varphi_{10})], \tag{5.4}$$

Initial phase

$$tg\Phi = \frac{A_1 \sin\Phi_1 + A_2 \sin\Phi_2}{A_1 \cos\Phi_1 + A_2 \cos\Phi_2}. \tag{5.5}$$

If the phase difference $\Delta\varphi_0 = \varphi_{02} - \varphi_{01}$ of the excited oscillations remains constant over time, the waves are called **coherent**.

The quantity $\Delta = S_2 - S_1$ is called the **optical path difference** and is equal to the the difference optical length $S_2 - S_1$. **Optical length** S of the wave is the product of a geometric path to the refractive index *n*:

$S = l \cdot n$,

$$n = \sqrt{\varepsilon\mu}, \tag{5.6}$$

$$v_{ph} = \frac{c}{n} \Rightarrow n = \frac{c}{v_{ph}}. \tag{5.7}$$

where *n* - refractive index shows how many times the speed of light in a vacuum (the speed of light $c = 3\ 10^8$ *m/s*), more than the speed of light in a given medium – v_{ph} - the phase velocity.

The intensity of the wave I is proportional to the square of the amplitude $I \sim A^2$, hence

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(k\Delta + \Delta\varphi), \tag{5.8}$$

because $\Delta\varphi = const$ for coherent waves, depending on the difference optical path Δ at some points will light amplification, and in others - it weakening.

If $I_1 = I_2$, then

$$I_{\max} = 4I_1, \tag{5.9}$$

$$I_{\min} = 0\,, \qquad (5.10)$$

i.e will be a redistribution of intensity (energy) waves in space.

Redistribution of light flux in space, in which at some points there are maximums and minimums in other intensity is called **interference**.

A necessary condition for the interference of waves is their coherence. However, due to the transverse electromagnetic waves their coherence condition is not sufficient to produce an interference pattern.

Necessary, in coherence to vibrations vectors $\vec{E}$ of electromagnetic fields, the interfering waves occurred along the same or a similar direction, ie requires that the interfering waves propagating in the same direction and the plane $\vec{E}$ of the waves were close.

Coherent are **monochromatic waves -** unlimited in space of one specific wavelength and strictly constant frequency. Because no real power does not strictly monochromatic light, the waves emitted by any source of light other than a laser, are incoherent. Therefore, the experiment did not observe the interference of light from independent sources of light, such as the two bulbs.

5.1.2 The interference of light in thin plane-plates

Consider a plane-parallel glass (or transparent) plate n = 1.5, thickness b (temporal coherence condition will be satisfied if $b < \dfrac{\lambda_0^2}{4\Delta\lambda_0}$, that is for $\lambda_0 = 5 \cdot 10^{-7}\ m$ and $\Delta\lambda = 20\ \overset{\circ}{A}$ and $b = 6 \cdot 10^{-8}\ m$). The plate is at an angle i plane $\vec{E}$ monochromatic wave. The plate is in the air $n_{air} = 1$.

The incident wave is partially reflected (~5°) from the upper surface of

the plate (beam 1) and partially refracted (beam *AO*). Refracted wave, reaching the bottom of the plate, and partially reflected (ray *OC*), and partially refracted (beam 2'). The same thing happens on the upper surface of the plate at the point with the ray *OC*, and the refracted wave (beam 2) is superimposed to the wave directly reflected from the top surface (beam 1). These two waves are coherent. The result of the interference depends on the Δ - optical path difference.

The path difference acquired by the beams 1 and 2 (Figure 5.1) before they meet in that *C* is

$$\Delta = nS_2 - S_1, \tag{5.11}$$

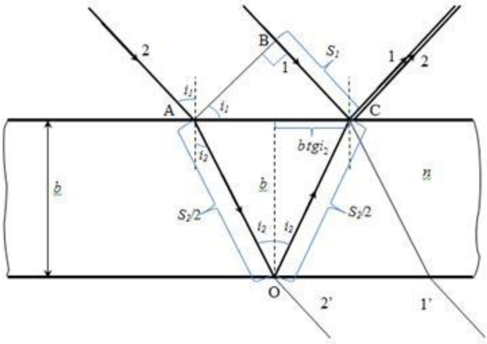


Figure 5.1

$$S_1 = |BC| = 2btgi_2 \cdot \sin i_1, \tag{5.12}$$

$$S_2 = (|AO| + |BC|)n = \frac{2b}{\cos i_2} n \cdot \tag{5.13}$$

In geometrical optics, we know the law of refraction

$$\frac{\sin i_1}{\sin i_2} = \frac{n_2}{n_1} = \frac{n}{1} \Rightarrow \sin i_1 = n \sin i_2 \cdot \tag{5.14}$$

of trigonometry

$$tgi_2 = \frac{\sin i_2}{\cos i_2}, \tag{5.15}$$

$$\cos i_2 = \sqrt{1 - \sin^2 i_2}, \tag{5.16}$$

$$n \cos i_2 = \sqrt{n^2 - n^2 \sin i_2} = \sqrt{n^2 - \sin^2 i_1}. \tag{5.17}$$

Then

$$\Delta = S_2 - S_1 = \frac{2b}{\cos i_2} n - 2btgi_2 \sin i_1 = 2b\left(\frac{n - \sin i_2 \sin i_1}{\cos i_2}\right) | \frac{\cdot n}{\cdot n} =$$

$$= 2b\left(\frac{n^2 - n \sin i_2 \sin i_1}{n \cos i_2}\right) = 2b\left(\frac{n^2 - \sin^2 i_1}{\sqrt{n^2 - \sin^2 i_1}}\right) = 2b\sqrt{n^2 - \sin^2 i_1}. \tag{5.18}$$

In calculating the phase difference $\Delta\varphi$ between oscillations in the beams 1 and 2 need besides the optical path difference $\Delta\varphi$ into account the phase change on reflection in p. *A*. Since in p. *A* that there is a reflection from the interface between the optically less dense medium to an optically denser medium ($n_2 > n_1$, because $n_{glass} > 1$), the phase of the wave changes in A on π. In that reflection occurs on the boundary between the media, the optically denser medium to an optically less dense, so that the phase change in p. O not occurs.

Thus, the phase change in p. A can be accounted for by adding to Δ (or subtracted from) the half wavelength in a vacuum - $\frac{\lambda}{2}$. Then, finally

$\Delta = 2b\sqrt{n^2 \sin^2 i_1} - \frac{\lambda_0}{2}$ - The optical path difference for the interference of the reflected beams 1 and 2.

$\Delta = 2b\sqrt{n^2 \sin^2 i_1}$ - the optical path difference for the interference of passing rays 1'and 2'.

5.2 THE DIFFRACTION OF LIGHT

5.2.1 The diffraction of light and the conditions for its observation. Huygens-Fresnel principle

If on the way of the light waves are opaque bodies or screens with apertures, then rough observations show that these bodies, form a region of shadows. This area may be outlined geometrically, assuming that light travels in straight lines, the light rays are straight lines.

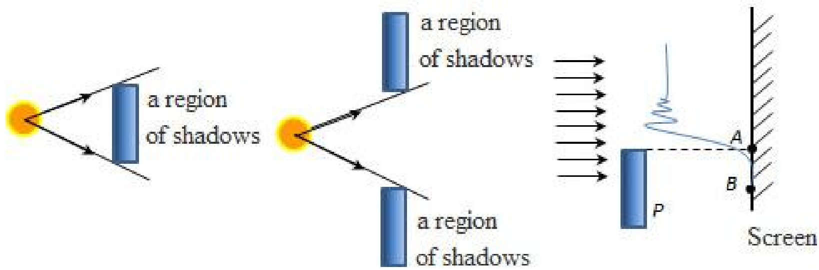


Figure 5.2

A more detailed observation shows that the light waves come into the geometric shadow, and the border between areas of light and shadow appear alternating high and low light, indicating some redistribution of light energy at this boundary. This rounding light waves borders opaque bodies to form an interference energy redistribution on different areas called diffraction waves. (or, a phenomenon that occurs when light in a medium with sharp irregularities, called diffraction of light.)

Diffraction phenomenon can be explained by using the Huygens principle: each point of the space to which comes a wave motion (ie, the wave front) is a source of secondary waves, the envelope which gives the position of the wave front in the next moment time.

In a homogeneous medium secondary waves will be of a hemisphere, the direction of propagation of the secondary waves coincides with the direction of the primary wave.

Problem of the distribution of energy along the wave front can be solved using the Huygens-Fresnel principle:
a) Huygens' principle, b) the sources of secondary waves are coherent, c) d*A* amplitude oscillations excited at p. *M* secondary sources proportional to the ratio of the area *dS* of the wave surface area *S* to the distance *r* from it to point *M*, and depends on the angle *α* between the external normal to the wave surface and the direction of the element d*S* at *M*.

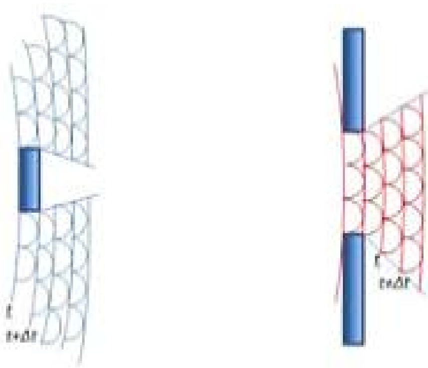


Figure 5.3

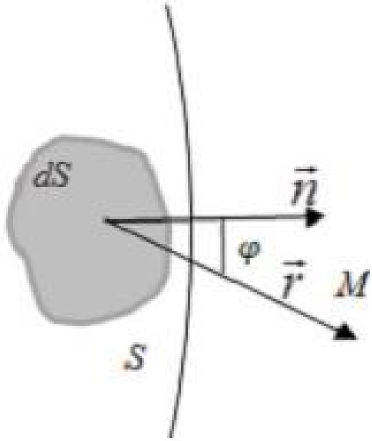


Figure 5.4

$$dE = K(\varphi)\frac{a_0 dS}{r}\cos(\omega t - kr + \alpha_0) \qquad (5.19)$$

$K(\varphi)$ - proportionality factor depending on the angle φ.

When $\varphi = 0$, $K(\varphi) = \max$, at $\varphi = \frac{\pi}{2}$ $K(\varphi) = 0$.

The resulting field in point *M* is a superposition of waves (5.19), taken for the entire wave surface:

$$E = \int_s K(\varphi)\frac{a_0 dS}{r}\cos(\omega t - kr + \alpha_0) \qquad (5.20)$$

– analytical account of the principle of Huygens - Fresnel.

5.2.2 Fraunhofer diffraction on a single slit

Fraunhofer diffraction (Figure 5.5) (or diffraction plane light waves, or parallel-ray diffraction) was observed in the case where the light source and the observation point is infinitely removed from the constraints of diffraction.

l- length, b - width. The path difference between the beams 1 and 2 in the direction φ

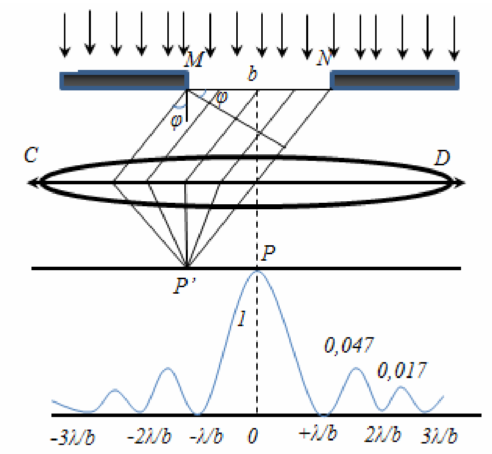


Figure 5.5

$$\Delta = NF = b\sin\varphi. \qquad (5.21)$$

We divide the wave surface at the site MN gap on the Fresnel zone, having a form of bands parallel to the edge of the M slots. The width of each band is selected to the path difference from the edges of these zones is equal to $\lambda/2$, i.e, in all on slit width go in $\Delta/(\lambda/2)$ zones. Because light is normally incident on the slit, the slit plane coincides with the wave front, so that all points in the plane of the front slot will vary in phase.

The amplitudes of the secondary waves in the plane of the slit will be equal, as the selected Fresnel zone have the same size and equally inclined to the direction of observation.

Number of Fresnel zones $\Delta/(\lambda/2)$ fit the width of the gap depends on the angle φ.

Minimum condition for Fresnel diffraction:

If an even number of Fresnel zones

$$\frac{\Delta}{\lambda/2} = \pm 2m, \qquad (5.22)$$

or

$$b\sin\varphi = \pm 2m\frac{\lambda}{2}, \qquad m = 1, 2, 3... \qquad (5.23)$$

then in p. P is observed diffraction minimum.

Maximum condition:

If an odd number of Fresnel zones

$$\frac{\Delta}{\lambda/2} = \pm(2m+1), \tag{5.24}$$

$$b\sin\varphi = \pm(2m+1)\frac{\lambda}{2}, \qquad m = 1, 2, 3..., \tag{5.25}$$

is observed diffraction maximum.

When $\varphi` = 0$, $\Delta = 0$ in the gap fits one Fresnel zone and, therefore, in point *P* the main (center) a maximum of zero order.

The main part of the light energy is concentrated in the main maximum: *m* = 0:1:2:3 ...; I = 1: 0.047: 0.017: 0.0083 ... (*m*-high order; *I*-intensity).

The narrowing gap leads to a broadening of the main peak and a decrease in its brightness (the same with the other peaks). With the broadening of the gap ($b > \lambda$) peaks will be brighter, but the diffraction bands are narrower, and the number of bands themselves – more. When $b >> \lambda$ in the center of the source image is sharp, ie have the rectilinear propagation of light.

When falling of white light is decomposed into its components. While violet light will deviate less blue – more, etc., red – maximum. The main maximum in this case will be white.

5.2.3 The diffraction grating

The diffraction grating is a collection of a large number *N* of identical width and parallel slits separated by opaque intervals, of the same as the width.

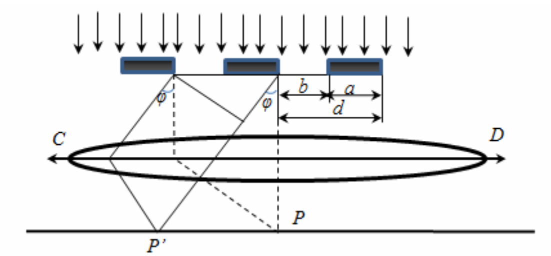


Figure 5.6

B – width of the gap;
a – the width of the opaque area;
d = *a* + *b* –period or lattice constant.

$$d = \frac{1}{N}. \tag{5.26}$$

The diffraction pattern on the lattice is defined as the mutual interference of the waves coming from all the cracks, ie a diffraction grating is multipath interference. Because slits are separated by the same distance, the differences of the rays coming from the two adjacent slots will be for the direction φ are identical across the entire grating.

$$\Delta = CF = (a+b)\sin\varphi = d\sin\varphi \,. \qquad (5.27)$$

In areas in which there is a minimum of one slit, and minimums will be in the case of *N* slots, ie the condition of the primary minimum of the diffraction grating is analogous to the condition for the minimum gap:

$$b\sin\varphi = \pm m\frac{\lambda}{2}, \qquad m = 1,\ 2,\ 3... \qquad (5.28)$$

- **the condition of the primary minimum**.

The maximum condition: the cases φ, which satisfy the maximum for the single slit can be either maxima or minima, as it all depends on the path difference between the beams. **The condition of the main maxima**:

$$d\sin\varphi = \pm k\frac{\lambda}{2}, \qquad k = 0,1,\ 2,\ 3... \qquad (5.29)$$

These peaks are located symmetrically relative to the center (zero $k = 0$) maximum.

For those angles φ, which is performed at the same time (5.28) and (5.29) the maximum will not, and will be a minimum (eg, $d = 2b$ for all even $k = 2p, \quad p = 1,\ 2,\ 3...$). Between the main peaks are additional very weak peaks, the intensity of which is much less than that of the main peaks (1/22 the intensity of the nearest main maximum). Additional peaks is *N* – 2, where *N* – number of strokes.

Additional maxima condition:

$$d\sin\varphi = \pm(2k`+1)\frac{\lambda}{2}\frac{1}{N}, \qquad k` = 1,\ 2,\ 3... \qquad (5.30)$$

$k` \neq N-1,\ N,\ N+1,\ 2N-1,\ 2N,\ 2N+1...$

Between the main peak will be located (*N* – 1) additional minima.

Additional minimum condition:

$$d\sin\varphi = \pm m`\frac{\lambda}{N}, \qquad m` = 1,\ 2,\ 3... \qquad (5.31)$$

$m` \neq 0,\ N,\ N,\ 2N,\ 3N...$

Thus, the diffraction pattern, at the diffraction on grating depends on *N* and the ratio *d*/*b*.

Let *N* = 5, *d/b* =4. Then the number of major peaks $(\sin\varphi = 1)\, k_{\max} < d/\lambda$. Between them on the $N - 2 = 3$ additional maxima and $N - 1 = 4$ additional minimum.

When $k/m = d/b = 2, 4, 8...$ - the main maxima will not, and will be the primary minimum.

Thus, the diffraction pattern with diffraction on grating will be:

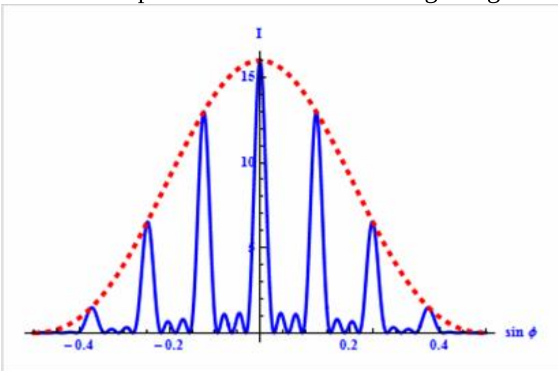


Figure 5.7

If the the diffraction grating light up a monochromatic white light, the image will be shown in Figure 5.7. If illuminated with white light, all peaks except center ($k = 0$) decompose in the range – a set of component colors, and purple lines are closer to the center and red on (because $\lambda_v < \lambda_r$ then $\varphi_r < \varphi_v$) Figure 5.8.

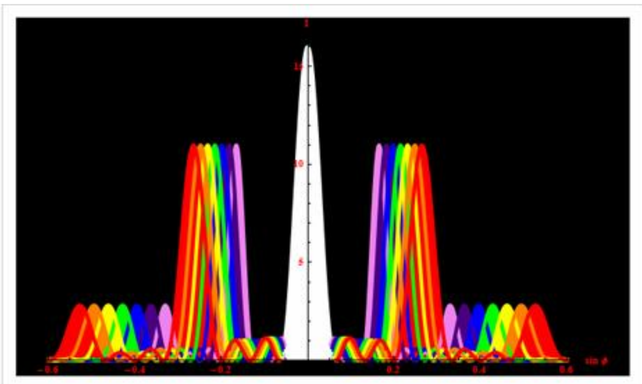


Figure 5.8.

5.3 Polarized light

The emission of a photon of light is due to the transition of an electron from the excited state to the ground. The electromagnetic waves emitted by this transition is transverse, that is a vector $\vec{E}$ and $\vec{H}$ are mutually perpendicular and perpendicular to the direction of propagation. Oscillations of the vector $\vec{E}$ in the same plane. Light, in which vector $\vec{E}$ oscillates in one direction is called a **plane-polarized light** (or electromagnetic wave). Called **polarized light**, in which the direction of oscillation of the vector $\vec{E}$ are ordered in some way.

Light is electromagnetic radiation of the total set of atoms. Atoms emit light waves independently of each other, so the light wave emitted by the body as a whole, is characterized by all sorts of vibrations of the light vector $\vec{E}$ equally. Light with all kinds of equally probable orientations vector $\vec{E}$ is **natural**. Light, which has a preferred direction of oscillation of the vector $\vec{E}$ and small amplitude oscillations of the vector $\vec{E}$ in the other direction is a **partially polarized**. In plane polarized light plane that varies vector $\vec{E}$, called the **plane ofpolarization**, plane, which varies vector $\vec{H}$, called the **plane of oscillation**.

Vector $\vec{E}$ is a light vector, because the action of light on the matter of primary importance is the electric component of the wave field acting on the electrons in the atoms of the substance.

The polarization of light refers to the direction of the electric field vector $\vec{E}$ of the wave. There are three options for $\vec{E}$:

(1) its direction and amplitude remains fixed in space - *linear* polarization,

(2) its direction rotates at angular frequency ω about the direction of propagation and the amplitude remains constant - *circular* polarization.

(3) its direction rotates at angular frequency ω and its amplitude varies between a maximum and minimum during each complete rotation - *elliptical* polarization.

For propagation in the *x*- direction the vector $\vec{E}$ may be resolved into two orthogonal components E_y and E_z. Each of the three polarization states is thus characterised by a fixed phase relationship between these components. If the phase is randomly varying the light is said to be *unpolarized.*

Polarization states

An electromagnetic wave travelling in the positive *x* direction has an electric field $\vec{E}$ with components E_y and E_z.

$$E_y = E_{0y}\cos(kx - \omega t)\vec{j}, \tag{5.32}$$

$$E_z = E_{0z}\cos(kx - \omega t - \delta)\vec{k}, \tag{5.33}$$

where δ is a relative phase. The light is polarized when δ is a constant.

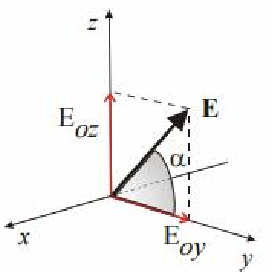


Figure 5.9 Electric field vector in light wave has components E_{0z} and E_{0y} in plane orthogonal to propagation direction along x-axis

Case 1: Linearly polarized light, $\delta = 0$.

The components are in phase. The resultant is a vector $\overrightarrow{E_p}$:

$$\overrightarrow{E_p} = \left\{E_{0y}\vec{j} + E_{0z}\vec{k}\right\}\cos(kx - \omega t), \tag{5.34}$$

at *a* fixed angle α to the *y*-axis

$$tg\alpha = \frac{E_{oz}}{E_{oy}}. \tag{5.35}$$

Case 2: Circularly polarized light, $\delta = \pm\frac{\pi}{2}$.

Consider $\delta = -\frac{\pi}{2}$ and

$$E_{oy} = E_{oz} = E_o$$

$$E_y = E_0 \cos(kx - \omega t)\vec{j}, \tag{5.36}$$

$$E_z = E_0 \sin(kx - \omega t)\vec{k}, \tag{5.37}$$

$$tg\alpha = \frac{\sin(kx - \omega t)}{\cos(kx - \omega t)} = tg(kx - \omega t). \tag{5.38}$$

The tip of the E-vector rotates at angular frequency ω at any position x on the axis, and rotates by 2π for every distance λ along the x-axis. What is the direction of rotation? Consider position $x = x_0$ and time $t = 0$.

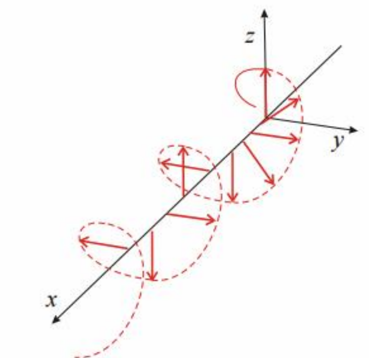


Figure 5.10 Circularly polarized light propagating in the positive x-direction

$$E_y = E_0 \cos(kx_0), \tag{5.39}$$

$$E_z = E_0 \sin(kx_0). \tag{5.40}$$

The vector is at some angle α.

At position $x = x_0$ and time $t = kx_0 / \omega$

$$E_y = E_0,\ E_z = 0\,. \tag{5.41}$$

The $\vec{E}$-vector has rotated clockwise as viewed back towards the source. See figure 5.11. This is Right Circularly Polarized light ($\delta = -\frac{\pi}{2}$). Right circularly polarized light advances like a *Left-handed* screw!

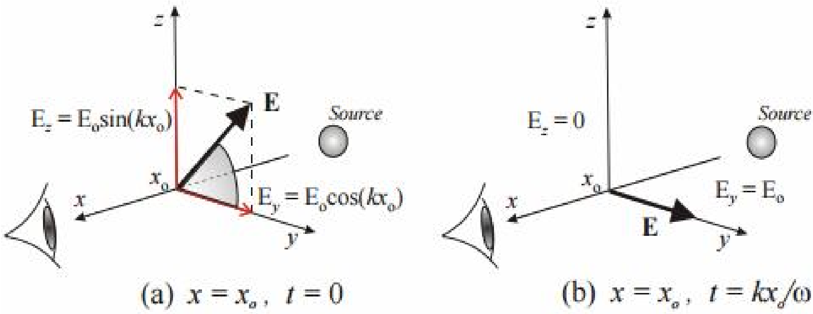


Figure 5.11 Direction of circular polarization is determined by looking back towards the source. (a) and (b) show $\vec{E}$-vector at a point $x = x_0$ at time $t = 0$, and a later time $t = kx_0 / \omega$. In this case the $\vec{E}$-vector has rotated clockwise and is denoted Right Circularly Polarized

Conversely, $\delta = +\frac{\pi}{2}$ is Left Circularly Polarized light: viewed towards the source the $\vec{E}$-vector rotates anti-clockwise. Thus the $\vec{E}$ -vector for right and left circular polarization is written:

$E_R = E_0[\cos(kx - \omega t)\vec{j} + \sin(kx - \omega t)\vec{k}$, (5.42)

$E_L = E_0[\cos(kx - \omega t)\vec{j} - \sin(kx - \omega t)\vec{k}$, (5.43)

Note that a linear superposition of E_R and E_L and gives linear or plane polarized light.

$\overrightarrow{E_p} = \overrightarrow{E_R} + \overrightarrow{E_L} = 2E_0\cos(kx - \omega t)\vec{j}$, (5.44)

If the components are of unequal amplitude then the resultant traces out an ellipse i.e the light is elliptically polarized.

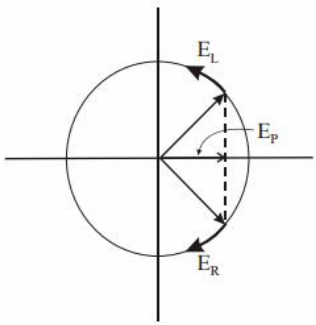


Figure 5.12

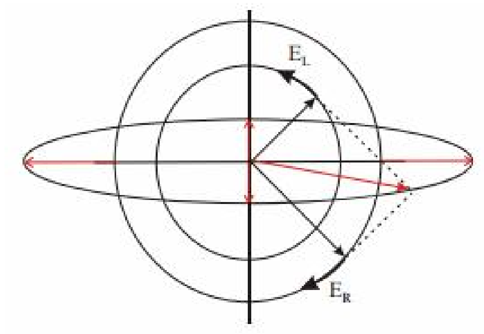


Figure 5.13 A superposition of right- and left-circularly polarized components of unequal magnitude gives elliptically polarized light

Case 3: Elliptically polarized light.

In general there is a relative phase δ between y and z components:

$$E_y = E_{0y}\cos(kx-\omega t), \tag{5.45}$$

$$E_z = E_{0z}\cos(kx-\omega t-\delta). \tag{5.46}$$

Writing

$$E_z = E_{0z}[\cos(kx-\omega t)\cos\delta - \sin(kx-\omega t)\sin\delta] \tag{5.47}$$

Substitute in (5.47) using

$$\cos(kx-\omega t) = \frac{E_y}{E_{0y}}, \text{ and } \sin(kx-\omega t) = \left[1-\left(\frac{E_y}{E_{0y}}\right)^2\right]^{1/2}, \tag{5.48}$$

we obtain:

$$\frac{E_y^2}{E_{0y}^2}+\frac{E_z^2}{E_{0z}^2}-2\frac{E_y}{E_{0y}}\frac{E_z}{E_{0z}}\cos\delta = \sin^2\delta \cdot \tag{5.49}$$

So for

$$\delta = \pm\frac{\pi}{2}, \; \frac{E_y^2}{E_{0y}^2}+\frac{E_z^2}{E_{0z}^2} = 1 \cdot \tag{5.50}$$

This is the equation for an ellipse with E_{0y}, E_{0z} as the major/minor axes, i.e. the ellipse is disposed symmetrically about the y / z axes.

For $\delta = \pm\frac{\pi}{2}$ the axes of symmetry of the ellipse are rotated relative to the y / z axes by an angle θ

$$tg\,2\theta = 2\frac{E_{0y}\cdot E_{0z}}{E_{0y}^2 - E_{0z}^2}\cos\delta. \tag{5.51}$$

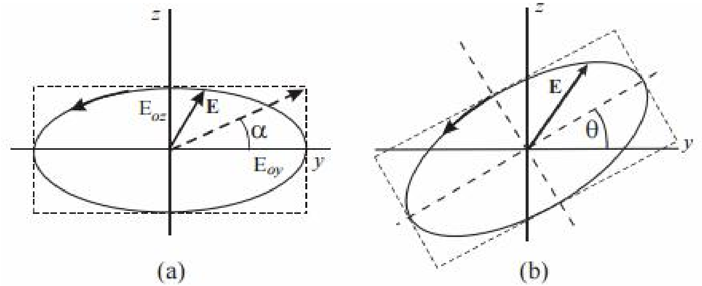


Figure 5.14 Elliptically polarized light (a) axes aligned with y, z axes, (b) with axes at angle θ relative to y, z axes

As δ varies from $0 \rightarrow 20\pi$ the polarization varies from linear to elliptical and back to linear. Thus we may transform te of pol the sta arization between linear and elliptical or vice-versa by altering the relative phase of the two components. This can be done using a material that has different refractive index for two different directions of polarization i.e. a birefringent material.

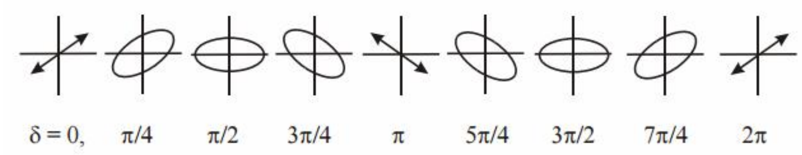


Figure 5.15 General elliptical state of polarization for different values of relative phase δ between the components

Most simply polarized light can be obtained from natural light by reflection of light waves from the boundary between two dielectrics.

If natural light is incident on the boundary between two dielectrics (eg, air-to-glass), then part of it is reflected and part is refracted and propagates in the second medium.

Brewster's law:

At an angle of incidence equal to the Brewster angle i_{Br}: 1. reflected from the boundary between two dielectric beam is completely polarized in the plane perpendicular to the plane of incidence, 2. the degree of polarization of the refracted beam reaches a maximum value less than unity, 3. refracted ray is partially polarized in the plane of incidence, 4. the angle between the reflected and refracted rays is equal to 90°; 4. Brewster angle is equal to the tangent of the relative refractive index

$$tgi_{Br} = n_{21} = \frac{n_2}{n_1} \tag{5.52}$$

-Brewster's law.

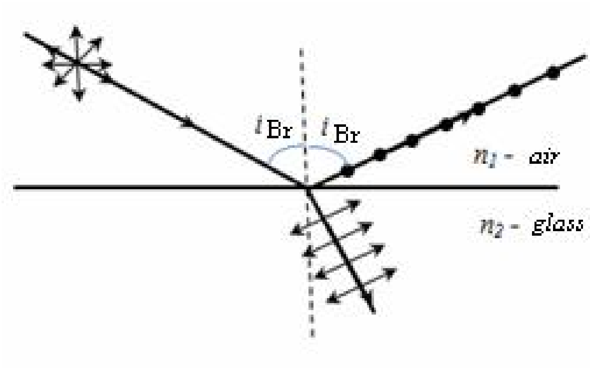


Figure 5.16

n_{12} - the refractive index of the second medium with respect to the first.

The angle of incidence (reflection) - the angle between the incident (reflected) ray and the normal to the surface. The plane of incidence - the plane through the incident ray and the normal to the surface.

The degree of polarization of the refracted light can be significantly enhanced by repeated refraction of the condition of the light on the boundary between at the Brewster angle. If the glass ($n = 1{,}53$) the degree of polarization of the refracted beam is ≈ 15%, after refraction at 8-10 overlapping glass plates, the light will be released almost completely polarized - ream Stoletov.

Polarized light can be obtained from a natural with polarizers - anisotropic crystals transmit light in only one direction (Iceland spar, quartz, tourmaline).

Chapter 6 QUANTUM AND ATOMIC PHYSICS

To begin our study of modern physics we have to address an old question: "Is light composed of waves or particles? Indeed, experiments on interference and diffracton early in the nineteenth century led physicists to decide in favour of the wave theory. But surprises were in store for them, beginning with a revolutionary new interpretation of the process of radiaton by a black body. (An ideal system that absorbs all incoming radiation is called a blackbody).

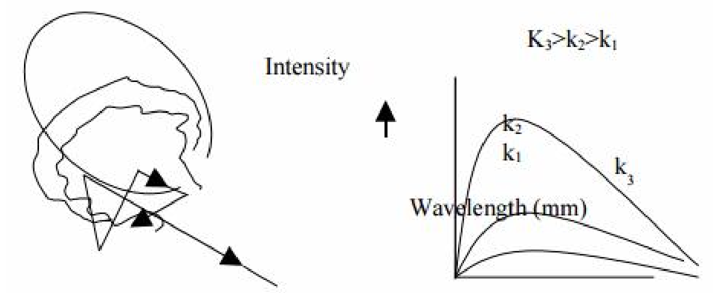
In this unit we will deal with some of the changes that occurred during the transition to Modern Physics. We shall introduce you to new experimental knowledge and to new theories about the atom and its constituent particles, about radiations, and about physical systems containing these things.

The ideas of physics you have already encountered generally apply in modern physics, but many of them must be reinterpreted . We shall still use the principles of conservation of energy and momentum, for example, and the concepts of velocity, mass, position and time. However, our literal everyday interpretation of these concepts often fails as in the atomic world.

6.1 THE PHOTON AND THE ATOM

6.1.1 Blackbody Radiation and the Quantum Theory

A major surprise to physicists at the end of the nineteenth century concerned the distribution of wavelength's emitted by a blackbody. As you know, most objects absorb some incoming radiation and reflect the rest. An ideal system that absorbs all the incoming radiation is called a blackbody.



a) b)

Figure 6.1

Physicists study blackbody radiation by observing a hollow object with a small opening as shown in figure 6.1,a. The system is a good approximation to a blackbody because it traps radiation. The light emitted by the small opening is in equilibrium with the walls of the object because it has been absorbed and re-emitted many times.

Figure 6.1,b shows the intensity of blackbody radiation at three dfferent temperatures. You can see that as the temperature increases, the total energy emitted by the body (the area under the curve) also increases. In addition, as the temperature increases, the peak of the distribution shifts to shorter wavelengths.

Scientist could not account for this experimental results with classical physics. Figure 6.2 compares an experimental plot of the blackbody radiation spectrum with the theoretical picture of what this curve should look like based on classical theories.

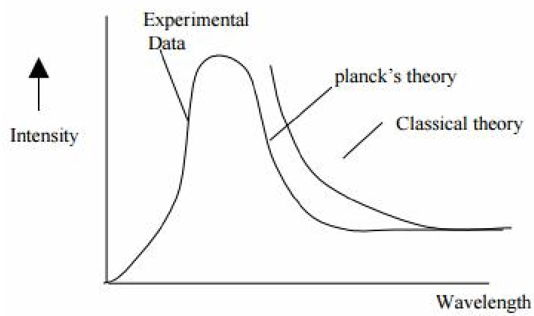


Figure 6.2

Classical theory predicts that as the wavelength approaches zero, the amount of energy being radiated should become infinte. This is contrary to the experimental data, which show that as the wavelength approaches zero, the amount of energy being radiated also approaches zero. This contradiction is often referred to as the ultraviolet catastrophe because the disagreement occurs at the ultraviolet end of the spectrum.

In 1900, Max Planck (1858 – 1947) developed a formula for blackbody radiation that was in complete agreement with experiment data at all wavelenghts.

Planck proposed that blackbody radiation was produced by submicroscopic electric oscillators, which he called resonators. He assumed that the walls of a glowing cavity were composed of billions of these resonators, all vibrating at different frequencies.While most scienctist naturally assumed that the energy of these resonators was continuous, Planck made the radical assumption that these resonators could only absorb and then re-emit certain discrete amounts of energy. With this method, planck found that the total energy of a resonator with frequency ν is an integral multiple of $h\nu$, as follows

$$E = nh\nu \quad (6.1)$$

where n is a positive integer called a quantum number, and the factor h is planck's constant.

Because the energy of each resonator comes in discrete units, it is said to be quantized, and the allowed energy states are called quantum states or energy levels. With the assumption that energy is quantized, planck was able to derive the curve shown in Figure 6.2.

According to planck's theory, the resonators absorb or emit energy in discrete units of light energy called quanta (now called photons) by "jumping" from one quantum state to another adjacent state. It follows from Eq. 6.1 that if the quantum number, n changes by one unit, the amount of energy radiated changes by $h\nu$. Hence, the energy of a light quantum, which corresponds to the energy difference between two adjacent levels, is given by.

$$E = h\nu \quad (6.2)$$

A resonator will radiate or absorb energy only when it changes quantum states. If a resonator remains in one quantum state, no energy is absorbed or emitted.

6.1.2 Photon Energy, Momentum and Wavelength

A beam of electromagnetic radiation, considered as an electromagnetic wave, is characterised by its frequency, ν or its wavelength λ wich are related by

$$\nu = \frac{c}{\lambda} \quad (6.3)$$

The same beam, considered as a stream of photons, is characterised by the energy E or the momentum p of the individual photons. A photon has no mass and travels at a speed, $c = 3.00 \times 10^8$ ms $^{-1}$. Its energy and momentum are related by

$$E = cp \quad (6.4)$$

The vital connecting link between these two descriptions of the same beam of radiation, proposed by Einstein, is that the photon energy E is proportional to the freqeuncy f of the electromagnetic wave.

$$E = h\nu \quad (6.5)$$

The constant of proportionality h is the planck's constant.

Electromagnetic radiation can be classified according to the energy of its photons, or the wavelength, or the frequency, whichever is most convenient. For example, from $\nu = \frac{c}{\lambda}$, Eq. 6.5 can be written to show the relationsgip between photon energy E and wavelength.

$$E = cp \quad (6.6)$$

Photon energies are usually specified in electrovolts, and wavelengths in angstrom unit. Inserting the numerical values of h and c and the required conversion factors, we write Eq. 6.6 in a form which is convenient for calculations:

$$E = \frac{12.4\times10^{3}\ \overset{0}{A}.eV}{\lambda} \tag{6.7}$$

which gives the photon energy E in electrovolts when the wavelength λ is in angstrom units.

Example

Find the energy of the photons in a beam whose wavelength is:

(a) 6.2 x 10^3 A^0 (orange light)

(b) 4.13 x 10^3 A^0 (violet light)

Solution

(a) A photon beam of arange light has an energy which is, from Eq. 3.7

$$E = \frac{12.4\times10^{3}\ \overset{0}{A}.eV}{6.2\times10^{3}\ \overset{0}{A}} = 2.0\ eV$$

(b) A photon of the beam of violet light has an energy

E = 12.4 x 10^3 A^0 eV = 3.0 eV

4.13 x 10^3 A^0

6.1.3 The Nuclear Atom

The model of the atom in the days of Newton was that of a tiny, hard indestructible sphere. This model was a good basis for the kinetic theory of gases. However, new models had to be devised when experiments revealed the electrical nature of atoms. The discovery of the electron in 1897 prompted J.J. Thomson (1856 – 1940) to suggest a new model of the atom. In Thomson's model, electrons are embedded in a sperical volume of positve charge like seeds in a watermelon, as shown in Figure 6.3.

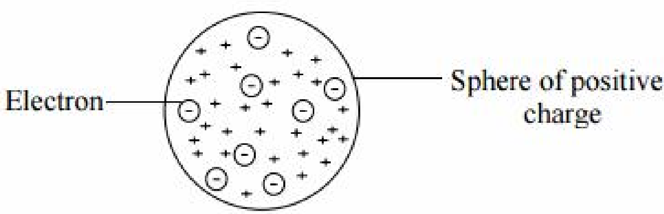


Figure 6.3

In 1911 a " planetary model", in which the electrons revolve like planets round a small, massive, positively charged nucleus resulted from

Rutherford’s experiments on the scattering of α -particles by gold or platinum foil.

The nucleus is now believed to consist of protons of mass 1 and charge + 1e, and neutrons of mass 1 and charge 0. Surrounding the nucleus are planetary electrons of mass 1/1840 and charge - 1e. The constitutions of the three lightest elements are as follows:

Table 3

	Nucleus	Planetary electrons
Hydrogen	Charge 1 mass (1 proton)	1
Helium	Charge 2 mass 4 (2 protons and 2 neutrons)	2
Lithium	Charge 3 mass 7 (3 protons and 4 neutrons)	3

The atoms of the remainder of the elements are built up in a similar way. The atomic number gives the charge on the nucleus and also the number of planetary electrons; the atomic mass minus the atomic number gives the number of neutrons in the nucleus.

6.1.4 The Bohr Atom

The difficulty with Ruthford’s model atom is that, according to the laws of classical physics, it cannot exist. The electrons rotating round the nucleus must have an acceleration towards the nucleus and consequently should radiate energy continuously, spiralling towards the nucleus to provide the energy.

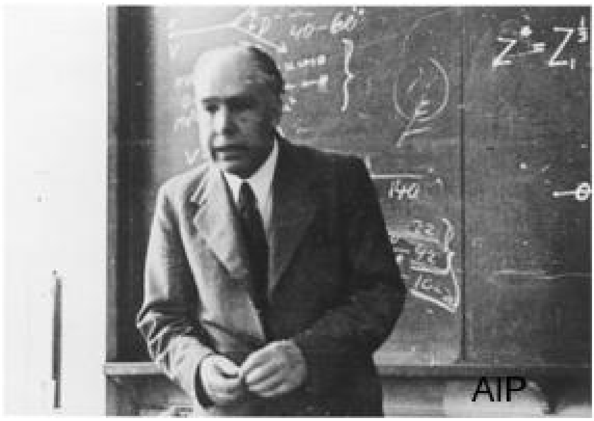


Figure 6.4

In 1913 Niels Bohr applied the quantum theory to the nuclear atom. Bohr assumed that the planetary electrons in an atom can exist only in a limited number of stable orbits or stationsary states having definite amount of energy but not emitting radiation, and that radiation occurs only when an electron jumps from one stable orbit to another. He assumed

$$h\nu = E_2 - E_1 \tag{6.8}$$

Where is the frequency of the energy radaited when the electron jumpls from orbit of energy E_2 to one of energy E_1

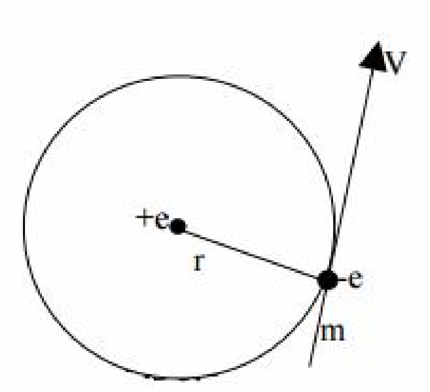


Figure 6.5

The hydrogen atom consists of a single electron of charge – *e* revolving round a nucleus of charge +*e*. Suppose the electron has a mass *m*, that it revolves in a circle, and that when the radius of its orbit is *r* its velocity is υ (Figure 6.5). The electrostatic altraction between the nucleus and the electron must equal the centrifugal force; thus

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m\upsilon^2}{r} \tag{6.9}$$

At this point Bohr made a further assumption which seems purely arbitrary; on the new wave mechanics, however, its meaning becomes more understandable. He assumed that the angular momentum of the electron, $m\upsilon r$, is always an exact multiple of $h/2\pi$; thus

$$m\upsilon r = \frac{nh}{2\pi}, \tag{6.10}$$

when n is an integer called the quantum number. The angular momentum is then said to be quantized.

The next step is to find the energy of the electron on its orbit. The potential energy of the electron is the work done in bringing it from infinity to its orbit and this is – ($\dfrac{e^2}{4\pi\, r\varepsilon_0}$); the negative sign indicates that work is done by the electron as it approaches the oppositely charged nucleus. The kinetic energy of the electron is $m\upsilon r$ and from Eq. 6.9 this is equal to ($\dfrac{1}{2}\dfrac{e^2}{4\pi\, r\varepsilon_0}$).

$\therefore$ Total energy of electcon =

$$k.e + p.e = \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r} - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{1}{2}\frac{e^2}{4\pi\varepsilon_0 r}$$

Form Eq. 6.9

$$m\upsilon^2 r = \frac{e^2}{4\pi\varepsilon_0}\cdot \tag{6.11}$$

Squaring Eq. 6.10

$$m^2\upsilon^2 r^2 = \frac{n^2h^2}{4\pi^2}\cdot \tag{6.12}$$

Dividing 6.12 by 6.11, $r = \dfrac{n^2h^2\varepsilon_0}{\pi e^2 m}$

Energy of electron

$$= -\frac{1}{2}\frac{e^2}{4\pi\varepsilon_0} = \frac{e^4 m}{8n^2h^2\varepsilon_0^2}, \tag{6.13}$$

$$h\nu = E_2 - E_1 = \frac{e^4 m}{8h^2\varepsilon_0^2}\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}, \tag{6.14}$$

$$\nu = \frac{e^4 m}{8h^3\varepsilon_0^2}\left\{\frac{1}{n_1^2} - \frac{1}{n_2^2}\right\}.$$

In 1884 Balmer, a swiss schoolmaster, had discovered a formula representin the series of visible spectral lines of hydrogen. The formula can be written in the form

$$\nu = R\left\{\frac{1}{2^2} - \frac{1}{m^2}\right\}, \tag{6.15}$$

where m is an integer greater than 2. The value R, called the Ry dberg constant, was known from the measured frequencies of the spectral lines. Bohr was able to calculate the value of the constant corresponding to R in his formular , i.e $= \dfrac{e^4 m}{8h^3\varepsilon_0^2}$, from the known values of e, m and h. The agreement was perfect within the limits of experimental error.

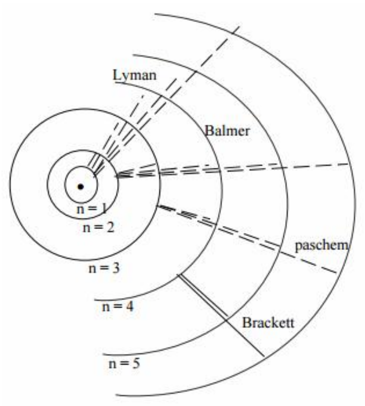


Figure 6.6

Bohr explained the emission of the visible spectral lines of hydrogen as due to electrons jumping from outer orbits to the second orbit (Figure 6.6). Other spectral series for hydrogen were also known, the Lyman series in the ultraviolet, the Paschen and the brackett series in the infra-red. These are due to electrons jumping into the first, third and fourth orbits respectively.

In the normal condition of the hydrogen atom the electron is in its innermost orbit, for which n = 1. As a result of collision, say in a discharge tube the electron may be knocked into orbits for which n = 2 or 3, etc; the atom is then said to be excited. The electron will jump back to the innermost orbit, possibly in one jump or in stages, given out the appropriate radiation. If the electron is completely removed the atom is said to be ionized.

When an atom emits a photon, the law of conservation of energy implies that the enrgy of the atom must change from an initial value E*u* (the subscript *u* denotes the upper energy level, as in Figure 6.7) to a lower value E_τ such that

$$E_{photon} = E_u - E_\tau \qquad (6.16)$$

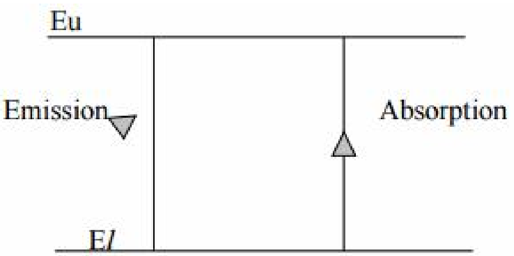


Figure 6.7

This "Bohr frequency condition" determines the photon frequency ν, since $E_{photon} = h\nu$. The empirically determined sequence of terms, whose differences determine the frequencies in the hydrogen atom spectrum, must then be proportional to the possible values of the energy of the hydrogen atom. These energies called the energy levels of the hydrogen atom, are given by (see Eq. 6.13)

$$E_n = \frac{-21.8\times 10^{-19}}{n^2} J = \frac{-13.6\, eV}{n^2}. \tag{6.17}$$

where n is the number of the level, or the principle quantum number. That is E_1 = -13.6 eV, E_2 = -3.40eV, E_3 = - 1.51eV, and so on, as shown in Figure 6.8.

For example, a hydrogen atom can exist for a shortwhile (10^{-8}s) in a state with energy E_3 = - 1.51 eV. If after the emissin of a photon, the atom is left in the state with the lower energy, E_2 = - 3.40 eV (Figure 6.8), the photon emitted must have an energy, according to Eq. 6.16, given by

$$E_{photon} = E_3 - E_2 = (-1.51 eV) - (-3.40\, eV) = 1.89\, eV$$

The wavelength of this photon is

$$\lambda = \frac{hc}{E} = \frac{12.4\times 10^3\, \overset{0}{A}.eV}{1.89\, eV} = 6.56\times 10^3\, \overset{0}{A}.$$

$$(h = 6.63\times 10^{-34}\, J.s,\, c = 3\times 10^8\, ms^{-1}, 1\, eV = 1.6\times 10^{-19}\, J, 1\, \overset{0}{A} = 10^{-10}\, m).$$

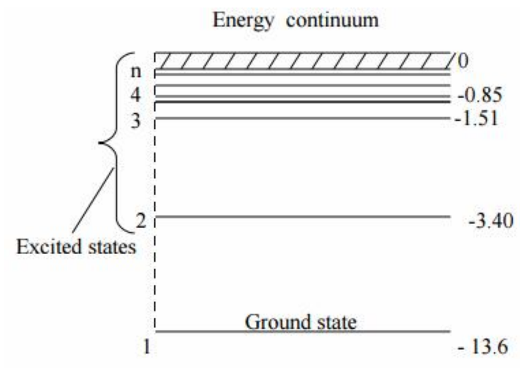


Figure 6.8

The novel idea thus advanced by Bohr is that the energy of the hydrogen atom (and infact, all atoms molecules, and any bound system) can have only certain discrete values (instead of a continous range of values) in its bound states. That is, there exist discrete energy levels.

The lowest of these energy levels is called the ground state, and all the higher levels are called excited states. The value E = 0 is the energy when the electron and the proton are completely separated and at rest. Since this energy level is 13.6 *eV* above the ground state, we see that 13.6 *eV* must be supplied to a hydrogen atom in its ground state in order to remove the electron, that is, to ionize the atom. In other words, the binding energy of a hydrogen atom against separation into a proton and an electron is 13.6 *eV*.

When the electron and proton are separated, they can have any amount of kinetic energy. Corresponding to these states, which are not bound states, the energy level diagram (Figure 6.8) shows a continous range of possible values of the energy of the electron proton system.

When an atom gives off energy it passes from an upper to a lower energy level. If an atom absorbs energy it passes from a lower to a higher level (Figure 6.6). For the absorption of a photon, the Bohr frequency condition still applies, but now the lower energy E_l is the initial energy of the atom.

6.1.5 X-rays. X-ray Spectra

X-rays are a form of electromagnetic radiation having short wavelengths and high frequencies, about $10^{18} - 10^{19}$ *Hz*.

Their production can be explained using the energy levels theory, since *X*-rays are produced by streams of high-energy electrons in collision with atoms of high atomic number, such as tangsten or molybdenum.

X-rays are produced, by two distinct processes when the electron hit a metal target.

1) The electrons suddenly lose energy when they collide with the target nuclei. A large percentage of the energy is converted into heat, but some is converted into *X*-ray photons. Each electron gains the same energy from being accelerated by the tube voltage, but varying fractions of this energy are converted into photons. As usual, the energy of the photon created $h\nu$. The maximum frequency (minimum wavelength) will be produced when all the energy gained by an electron is converted into a photon. A continous range of smaller frequencies (greater wavelengths) is created by smaller fractions of the electrons energies being converted into photons. This continous *X*-ray spectrum is typical of the tube voltage but independent of the target material.

2) The electron gives some of its energy to an electron in a target atom. The electron in the atom ‘jumps’up to a higher level and X-rays are emitted as the electron falls back to the lower level. The energy $h\nu$ of the photons produced is equal to the energy level difference. The energy of an *X*-ray photon is very large, and so the energy levels involved must have a large separation. The frequencies (and wavelength’s) emitted have discrete values, which are typical of the target material. This gives rise to the peaks in Figure 6.9.

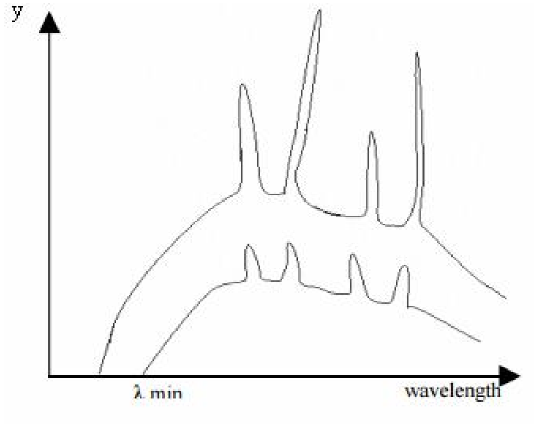


Figure 6.9

6.1.6 The X-ray Tube

Figure 6.9 shows the essentials of the *X*-rays tube invented by Coolidge in 1913. the tube is exhausted as highly as possible so that no discharge would pass through it if it were used as a gas tube. The electrons are emitted by a tungsten filament C and the rate of their emission, which determines the intensity of the X-rays, can be controlled by the heating current through the filament.

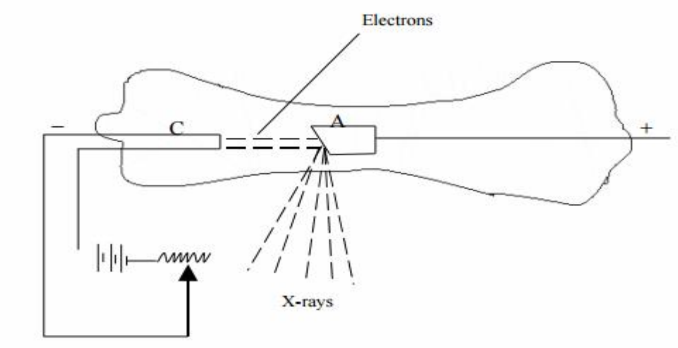


Figure 6.10

The hardness of the *X*-rays is controlled separately by the p.d. between *A* and *C*. Their hardness depends on their frequency and if this is greater the faster the speed of the electrons hitting the target. The phenomenon is a kind of reverse photoelectric effect and a similar relation holds:

$$V_e = h\nu \tag{6.18}$$

where V is the p.d. through which the electrons fall, e = charge of the electron, h = Planck's constant, ν = maximum frequency of the X-rays emitted.

As mentioned earlier, only about 1% of the energy of the electrons is converted into X-rays. The rest of the energy appears as heat, and hence the target is made of a metal of high melting – point, such as tungsten or molybdenun, embedded in a solid block of copper which is a good thermal conductor.

6.1.7 Nature and Properties of X-rays

X-rays are electromagnetic waves, like light, having the same speed of 3x $10^8 ms^{-1}$, but having a wavelength about a thousand times shorter than that of light waves. Their wavelengths range from 0.06 to 100 Angstrom units againsts 3,900 to 7,600 Angstroom units of visible light.

(1) X-rays travel in free space with a speed of 3 x $10^8 ms^{-1}$, the same that of light.

(2) They affect a photographic plate much more intensely than light because of their very much shorter wavelength.

(3) They are unaffected by magnetic and electric fields, clearly showing that they are not a stream of charged particles.

(4) Like light, they liberate photoelectrons when allowed to fall on certain metals.

(5) Scattered X-rays show a marked degree of polarization like the scattered sky light.

(6) They undergo reflection, refractions, dispersion and diffraction.

(7) They can ionise a gas (or air) through which they are allowed to pass.

(8) They cause fluorescence in substances like barium plastinocyanide, zinc sulphide and cadmium tun state.

(9) As a direct consequence of their extemely small wavelenght, they can easily pass through matter, opaque to ordinary light, as for example, paper, card board, wood, thin sheets of metal and also through human flesh. They are, however, absorbed by denser substances, like iron, lead and bones (due to their high calcium content) and therefore cast their shadows on fluorescent screens or photographic plates.

Thus if I_o and I are the respective intensities of X-rays before and after passing through a material of thickness x (where intensity is the amount of energy carried in unit time across unit area, perpendicular to the direction of flow of enrgy), we have

$$I = I_0 \cdot e^{mx} . \qquad (6.\ 19)$$

Where u is called the linear absorption coefficient of the material and its dimensions are therfore, those of reciprocal length.

6.1.8 Uses of X-rays. The usefulness of *X*-rays is largely due to their penetrating power

Medicine* .** Radiographs or *X*-ray photographs are used for a variety of purposes. As mentioned earlier, *X*-rays can pass through flesh but not through bones. Therefore, sharp dark shadows of the bony parts of the body are obtained against a lighter background on a flourescent screen or a photographic plate, if these be interposed in the path of the ***X-rays. Such ***X***-ray photographs are called radiographs. Dislocation, fractures and the presence of foreign bodies like pins, bullets, etc. inside the human body can thus be easily deteted. In radio therapy, periodic ***X***-ray exposures, in properly controlled doses, are given for treatment of obstinate skin diseases and malignant or cancerous growths or tumours.

Industry. Casting and welded joints can be inspected for internal imperfections using X-rays. A complete machine may also be examined from a radiograph without having to be dismantled.

X-ray crystallography. The study of crystal structure by ***X***-rays is now a powerful method of scientific research. The first crystals to be analysed were of simple compounds such as sodium chloride but in current years the structure of very complex organic molecules has been unravelled.

6.2 RADIOACTIVITY AND THE NUCLEAR ATOM

6.2.1 Radioactivity and the Nuclear Atom

In unit 19 we saw that the nucleus of the hydrogen atom consists of a single particle, called proton, with just a single electron going round it in a circular orbit. The mass of a proton is about 1836 times that of an electron and it carries a positive charge (+e) equal in magnitude to the negative charge (-e) on an electron. The atom is thus electrically neutral, with practically the whole of it mass concentrated in the nucleus. In this unit we shall consider the structure of the atoms of other elements. In particular, we will see that the occurreance of discrete energy levels in a hydrogen atom is only a particular instant of a very general phenomenon. The energy of any bound system is restricted to certain discrete values which are called the energy levels of the system. For instance, the nucleus 60/28N_i has a stable ground state and many discrete energy levels corresponding to excited states. We will also see that the excited states of an atom are unstable with a characteristic mean life τ.

The spontaneous of a nucleus is called radioactivity and is classified according to the particle emitted in α decay an α-particle $({}^{4}_{2}H_{e})$ is emitted, are often referred to as γ rays.

The protons and neutron both being constituents of the nucleus are called nucleons and their total number gives what is called the mass number A of the atom. The number of protons in the nucleus (because this is equal to the number of electrons) gives the atomic number, Z of the atom. Therefore, the number of neutrons in the nucleus, $N = A - Z$, i.e equal to the difference between the mass number and the atomic number of the atom. To summarize,

Atomic number of an atom, Z = number of protons or number of electrons in the atom.

Mass number of an atom, A = number of nucleous in the atom = number of protons + number of neutrons in the nuleus.

Number of neutrons in an atom, $N = A - Z$ = mass number minus atomic number.

Nuclei with identical number of protons (i.e. with the same value of Z) or identical number of neutrons (i.e. the same value of N) belong to the same species and a nuclear species is called a nuclide. The notation $_zX^A$, where X stands for the chemical symbol of the atom or the element, the subcript A for the mass number and the subscript Z for the atomic number of the atom.

Nuclides with the same atomic number, Z (i.e with the same number of protons) are called isotopes; those with the same value of mass number A (i.e. with the same number of nucleous) are called isobars and those with the same value of $N = A\text{-}Z$ (i.e with the same number of neutrons) are called isotoes. Thus, for example, $_{17}Cl^{37}$ is an isotope of Cl^{35}, because Z = 17. It is an isobar of $_{16}S^{37}$, because for both A = 37 and it is an isotope of $_{19}K^{39}$ because for both $N = A\text{-}Z$ = 20 (i.e. 37 – 17 = 39 – 19 = 20).

6.2.2 Atomic Mass Unit

Hitherto, we have given the masses of protons and neutrons in kilogram. It will interest you to learn that the International Union of Pure and Applied Physics (IUPAP) decided in 1960 to adopt a new masss scale for the measurement of mases in nuclear Physics. It is called the atomic mass scale and the atomic unit on this scale (written as amu) is 1 /12 of the mass of $_6C^{12}$, the most abundant and the most stable isotope of carbon. It is always preferable to express the masses of atoms on the atomic mass scale rather than in kilogram, because it is more suitable for the magnitude of atomic masses and is far more accurate, since atomic masses can be determined very accurately relative to the carbon atom $_6C^{12}$.

Now, since the mass of an atom is equal to its atomic weight divided by Avogadro number (6.02 x 10^{23}), we have

$$1\,amu = \frac{(1/2 \times 12)}{6.02\times 10^{23}}\,g = 1.66\times 10^{-27}\,kg$$

which is nearly the mass of a hydrogen atom.

In accordance with Einstein's mass energy equation $E_0 = m_0 c^2$, where E_0 is the energy of a resit mass m_0 (c being the velocity of light in free space = 3.0 x $10^8 ms^{-1}$).

Hence $1\,amu = 1.66 \cdot 10^{-27} \cdot (3 \cdot 10^8)^2 = 1.49 \cdot 10^{-10}\,J$.

And since $1.602 \cdot 10^{-19}\,J = 1\,eV$, we have

$$1\,amu = \frac{1.49 \cdot 10^{-10}}{1.602 \cdot 10^{-19}} = 9.31 \cdot 10^8\,eV = \frac{9.31 \cdot 10^8}{10^6} = 931\,MeV \cdot$$

Thus

$$1\,amu = \frac{1}{12}{}_6C^{12} = 1.66 \cdot 10^{-27}\,kg = 1.49 \cdot 10^{-10}\,J = 9.31 \cdot 10^8\,eV = 931\,MeV \cdot$$

The atomic mass scale is also referred to as the isotropic mass scale and hence the mass of an atom in amu is its isotropic mas.

6.2.3 Nuclear Binding Energy

The mass, M of an atom can be determined directly by the mass spectrograph. (Here M stands for the mass in *kg* of an individual atom).

The mass that an atom ought to have as an assembly of neutrons and protons and electrons can be calculated, for there are Z protons (mass M_p), Z electrons (mass M_e) and $N = (A - Z)$ neutrons (mass M_n), giving a total mass of $Zm_p + Zm_c + NM_n$.

But the measured mass M is less than this by a differene $\Delta M = (Zm_p + Zm_c + Nm_n - M)$, which is called the mass defect.

The mass defect ΔM represents the energy ΔMc^2 that would be liberated if the nucleons and the electrons were assembled, and it is therefore the energy which would have to be supplied in order to dismember the atom again. So the greater the value of ΔM, the greater is the stability of the atom against this kind of breaking – up. A better stability criterion is the mass deflect per nucleon, $\Delta M/A$, which represents the binding energy per nucleon.

The mass of a nucleon, about 1.6 x 10^{-27}*kg*, is roughly 1.5 x 10^{-10} *J*, say about 1 *eV*; one millionth of this is a difference of 1 *KeV* per nucleon. Therefore, we need to used very accurate mass values if the calculation is to Mean much.

As we saw in the last section, masses are expressed in terms of the unified mass constant, M_u, which is the mass of the ^{12}C atom (amu), and some relevant values are:

Unifield atomic mass constant, $Mu = 1.66 \cdot 10^{-27}\,kg = 931.478 MeV$

Mass of proton, $M_p = 1.00727647\,Mu$

Mass of electron, $M_e = 5.485803 \cdot 10^{-4}\ Mu$

Mass of neutron, $M_n = 1.008665\ Mu$

The graph of Fig. 6.1 shows how the binding energy per nucleon varies with *A*. The only three points that do not lie on the smooth curve are the particularly stable (even-evn and multiple of 4) nuclides ^4He, ^{12}C and ^{16}C. The curve has a maximum at about A = 50, but over the range *A* = 80 to *A* = 250 the gradient is almost uniform and is not very considerable. The higher a nuclide's place on the curve, the more stable it is, and any nuclear change which ascends the curve librates energy. The two ways up are fusion from the left-hand-side, and fission (or on a smaller scale the disintegratioin that gives α-emission) form the right –hand side.

6.2.4 Nuclear Forces

Now that we know that a nucleus consists of protons carrying +*e* charges) and neutron (carrying no charge), the question arises as to what keeps the nucleus from falling apart in view of the fairly large force of repulsion between the protons. Definitely, the gravitational force of attraction between the nucleous is much too weak to hold them together. There must, therefore be some other very strong force of attraction, binding the protons and neutrons so compactly together, quite different from the forces with which we are familiar in classical physics.

Various experiments on scaltering of nuclei by one another, on collision, clearly show that there are indeed very strong attractive forces which are effective only within a very small range of the order of 10^{-15} *m*. It is, therefore, not a mere coincidence that the radius of a nucleus too is of the same order (10^{15} *m*). These short range forces are called nuclear forces and are effective only when two nuclei just touch each other and fall to zero as soon as they are separated.

Another significant point about these attractive forces is that they are the same between protons and protons (*p*-*p* force), between protons and neutrons (*p*-*n* forces) and between neutrons and neutrons (*n*-*n* forces), in spite of the fact that there is also a repulsive force between protons and protons. This latter force must obviously be negligible compared to the attractive nuclear force between them. Hence, so far as nuclear forces are concerned, protons and neutrons are one and the same thing, the positive charge on the protons being of no consequence at all. This fact is referred to as the chargeindependence character of the nuclear forces.

6.2.5 Three Main Types of Rediation

Alpha Radiation: This is a particle, comprising two protons and two neutrrons. Hence it has a mass about 8000 times that of the electron and a charge of +3.2 x 10^{-19}C.

Beta Radiation: There are, infact, two B particles, the B– and the B+. The B- is the B- particle normally referred to in Nuclear Physics and it is an electron. Electrons do not infact exist in the nucleus, but the beta particle is created and ejected from the nucleus when a neutron changes into a proton. The B+ particle (a position same mass as electron, same charge as proton) is created and ejected when a proton changes into a neutron.

Gamma Radaition: This is a photon of electromagnetic radiation sometimes ejected by nuclei following beta or a alpha emission, when the nucleus adjusts its energy levels. It has no mass and no charge.

Radioactive Decay

When the nucleus of a radioactive atom disintegrates, it may emit an alpha particle or a beta particle. Gamma rays may precide or follow either kind of particle. When an alpha particle is emitted, the mass number A decreases by 4 and the atomic number *Z* by 2, because the positively charged alpha particle carries off two electronic units of charge, leaving the positve nuclear charge less by two electron units (conservation of energy). Emitting a Pparticle does not alter the mass number, it increases the atomic number by one, because the negatively charged B-particle carries off one electronic unit of charge, leaving the positive nuclear charge greater by one electronic unit.

The disintegration of an individual nucleus is a random event. The word decay (or rate of decay) is used for the rate at which the number N of serviving nuclei in a given sample of a pure radiative nuclide diminishes with time. Since the decay is andom, this rate depends only on itself. The rate - dN/dt at any given time is proportional to the number of surviving nuclei at that time. So $-dN/dt = \lambda N$ where λ is a constant which depends on the nuclide called the decay constant.

Integrating gives loge $N/N_0 = -\lambda t$, where $N = N_0$ at $t = 0$, so that at time *t*.

$$N = N_0 \cdot e^{-\lambda t} \quad . \tag{6.20}$$

The number of nuclei that have disintegrated at time t is given by

$$N_0 - N = N_0 \cdot (1 - e^{-\lambda t}) \,. \tag{6.21}$$

The half-life $T_{1/2}$, of a radioactive nuclide is defined as the time, from the original observation, for the number of surviving nucleir to be reduced to one-half. Thus, for $N/N_0 = 1/2 = -\log_e 2 = -\lambda T_{1/2}$, and

$$T_{1/2} = (\log_e 2)/\lambda = \frac{0.693}{\lambda} \,. \tag{6.22}$$

The quantity that is actually observed as the ‘activity’ is a count-rate, or the equivalent of an ionisation current, which gives the rate of decay – dN/dt at that instant. But dN/dt is proportional to *N*, whence

$$- dN / dt = -(dN / dt) \cdot e^{-\lambda t} \quad (6.23)$$

and the half-life is therefore also the time, from the initial observation, for the activity to be reduced to one-half. However, unless the product is a stable nuclide, or one with a very long half-life, there is more than one contribution to the activity. So it is only in suitable cases that the measured activity enables the half-life to be found directly.

It follows from Eq. 6.23 that a large λ means a short half-life because at any particular time there is a large rate of decay for a given number of atoms. After one half-life both the number of atoms and the activity have halved. After two half-lives they both have quartered, and so on. Always remember that the half-life of a nuclide is the average time it takes for half its atoms to decay.

An activity of 1 disintegration per second is 1 *Bq* (becquerel) Half-lives vary from millionths of a second to thousands of millions of years. Radium 226 has a half-life of 1622 years, therefore starting with 1 g of pure radium, ½ g remains as radium after 1622 years, ¼ g after 3244 years and so on. An exponental decay curve, like that for the discharge of a capacitor through a high resistor, is shown in figure 6.11 to illustrate the idea of half-lives.

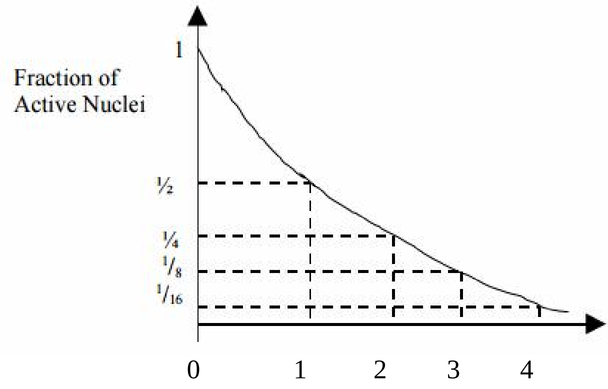


Figure 6.11

The first member of a series decays into a daughter product, but this in turn will probably itself decay until a stable non-decaying isotope is produced. The list of all the members of the family is called a radioactive series. Some time after the production of the original source all the members of the series will be in equilibrium, i.e they will be produced from their parent at the same rate at which they are decaying, i.e $N_1\lambda_1 = N_2\lambda_2 = N_3\lambda_3 ....$, where N_1, N_2 are the equilibrium numbers of atoms of each memebr of the series.

An example of a series is:

$${}^{23}_{90}\mathrm{Th} \xrightarrow{\alpha} {}^{228}_{88}\mathrm{Ra} \xrightarrow{\beta} {}^{228}_{89}\mathrm{Ac} \xrightarrow{\beta} {}^{228}_{90}\mathrm{Th} \xrightarrow{\alpha} {}^{224}_{88}\mathrm{Ra} \xrightarrow{\alpha} {}^{220}_{80}\mathrm{Ra} \xrightarrow{\alpha} {}^{216}_{84}\mathrm{Po}$$

Note the effects on *Z* and *A* of the emission of alphas and betas. Note also that $_{232}$Th and $_{228}$Th are isotopes.

Example

If 8 x 10^{10} atoms of radon are separated from radium, how many disintegrations will occur in 11.46 days ? (Half-life radon = 3.82 days).

Solution

First, notce that 11.46 days is three times the half-life of radon. We could solve the problem as follows: during the first 3.82 days, half of the radon atoms will decay, leaving 4 x 10^{10}; during the second 3.82 days, half of these will decay, leaving 2 x 1010 and during the final 3.82 days, half of these will decay, leaving 1 x 10^{10}. Thus 7 x 10^{10} disintegrations would occur in 11.46 days.

Alternatively,

Using the formular fo radioactive decay (Eq. 6.20), we have N = 8 x 10^{10} e - (0.693/3.82 days) (11.46 days)

$$= 8 \cdot 10^{10} e^{-2.079} = 8 \cdot 10^{10} \cdot 0.125 = 1 \cdot 10^{10}$$

Hence the number of disintegrations is $7 \cdot 10^{10}$.

6.2.6 Nuclear Stability

Whilst the chemical properties of an atom are governed entirely by the number of protons in the nucleus (i.e the atomic number *Z*), the stability of an atom appears to depend on both the number of protons and the number of neutrons.

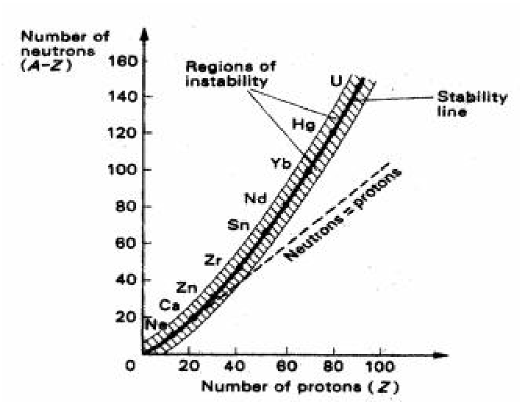


Figure 6.12

In Figure 6.12 the number of neutrons ($A - Z$) has been plotted against the number of protons for all known nuclides, stable and unstable, natural and man-made. A continous line has been drawn approximately through the stable nuclides (only a few are labelled) and the shading on either side of this line shows the region of unstable nuclides.

For stable nuclides the following points emerge:

1) The lightest nuclides have almost equal numbers of protons and neutrons.

2) The heavier nuclides require more neutrons than protons, the heaviest having about 50 per cent more.

3) Most nuclides have both an even number of protons and an even number of neutrons. The implication is that two protons and two neutrons. i.e an alpha particle, form a particularly stable combination and in this connection, it is worth noting that oxygen ($^{16}{}_{8}O$), silicon ($^{14}{}_{28}Si$) and iron ($^{56}{}_{28}Fe$) together account for over three quarters of the earth's crust.

For unstable nuclides the following points can be made:

1) Disintegrations tend to produce new nuclides nearer the 'stability' line and continue until a stable nuclide is formed.

2) A nuclide above the line decays so as to give an increase in atomic number, i.e by beta emission (in which a neutron changes to a proton and an electron). Its neutro-to-proton ratio is thereby increased.

3) A nuclide below the line disintegates in such a way that its atomic number decreases and its neutron-to-proton ratio increases. In heavy nuclides this can occur by alpha emission.

6.2.7 Nuclear Fission and Fusion

If a nucleus of large mass splits (fissions) into two nuclei of smaller mass, then bearing in mind that the total number of nucleous remains constant, the total energy in the nuclei is less, and the energy difference is released as kinetic energy of the fragment. In $_{235}U$, spontaneous fission does not occur, but fission can be caused by bombarding it with thermal (low energy) neutrons.

$$^{235}U + ^{1}n \rightarrow X + Y + K^{1}n, \tag{6.24}$$

X and *Y* represent the fission fragments whose proton and nucleon numbers are not same values for each fission; *K* is the number of neutrons released in the process, *K* is not always the same, but the total number of protons and nucleons must be the same on both sides of the equation. *K* is usually 2 or 3 with an average value of 2.47. The phenomenon may be represented as in figure 6.13.

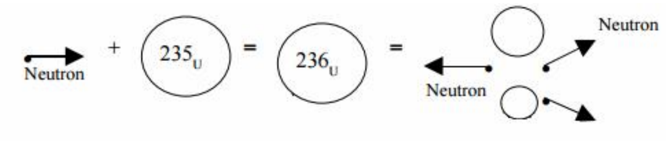


Figure 6.13

These neutrons can be used to produce further fissions, so producing a chain reaction which will run out of control unless the number of neutrons produced is kept under control. Infact, the neutrons produced in a fission reaction have considerable energies and are known as ‘fast neutrons which do not fision $_{234}$U. The neutrons have to be slowed down to thermal energies.

Several different nuclei have been identified as the result of fision, and all that can be said is that the nucleus splits into parts with masses in the approximate ratio 5:7.

Energy can also be produced by the fusion of two nuclei of small mass to produce a more massive nucleus e.g.

$$ {}_1^2H + {}_1^2H \rightarrow {}_2^3H + {}_0^1n \, . \tag{6.25} $$

This reaction takes place in the sun. The difficulty arises in providing the very high temperatures needed to give the two positve nuclei sufficient kinetic energy to overcome their electrostatic repulsion.

In physics, the **fundamental interactions**, also known as **fundamental forces**, are the interactions that do not appear to be reducible to more basic interactions. There are four conventionally accepted fundamental interactions—gravitational, electromagnetic, strong nuclear, and weak nuclear. Each one is described mathematically as a *field*. The gravitational force is modelled as a continuous classical field. The other three, part of the Standard Model of particle physics, are described as discrete quantum fields, and their interactions are each carried by a *quantum*, an *elementary particle*.

The two nuclear interactions have short ranges, producing forces at minuscule, subatomic distances. The strong nuclear interaction, which is carried by the gluonparticle, is responsible for the binding of quarks together to form hadrons, such as protons and neutrons; as a residual effect, it binds the latter particles to formatomic nuclei. The weak nuclear interaction, which is carried by the *W* and *Z* particles, also acts on the nucleus, mediating radioactive decay. The other two, electromagnetism and gravity, produce significant forces at macroscopic scales where the effects can be seen directly in everyday life. The electromagnetic force, carried by the photon, creates electric and magnetic fields, which are responsible for chemical bonding and are used in electrical technology.

Electromagnetic forces tend to cancel each other out when large collections of objects are considered, so over the largest distances (on the scale of planets and galaxies), gravity tends to be the dominant force.

All four fundamental forces are believed to be related, and to unite into a single force at high energies on a minuscule scale, the Planck scale, but particle accelerators cannot produce the enormous energies required to experimentally probe this. A goal of theoretical physicists working beyond the Standard Model is toquantize the gravitational field, yielding a theory of quantum gravity (QG) which would unite gravity in a common theoretical framework with the other three forces. Other theorists seek to unite the electroweak and strong fields within a Grand Unified Theory (GUT). Some theories, notably string theory, seek both QG and GUT within one framework, unifying all four fundamental interactions along with mass generation within a theory of everything (ToE).

A few researchers have interpreted various anomalous observations in physics as evidence for a fifth force, but this is not widely accepted.

The four fundamental interactions of nature

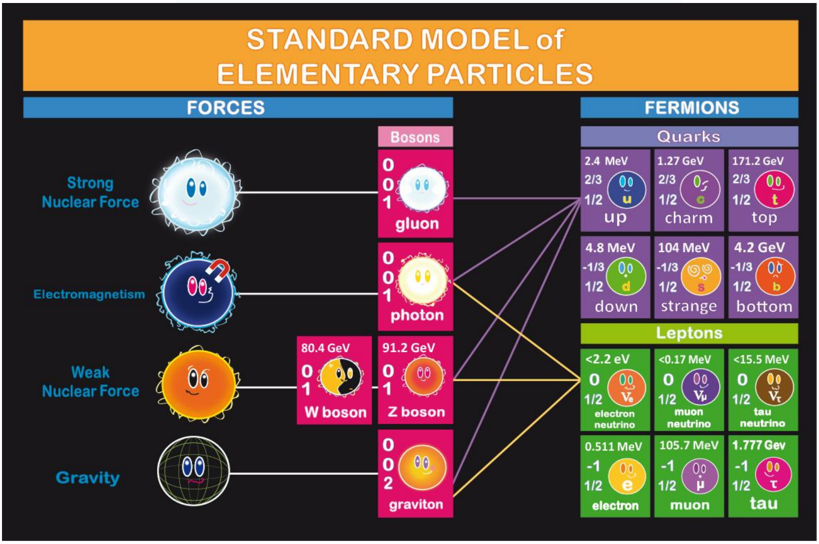


Figure 6.14

6.2.8 Overview of the fundamental interactions

In the conceptual model of fundamental interactions, matter consists of fermions, which carryproperties called charges and spin $\pm\frac{1}{2}$ (intrinsic angular momentum $\pm^{\hbar}/_{2}$, where ħ is the reduced Planck constant). They attract or repel each other by exchanging bosons. The interaction of any pair of fermions in perturbation theory can then be modelled thus:

Two fermions go in → *interaction* by boson exchange → Two changed fermions go out.

The exchange of bosons always carries energy and momentum between the fermions, thereby changing their speed and direction. The exchange may also transport a charge between the fermions, changing the charges of the fermions in the process (e.g., turn them from one type of fermion to another). Since bosons carry one unit of angular momentum, the fermion's spin direction will flip from $+^{1}/_{2}$ to $-^{1}/_{2}$ (or vice versa) during such an exchange (in units of the reduced Planck's constant).

Because an interaction results in fermions attracting and repelling each other, an older term for "interaction" is force.

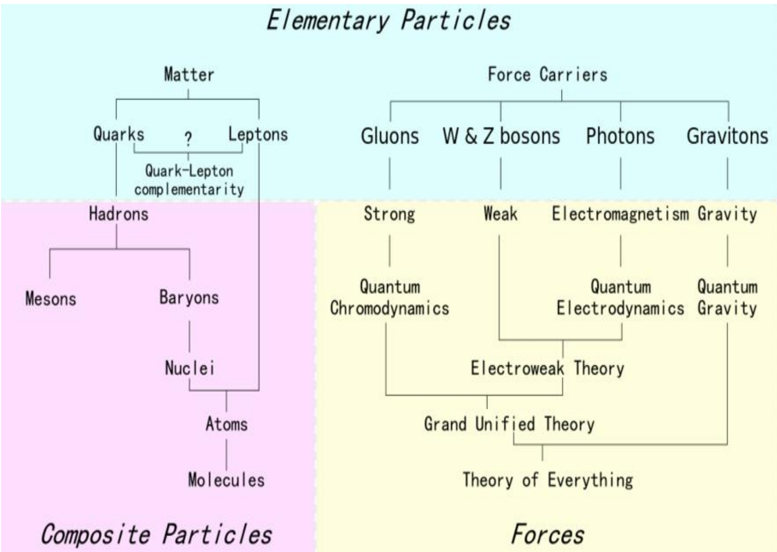


Figure 6.15 An overview of the various families of elementary and composite particles, and the theories describing their interactions. Fermions are on the left, and Bosons are on the right

According to the present understanding, there are four fundamental interactions or forces:gravitation, electromagnetism, the weak interaction, and the strong interaction. Their magnitude and behaviour vary greatly, as described in the table below. Modernphysics attempts to explain every observed physical phenomenon by these fundamental interactions. Moreover, reducing the number of different interaction types is seen as desirable. Two cases in point are the unification of:

- Electric and magnetic force into electromagnetism;
- The electromagnetic interaction and the weak interaction into the electroweak interaction; see below.

Both magnitude «relative strength» and «range», as given in the table, are meaningful only within a rather complex theoretical framework. It should also be noted that the table below lists properties of a conceptual scheme that is still the subject of ongoing research.

The modern (perturbative) quantum mechanical view of the fundamental forces other than gravity is that particles of matter (fermions) do not directly interact with each other, but rather carry a charge, and exchange virtual particles (gauge bosons), which are the interaction carriers or force mediators. For example, photons mediate the interaction of electric charges, and gluons mediate the interaction of color charges.

Gravity

Gravitation is by far the weakest of the four interactions. The weakness of gravity can easily be demonstrated by suspending a pin using a simple magnet (such as a refrigerator magnet). The magnet is able to hold the pin against the gravitational pull of the entire Earth.

Yet gravitation is very important for macroscopic objects and over macroscopic distances for the following reasons. Gravitation:

- Is the only interaction that acts on all particles having mass, energy and/or momentum.
- Has an infinite range, like electromagnetism but unlike strong and weak interaction.
- Cannot be absorbed, transformed, or shielded against.
- Always attracts and never repels.

Even though electromagnetism is far stronger than gravitation, electrostatic attraction is not relevant for large celestial bodies, such as planets, stars, and galaxies, simply because such bodies contain equal numbers of protons and electrons and so have a net electric charge of zero. Nothing "cancels" gravity, since it is only attractive, unlike electric forces which can be attractive or repulsive. On the other hand, all objects having mass are subject to the gravitational force, which only attracts. Therefore, only gravitation matters on the large-scale structure of the universe.

The long range of gravitation makes it responsible for such large-scale phenomena as the structure of galaxies and black holes and it retards the expansion of the universe. Gravitation also explains astronomical phenomena on more modest scales, such as planetary orbits, as well as everyday experience: objects fall; heavy objects act as if they were glued to the ground, and animals can only jump so high.

Gravitation was the first interaction to be described mathematically. In ancient times, Aristotle hypothesized that objects of different masses fall at different rates. During the Scientific Revolution, Galileo Galilei experimentally determined that this was not the case - neglecting the friction due to air resistance, and buoyancy forces if an atmosphere is present (e.g. the case of a dropped air-filled balloon vs a water-filled balloon) all objects accelerate toward the Earth at the same rate.Isaac Newton's law of Universal Gravitation (1687) was a good approximation of the behaviour of gravitation. Our present-day understanding of gravitation stems from Albert Einstein's General Theory of Relativity of 1915, a more accurate (especially for cosmological masses and distances) description of gravitation in terms of the geometry of spacetime.

Merging general relativity and quantum mechanics (or quantum field theory) into a more general theory of quantum gravity is an area of active research. It is hypothesized that gravitation is mediated by a massless spin-2 particle called the graviton.

Although general relativity has been experimentally confirmed (at least for weak fields) on all but the smallest scales, there are rival theories of gravitation. Those taken seriously by [citation needed] the physics community all reduce to general relativity in some limit, and the focus of observational work is to establish limitations on what deviations from general relativity are possible.

Proposed extra dimensions could explain why the gravity force is so weak [32].

Electroweak interaction

Electromagnetism and weak interaction appear to be very different at everyday low energies. They can be modeled using two different theories. However, above unification energy, on the order of 100 GeV, they would merge into a single electroweak force.

Electroweak theory is very important for modern cosmology, particularly on how the universe evolved. This is because shortly after the Big Bang, the temperature was approximately above 10^{15} K. Electromagnetic force and weak force were merged into a combined electroweak force.

For contributions to the unification of the weak and electromagnetic interaction between elementary particles, Abdus Salam, Sheldon Glashow and Steven Weinbergwere awarded the Nobel Prize in Physics in 1979. [33; 34].

Electromagnetism

Electromagnetism is the force that acts between electrically charged particles. This phenomenon includes the electrostatic force acting between charged particles at rest, and the combined effect of electric and magnetic forces acting between charged particles moving relative to each other.

Electromagnetism is infinite-ranged like gravity, but vastly stronger, and therefore describes a number of macroscopic phenomena of everyday experience such asfriction, rainbows, lightning, and all human-made devices using electric current, such as television, lasers, and computers. Electromagnetism fundamentally determines all macroscopic, and many atomic levels, properties of the chemical elements, including all chemical bonding.

In a four kilogram (~1 gallon) jug of water there are

$$4000\,g\ H_2O \cdot \frac{1mol\ H_2O}{18\,g\ H_2O} \cdot \frac{10\,mol\ e^-}{1\,mol\ H_2O} \cdot \frac{96{,}000\,C}{1\,mol\ e^-} = 2.1 \cdot 10^8\ C$$

of total electron charge. Thus, if we place two such jugs a meter apart, the electrons in one of the jugs repel those in the other jug with a force of

$$\frac{1}{4\pi\varepsilon_0} \frac{(2.1 \cdot 10^8)^2}{(1m)^2} = 4.1 \cdot 10^{26}\ N.$$

This is larger than the planet Earth would weigh if weighed on another Earth. The atomic nuclei in one jug also repel those in the other with the same force. However, these repulsive forces are canceled by the attraction of the electrons in jug *A* with the nuclei in jug *B* and the attraction of the nuclei in jug *A* with the electrons in jug *B*, resulting in no net force. Electromagnetic forces are tremendously stronger than gravity but cancel out so that for large bodies gravity dominates.

Electrical and magnetic phenomena have been observed since ancient times, but it was only in the 19th century that it was discovered that electricity and magnetism are two aspects of the same fundamental interaction. By 1864, Maxwell's equations had rigorously quantified this unified interaction. Maxwell's theory, restated usingvector calculus, is the classical theory of electromagnetism, suitable for most technological purposes.

The constant speed of light in a vacuum (customarily described with the letter «*c*») can be derived from Maxwell's equations, which are consistent with the theory of special relativity. Einstein's 1905 theory of special relativity, however, which flows from the observation that the speed of light is constant no matter how fast the observer is moving, showed that the theoretical result implied by Maxwell's equations has profound implications far beyond electromagnetism on the very nature of time and space.

In another work that departed from classical electro-magnetism, Einstein also explained the photoelectric effect by hypothesizing that light was transmitted in quanta, which we now call photons. Starting around 1927, Paul Dirac combined quantum mechanics with the relativistic theory of electromagnetism. Further work in the 1940s, by Richard Feynman, Freeman Dyson, Julian Schwinger, and Sin-Itiro Tomonaga, completed this theory, which is now called quantum electrodynamics, the revised theory of electromagnetism. Quantum electrodynamics and quantum mechanics provide a theoretical basis for electromagnetic behavior such as quantum tunneling, in which a certain percentage of electrically charged particles move in ways that would be impossible under the classical electromagnetic theory, that is necessary for everyday electronic devices such as transistors to function.

Weak interaction

The *weak interaction* or *weak nuclear force* is responsible for some nuclear phenomena such as beta decay. Electromagnetism and the weak force are now understood to be two aspects of a unified electroweak interaction - this discovery was the first step toward the unified theory known as the Standard Model. In the theory of the electroweak interaction, the carriers of the weak force are the massive gauge bosons called the W and Z bosons. The weak interaction is the only known interaction which does not conserve parity; it is left-right asymmetric. The weak interaction even violates *CP* - symmetry but does conserve *CPT*.

Strong interaction

The *strong interaction*, or *strong nuclear force*, is the most complicated interaction, mainly because of the way it varies with distance. At distances greater than 10 fem to meters, the strong force is practically unobservable. Moreover, it holds only inside the atomic nucleus.

After the nucleus was discovered in 1908, it was clear that a new force, today known as the nuclear force, was needed to overcome the electrostatic repulsion, a manifestation of electromagnetism, of the positively charged protons. Otherwise, the nucleus could not exist. Moreover, the force had to be strong enough to squeeze the protons into a volume that is about 10^{-15} m, much smaller than that of the entire atom. From the short range of this force, Hideki Yukawa predicted that it was associated with a massive particle, whose mass is approximately 100 MeV.

The 1947 discovery of the pion ushered in the modern era of particle physics. Hundreds of hadrons were discovered from the 1940 s to 1960 s, and an extremely complicated theory of hadrons as strongly interacting particles was developed. Most notably:

- The pions were understood to be oscillations of vacuum condensates;
- Jun John Sakurai proposed the rho and omega vector bosons to be force carrying particles for approximate symmetries of isospin and hypercharge;
- Geoffrey Chew, Edward K. Burdett and Steven Frautschi grouped the heavier hadrons into families that could be understood as vibrational and rotational excitations of strings.

While each of these approaches offered deep insights, no approach led directly to a fundamental theory.

Murray Gell-Mann along with George Zweig first proposed fractionally charged quarks in 1961. Throughout the 1960 s, different authors considered theories similar to the modern fundamental theory of quantum chromodynamics (QCD) as simple models for the interactions of quarks. The first to hypothesize the gluons of QCD were Moo-Young Han and Yoichiro Nambu, who introduced the quark color charge and hypothesized that it might be associated with a force-carrying field. At that time, however, it was difficult to see how such a model could permanently confine quarks. Han and Nambu also assigned each quark color an integer electrical charge, so that the quarks were fractionally charged only on average, and they did not expect the quarks in their model to be permanently confined.

In 1971, Murray Gell-Mann and Harald Fritzsch proposed that the Han/Nambu color gauge field was the correct theory of the short-distance interactions of fractionally charged quarks. A little later, David Gross, Frank Wilczek, and David Politzer discovered that this theory had the property of asymptotic freedom, allowing them to make contact with experimental evidence. They concluded that QCD was the complete theory of the strong interactions, correct at all distance scales. The discovery of asymptotic freedom led most physicists to accept QCD since it became clear that even the long-distance properties of the strong interactions could be consistent with experiment if the quarks are permanently confined.

Assuming that quarks are confined, Mikhail Shifman, Arkady Vainshtein and Valentine Zakharov were able to compute the properties of many low-lying hadrons directly from QCD, with only a few extra parameters to describe the vacuum. In 1980, Kenneth G. Wilson published computer calculations based on the first principles of QCD, establishing, to a level of confidence tantamount to certainty, that QCD will confine quarks. Since then, QCD has been the established theory of the strong interactions.

QCD is a theory of fractionally charged quarks interacting by means of 8 bosonic particles called gluons. The gluons interact with each other, not just with the quarks, and at long distances the lines of force collimate into strings. In this way, the mathematical theory of QCD not only explains how quarks interact over short distances but also the string-like behavior, discovered by Chew and Frautschi, which they manifest over longer distances.

6.3 ELEMENTARY PARTICLES. QUARKS

A **quark** is an elementary particle and a fundamental constituent of matter. Quarks combine to form composite particles called hadrons, the most stable of which are protons and neutrons, the components of atomic nuclei [32]. Due to a phenomenon known as *color confinement*, quarks are never directly observed or found in isolation; they can be found only within hadrons, such as baryons (of which protons and neutrons are examples) and mesons.[33; 34]. For this reason, much of what is known about quarks has been drawn from observations of the hadrons themselves.

Quarks have various intrinsic properties, including electric charge, mass, color charge, and spin. Quarks are the only elementary particles in the Standard Model of particle physics to experience all four fundamental interactions, also known as *fundamental forces* (electromagnetism, gravitation, strong interaction, and weak interaction), as well as the only known particles whose electric charges are not integer multiples of the elementary charge.

There are six types of quarks, known as *flavors*: up, down, strange, charm, top, and bottom. Up and down quarks have the lowest masses of all quarks. The heavier quarks rapidly change into up and down quarks through a process ofparticle decay: the transformation from a higher mass state to a lower mass state. Because of this, up and down quarks are generally stable and the most common in the universe, whereas strange, charm, bottom, and top quarks can only be produced in high energy collisions (such as those involving cosmic rays and in particle accelerators).

For every quark flavor there is a corresponding type of antiparticle, known as an *antiquark*, that differs from the quark only in that some of its properties have equal magnitude but opposite sign.

The quark model was independently proposed by physicists Murray Gell-Mann and George Zweig in 1964 [32]. Quarks were introduced as parts of an ordering scheme for hadrons, and there was little evidence for their physical existence until deep inelastic scattering experiments at the Stanford Linear Accelerator Center in 1968 [33; 34]. Accelerator experiments have provided evidence for all six flavors. The top quark was the last to be discovered at Fermilab in 1995 [32].

Classification

The Standard Model is the theoretical framework describing all the currently known elementary particles. This model contains six flavors of quarks (*q*), named up (*u*), down (*d*), strange (*s*), charm (*c*), bottom (*b*), and top (*t*) [31]. Antiparticles of quarks are called *antiquarks,* and are denoted by a bar over the symbol for the corresponding quark, such as *u* for an up antiquark. As with antimatter in general, antiquarks have the same mass, mean lifetime, and spin as their respective quarks, but the electric charge and other charges have the opposite sign [35].

Quarks are spin-$\frac{1}{2}$ particles, implying that they are fermions according to the spin-statistics theorem. They are subject to the Pauli exclusion principle, which states that no two identical fermions can simultaneously occupy the same quantum state. This is in contrast to bosons (particles with integer spin), any number of which can be in the same state [36]. Unlike leptons, quarks possess color charge, which causes them to engage in the strong interaction. The resulting attraction between different quarks causes the formation of composite particles known as *hadrons* (see «Strong interaction and color charge» below).

The quarks that determine the quantum numbers of hadrons are called *valence quarks*; apart from these, any hadron may contain an indefinite number of virtual (or *sea*) quarks, antiquarks, and gluons, which do not influence its quantum numbers [37]. There are two families of hadrons: baryons, with three valence quarks, and mesons, with a valence quark and an antiquark [38]. The most common baryons are the proton and the neutron, the building blocks of the atomic nucleus [39]. A great number of hadrons are known, most of them differentiated by their quark content and the properties these constituent quarks confer. The existence of «exotic» hadrons with more valence quarks, such as tetraquarks (qqqq) and pentaquarks (qqqqq), has been conjectured [40] but not proven [40; 41; 42]. However, on 13 July 2015, the LHCb collaboration at CERN reported results consistent with pentaquark states [42].

Elementary fermions are grouped into three generations, each comprising two leptons and two quarks. The first generation includes up and down quarks, the second strange and charm quarks, and the third bottom and top quarks. All searches for a fourth generation of quarks and other elementary fermions have failed,[41] and there is strong indirect evidence that no more than three generations exist [42; 43; 44]. Particles in higher generations generally have greater mass and less stability, causing them to decay into lower-generation particles by means of weak interactions. Only first-generation (up and down) quarks occur commonly in nature. Heavier quarks can only be created in high-energy collisions (such as in those involving cosmic rays), and decay quickly; however, they are thought to have been present during the first fractions of a second after the Big Bang, when the universe was in an extremely hot and dense phase (the quark epoch). Studies of heavier quarks are conducted in artificially created conditions, such as in particle accelerators 44.

Having electric charge, mass, color charge, and flavor, quarks are the only known elementary particles that engage in all four fundamental interactions of contemporary physics: electromagnetism, gravitation, strong interaction, and weak interaction [39]. Gravitation is too weak to be relevant to individual particle interactions except at extremes of energy (Planck energy) and distance scales (Planck distance). However, since no successful quantum theory of gravity exists, gravitation is not described by the Standard Model.

See the table of properties below for a more complete overview of the six quark flavors' properties.

Mass

Two terms are used in referring to a quark's mass: *current quark mass* refers to the mass of a quark by itself, while *constituent quark mass* refers to the current quark mass plus the mass of the gluon particle field surrounding the quark [46]. These masses typically have very different values. Most of a hadron's mass comes from the gluons that bind the constituent quarks together, rather than from the quarks themselves. While gluons are inherently massless, they possess energy – more specifically, quantum chromodynamics binding energy (QCBE) – and it is this that contributes so greatly to the overall mass of the hadron (see mass in special relativity). For example, a proton has a mass of approximately 938 MeV/c^2, of which the rest mass of its three valence quarks only contributes about 9 MeV/c^2; much of the remainder can be attributed to the field energy of the gluons [45; 46]. See Chiral symmetry breaking.

The Standard Model posits that elementary particles derive their masses from the Higgs mechanism, which is associated to the Higgs boson. It is hoped that further research into the reasons for the top quark's large mass of ~173 GeV/c^2, almost the mass of a gold atom [45; 46], might reveal more about the origin of the mass of quarks and other elementary particles [47].

Table 4

Quark flavor properties [45]

Name	Symbol	Mass (MeV/c^2)*	J	B	Q (e)	I_3	C	S	T	B'	Antiparticle	Antiparticle symbol
First generation												
Up	u	2.3 ±0.7 ± 0.5	½	+⅓	+⅔	+½	0	0	0	0	Antiup	$\bar{u}$
Down	d	4.8 ±0.5 ± 0.3	½	+⅓	−⅓	−½	0	0	0	0	Antidown	$\bar{d}$
Second generation												
Charm	c	1275 ±25	½	+⅓	+⅔	0	+1	0	0	0	Anticharm	$\bar{c}$
Strange	s	95 ±5	½	+⅓	−⅓	0	0	−1	0	0	Antistrange	$\bar{s}$
Third generation												
Top	t	173 210 ±510 ± 710	½	+⅓	+⅔	0	0	0	+1	0	Antitop	$\bar{t}$
Bottom	b	4180 ±30	½	+⅓	−⅓	0	0	0	0	−1	Antibottom	$\bar{b}$

The following table summarizes the key properties of the six quarks. Flavor quantum numbers (isospin (I_3), charm (C), strangeness S, not to be confused with spin), topness (T), and bottomness (B')) are assigned to certain quark flavors, and denote qualities of quark-based systems and hadrons. The baryon number (B) is $+{}^1\!/_3$ for all quarks, as baryons are made of three quarks. For antiquarks, the electric charge (Q) and all flavor quantum numbers (B, I_3, C, S, T, and B') are of opposite sign. Mass and total angular momentum (J; equal to spin for point particles) do not change sign for the antiquarks.

J - total angular momentum, B - baryon number, Q - electric charge, I_3 - isospin, C - charm, S - strangeness, T - topness, B' - bottomness.

* Notation such as 173210±510±710 denotes two types of measurement uncertainty. In the case of the top quark, the first uncertainty is statistical in nature, and the second is systematic.

Lepton

A **lepton** is an elementary, half-integer spin (spin ${}^1\!/_2$) particle that does not undergo strong interactions [30]. Two main classes of leptons exist: charged leptons (also known as the *electron-like* leptons), and neutral leptons (better known asneutrinos). Charged leptons can combine with other particles to form various composite particles such as atoms and positronium, while neutrinos rarely interact with anything, and are consequently rarely observed. The best known of all leptons is the electron.

Table 5

Properties of leptons

Particle/antiparticle name	Symbol	Q (e)	S	L_e	L_μ	L_τ	Mass (MeV/c^2)	Lifetime (s)	Common decay
Electron / Positron	e^-/e^+	−1 / +1	½	+1 / −1	0	0	0.510 998 910(13)	Stable	Stable
Muon / Antimuon	μ^-/μ^+	−1 / +1	½	0	+1 / −1	0	105.658 3668(38)	$2.197\,019(21) \times 10^{-6}$	$e^- + \bar{\nu}_e + \nu_\mu$
Tau / Antitau	τ^-/τ^+	−1 / +1	½	0	0	+1 / −1	1 776.84(17)	$2.906(10) \times 10^{-13}$	See τ^- decay modes
Electron neutrino / Electron antineutrino	$\nu_e/\bar{\nu}_e$	0	½	+1 / −1	0	0	< 0.000 0022	Unknown	
Muon neutrino / Muon antineutrino	$\nu_\mu/\bar{\nu}_\mu$	0	½	0	+1 / −1	0	< 0.17	Unknown	
Tau neutrino / Tau antineutrino	$\nu_\tau/\bar{\nu}_\tau$	0	½	0	0	+1 / −1	< 15.5	Unknown	

Leptons have various intrinsic properties, including electric charge, spin, and mass. Unlike quarks however, leptons are not subject to the strong interaction, but they are subject to the other three fundamental interactions: gravitation,electromagnetism (excluding neutrinos, which are electrically neutral), and the weak interaction.

For every lepton flavor there is a corresponding type of antiparticle, known as an antilepton, that differs from the lepton only in that some of its properties have equal magnitude but opposite sign. However, according to certain theories, neutrinos may be their own antiparticle, but it is not currently known whether this is the case or not.

Chapter 7 PRACTICAL TASKS

7.1 KINEMATICS

1. The airspeed of the aircraft is $\upsilon_0 = 800\, km/h$. The West to East wind speed $u = 15\, m/s$. What ground speed υ of the aircraft and the angel α to the meridium should be kept so that the direction was to a) the South; b) the North; c) the West; d) the East? ($a)\ \alpha = 3.87^0, \upsilon = 798.18\, km/h$; $b)\ \alpha = 3.87^0, \upsilon = 798.18\, km/h$; $c)\ \alpha = 90^0, \upsilon = 746\, km/h$; $d)\ \alpha = 90^0, \upsilon = 854\, km/h$).

2. The aircraft flies from point *A* to point *B*, located at a distance $l = 300\, km$ to the east. What the duration *t* of the flight will be when a) wind is calm; b) S-N wind; c) W-E wind. Wind speed $u = 20\, m/s$, airspeed $\upsilon_0 = 600 km/h$. (a) $t = 30$ min b) $t = 30,\ 218$ min c) $t = 26,\ 786$ min).

3. The object 1 accelerations, at an initial velocity υ_{10} and an acceleration a_1. Simultaneously object 2 stars to move slowler and slowler, having an initial velocity υ_{20} and acceleration a_1. Calculate the time *t* after starting of the movement, when both objects have the equal same speed? ($t = \dfrac{\upsilon_{20} - \upsilon_{10}}{a_1 + a_2}$).

4. The dependence of the distance S passed by the object on the time t is given by the equation $S = A - Bt + Ct^2$, where $A = 6\, m$, $B = 3\, m/s$ and $C = 2\, m/s^2$. What is average velocity and the average acceleration of the object for the time interval $1 \le t \le 4\, s$. Make a graph of the dependence of the path s, speed v and acceleration a on time t for a time interval $0 \le t \le 5\, s$ in 1 *s*. ($\overline{\upsilon} = 7\, m/s; \overline{a} = 4\, m/s^2$).

5. The aircraft must take off for speed $\upsilon = 250\, km/h$. What will the acceleration be if this speed is reached at the end of the runway with the length $l = 1000\, m$? What is the acceleration of the aircraft? What is the average speed of the aircraft on this ley? The movement of the aircraft is assumed to be equally accelerated. ($t = 28.8\, s;\ a = 2.41\, m/s^2;\ \upsilon_{av} = 34.72\, m/s$).

6. The aircraft takes off from the aerodrome at an angle $\alpha = 30^0$ to the horizon with a constant speed $\upsilon = 60\, m/s$. What will be the height in $t = 10\, s$ and at what the distance will be (in the horizontal direction) from the take-off point? ($S = 519.5\, m$).

7. What is the landing speed available for the aircraft when the runway length $l = 800\ m$ and braking action with acceleration $a = 5\ m/s^2$? ($\upsilon_0 = 89.44\ m/s$).

8. The engines of the rocket, launched vertically up from the Earth's surface, opereted 1 *min* and reported a constant acceleration $a = 3g$ to the rocket. What is the maximum height reached by the rocket? The acceleration of gravity g is assumed to be constant and equal to $9.8\ m/s^2$. Resistance to air is neglected. ($h = 194.04\ m$).

9. What is the lifting height and range of the flare missile, launched at a speed of $40\ m/s$ at an angle of 60^0 to the horizon. ($h = 61.1\ m;\ S = 141.2\ m$).

10. The rotation speed of the aircraft propeller is $1500\ rp/m$.What is the number revolutions made be the screw at a distance does $90\ km$ and a speed of $180\ km/h$? (45 000 *rpm*).

11. The dependence of the path *s* made by the object on time *t* is given by the equation $S = A + Bt + Ct^2$, where $A = 3\ m$, $B = 3\ m/s$ and $C = 1\ m/s^2$. What is the average velocity v and the average acceleration a in the first, second and third seconds of its motion. ($\overline{\upsilon}_1 = 3\ m/s; \overline{\upsilon}_2 = 5\ m/s; \overline{\upsilon}_3 = 7\ m/s; \overline{a_1} = \overline{a_2} = \overline{a_3} = 2\ m/s^2$).

12. The dependence of the path *s* made by the object on time *t* is given by the equation $S = A + Bt + Ct^2 + Dt^3$, where $C = 0.14\ m/s^2$ and $D = 0.01\ m/s^3$. What time t after the beginning of motion the body will have an acceleration $a = 1\ m/s^2$? What is the average acceleration a of the body during this time interval. ($\overline{a} = 0.64\ m/s^2;\ t = 12\ s$).

13. From the tower, which height is $h = 25\ m$ stone was throun horizontally with a speed $\upsilon_x = 15\ m/s$. What time will the stone be in motion? What distance l from the base of the tower will be when it falls to the ground? What will the speed on the stone be when is falls on the ground? What angle φ will be the trajectory of the stone with the horizon at the point of its fall to the ground? ($t = 2.26\ s;\ l = 33.9\ m;\ \upsilon = 26.7\ m/s;\ \varphi = 55^0 48`$).

14. A stone thrown horizontally fell to the ground in time $t = 0.5\ s$ at a distance $l = 5\ m$ horizontally from the throwing point. What height h the stone was thrown? What will the speed on the stone be thrown? What will the speed on the stone be when is falls on the ground? What angle φ will be the trajectory of the stone with the horizon at the point of its fall to the ground? ($\upsilon = 111.1\ m/s;\ h = 1.22\ m;\ \upsilon_x = 10\ m/s;\ \varphi = 26^0 12`$).

15. The ball, thrown horizontally, hits the wall, located at a distance $l = 5\ m$ from the throwing place. The height of the place where the ball hits the wall is $\Delta h = 1\ m$ less than the height h from which the ball is thrown. What is the speed the ball was thrown? What angle φ does the ball fly to the wall surface? ($\upsilon_x = 11.1\ m/s; \varphi = 68^0 12`$).

16. A stone thrown horizontally, in time $t = 0.5\ s$ after the beginning of the movement, had a velocity υ, 1.5 times greater than the speed υ_x at the time of the thrown. What is the speed the stone was thrown? ($\upsilon_x = 4.4\ m/s$).

17. The stone is thrown horizontally at a speed $\upsilon_x = 15\ m/s$. Calculate the normal a_n and the tangential acceleration a_τ of the stone in a time $t = 1\ s$ after starting of the motion. ($a_n = 8.2\ m/s^2; a_\tau = 5.4\ m/s^2$).

18. The stone is thrown horizontally at a velocity $\upsilon_x = 10\ m/s$. Calculate the radius of curvature of the trajectory of the stone in a time $t = 3\ s$ after starting of the movement.($R = 305\ m$).

19. Object 1 accelerates, uniformly at speed $\upsilon_{10} = 2\ m/s$ and acceleration a. In $t = 10\ s$ after beginning of the motion of the object 1, the object 2 begins to move from the same point equally, at an initial speed $\upsilon_{20} = 12\ m/s$ and the same accelerating a. Calculate the acceleration a, at which the object 2 is able to reach object 1. ($a = 1\ m/s^2$).

20. The dependence of the distance S passed by the object on the time t is given by the equation $S = At - Bt^2 + Ct^3$, where $A = 2\ m/s$, $B = 3\ m/s^2$ and $C = 4\ m/s^3$. Calculate: a) the dependence of the velocity υ and acceleration a on the time t; b) the distance S passed by the object, the velocity υ and the acceleration a in a time $t = 2\ s$ after the starting of the motion. Draw a graph of the dependence of the path S, velocity υ and acceleration a on the time t for a time interval $0 \le t \le 3s$ in 0.5 s. ($a)\ \upsilon = (2 - 6t + 12t^2)\ m/s, a = (-6 + 24t)\ m/s^2; b)\ s = 24\ m, \upsilon = 38\ m/s, a = 42\ m/s^2$).

7.2 DYNAMICS

21. An object with mass $m = 0.5\ kg$ moves so that the dependence of the path S passed from the object on time t is given by the equation $S = A \sin \omega t$, where $A = 5\ cm$ and $\omega = \pi\ rad/s$. Find the force F acting on the object through the time $t = 1/6\ s$ after the motion begins. ($F = -0.123\ H$).

22. An object with a mass $m = 0.5\,kg$ moves rectilinearly, and the dependence of the path S passed on time t is described by the equation $x = A - Bt + Ct^2 - Dt^3$, where $C = 5\,m/s^2$ and $D = 1\,m/s^3$. Find the force F acting on the body at the end of the first second of the motion. ($F = 2\,H$).

23. The aircraft rises and at a height of $h = 5\,km$ reaches a speed of $\upsilon = 360\,km/h$. How many times does the work A_1, performed during lifting against gravity, exceed the work of A_2, which is to increase the speed of the aircraft? ($A_1 / A_2 = 10$).

24. Find the efficiency factor of engine if it is known that at a speed of $\upsilon = 40\,km/h$ the engine consumes a volume of $V = 13.5$ liters of gasoline on the path $s = 100\,km$ and develops a power of $N = 12\,kW$. The density of gasoline is $\rho = 0.8 \cdot 10^3\,kg/m^3$, the specific heat of combustion of gasoline is $q = 46\,MJ/kg$. ($\eta = 0.22$).

25. A person weighing $m_1 = 60\,kg$, running at a speed of $\upsilon_1 = 8\,km/h$, overtakes a trolley weighing $m_2 = 80\,kg$, moving at a speed of $\upsilon_2 = 2.9\,km/h$, and jumps on it. How fast will the trolley move? How fast will the trolley move if the person ran to meet it? ($u = 5.14\,km/h, u` = 1.71\,km/h$).

26. An airplane flying at a speed of $\upsilon = 900\,km/h$ makes a «dead loop». What should be the radius of the «dead loop» R, so that the greatest force F, pressing the pilot to the seat, is equal to: a) the fivefold force of gravity acting on the pilot; b) the tenfold force of gravity acting the on the pilot? ($a)\ R = 1600\,m, b)\ R = 711\,m$).

27. At the lower end of the spring, suspended vertically, another spring is attached to the end of which the load is attached. The spring stiffnesses are equal to k_1 and k_2. Neglecting the mass of the springs in comparison with the weight of the load, find the ratio W_{n_1} / W_{n_2} of the potential energies of these springs. ($W_{p1} / W_{p2} = k_2 / k_1$).

28. Find the force of gravitational interaction F between two protons, located at a distance $r = 10^{-16}\,m$ from each other. The proton mass is $m = 1.67 \cdot 10^{-27}\,kg$. ($F = 1.86 \cdot 10^{-44}\,N$).

29. What linear velocity v will the artificial satellite of the Earth move along a circular orbit: a) at the surface of the Earth; b) at an altitude $h = 200\,km$ and $h = 7000\,km$ from the surface of the Earth? Find the period

of revolution T of the Earth's satellite under these conditions. ($a) h = 0\ km, \upsilon = 7.91\ km/s, T = 1\ h\ 25\min; b)\ h = 200\ km,$ $\upsilon = 7.79\ km/s, T = 1h\ 28\min; h = 7000\ km, \upsilon = 5.46, T = 4\ h\ 16\min.$).

30. How many times the kinetic energy W_k of an artificial Earth satellite moving in a circular orbit is less than its gravitational potential energy W_p? ($W_p / W_k = 2$).

31. Find the second cosmic velocity υ_2, i.e. speed, which must be reported to the body at the surface of the Earth, so that it overcomes the gravity of the earth and forever removed from the Earth. ($\upsilon_2 = 11.2\ km/s$).

32. A space rocket flies to the moon. What point of the straight line connecting the centers of mass of the Moon and Earth, the rocket will be gravitated by the Earth and the Moon with the same force? ($r = 3.4 \cdot 10^5\ km$).

33. Special centrifuges are used to prepare cosmonauts for overloads. What frequency of rotation of a centrifuge of radius $R = 5\ m$, the seat back presses on the pilot with the same force that occurs when the missile is accelerated $a = 3g$? ($n = 0.386\ rot/s$).

34. A wagon of mass $m = 20\ t$ moves equally slowly, having an initial speed $\upsilon_0 = 54\ km/h$ and an acceleration $a = -0.3\ m/s^2$. What is the braking force F acting on the car? How long will the car stop? What distance s car will pass to the stop? ($F = 6\ kN, t = 0.5\ s, S = 375\ m.$).

35. The object of mass $m = 0.5\ kg$ moves rectilinearly, and the dependence of the path S passed on time t is described by the equation $x = A - Bt + Ct^2 - Dt^3$, where $C = 5\ m/s^2$ and $D = 1\ m/s^3$. Find the force F acting on the object at the end of the first second of the motion. ($F = 2\ N$).

36. Under the action of the force $F = 10\ N$, the object moves rectilinearly so that the dependence of the path S passed on time t is given by the equation $S = A - Bt + Ct^2$, where $C = 1\ m/s^2$. Find the mass m of the object. ($m = 4.9\ kg$).

37. A molecule of mass $m = 4.65 \cdot 10^{-26}\ kg$, flying along the normal to the wall of the vessel at a velocity $\upsilon = 600\ m/s$, strikes against the wall and elastically jumps away from it without loss of speed. Find the momentum of the force $F\Delta t$ obtained by the wall during the impact. ($F\Delta t = 5.6 \cdot 10^{-23}\ N \cdot s$).

38. A ball of mass $m = 0.1\ kg$, falling from a certain height, strikes against the inclined plane and elastically bounces off from it without loss of speed. The angle of inclination of the plane to the horizon is $\alpha = 30^0$.

During the impact, the plane receives a momentum of the force $F\Delta t = 1.73\ N \cdot s$. What time t will pass from the moment the ball hits the plane to the moment when it will be at the highest point of the trajectory? ($t = 0.51\ s$).

7.3 ROTATIONAL MOTION

39. A homogeneous rod of length $l = 1\ m$ and mass $m = 0.5\ kg$ rotates in a vertical plane about a horizontal axis passing through the middle of the rod. What angular acceleration does the rod rotate, if the moment of forces $M = 98.1\ mN \cdot m$ acts on it? ($\varepsilon = 2.3544\ rad/s^2$).

40. A tangential force $F = 98.1\ N$ is applied to the rim of a homogeneous disk of radius $R = 0.2\ m$. When the disk rotates, the torque of frictional forces $M_{mp} = 98.1\ N \cdot m$ acts. Find the mass m of the disk if it is known that the disk rotates with an angular acceleration $\varepsilon = 100\ rad/s^2$. ($m = 7.36\ kg$).

41. A homogeneous disk of radius $R = 0.2\ m$ and mass $m = 5\ kg$ rotates about an axis passing through its center perpendicular to its plane. The dependence of the angular velocity ω of the disc rotation on time t is given by the equation $\omega = A + Bt$, where $B = 8\ rad/s^2$. Find the tangential force F applied to the rim of the disk. Friction is neglected. ($\varepsilon = 2.3544\ rad/s^2$).

42. A cord is wound on a drum of radius $R = 0.5\ m$, at the end of which a weight of $m = 10\ kg$ is attached. Find the moment of inertia J of the drum if it is known that the load falls with an acceleration $a = 2.04\ m/s^2$. ($J = 9.5\ kg \cdot m^2$).

43. A ball of diameter $D = 6\ cm$ and a mass $m = 0.25\ kg$ rolls without sliding along the horizontal plane with a speed of $n = 4\ rev/s$. Find the kinetic energy W_k of the ball. ($W_k = 0.0995\ J$).

44. The fan rotates at a frequency of $n = 900\ rpm$. After switching off the fan, rotating slower and slower, made a stop $N = 75\ rev$. The work of the braking forces $A = 44.4\ J$. Find the moment of inertia J of the fan and the moment of the braking forces M. ($J = 0.01\ kg \cdot m^2$; $M = 94.22 \cdot 10^{-3}\ N \cdot m$).

45. A horizontal platform weighing $m = 100\ kg$ rotates about a vertical axis passing through the center of the platform, with a frequency of $n_1 = 10\ rpm$. A person weighing $m = 60\ kg$ is at the same time on the edge of the platform. What frequency n_2 will the platform begin to rotate, if a person moves from the edge of the platform to its center? Consider the

platform as a homogeneous disk, and a human as a point mass. ($n_2 = 0.367\, rot/s = 22\, rot/\min$).

46. The object 1 moves uniformly accelerated, having an initial velocity $\upsilon_{10} = 2\, m/s$ and acceleration a. After a time $t = 10\, s$ after the beginning of the motion of the object 1, the object 2 begins to move from the same point equally fast, having an initial velocity $\upsilon_{20} = 12\, m/s$ and the same acceleration a. Find the acceleration a, at which the object 2. ($a = 1\, m/s^2$).

47. A homogeneous rod of length $l = 0.5\, m$ makes small oscillations in the vertical plane near the horizontal axis passing through its upper end. Find the period of oscillation T of the rod. ($T = 1.15879\, s$).

48. Find the oscillation period T of the rod of the previous problem if the axis of rotation passes through a point located at a distance $d = 10\, cm$ from its upper end. ($T = 1.16\, s$).

49. Two loads are secured at the ends of the vertical shaft. The center of mass is below the middle of the rod at a distance $d = 10\, cm$. Find the length of the rod *l* if it is known that the period of small oscillations of the rod with loads around the horizontal axis passing through its center is $T = 2\, s$. The mass of the rod is neglected in comparison with the mass of the goods. ($l = 0.446\, m$).

50. A hoop with a diameter $D = 56.5\, cm$ hangs on a nail driven into the wall, and makes small oscillations in a plane parallel to the wall. Find the period of oscillation T of the hoop. ($T = 1.5\, s$).

51. What is the smallest length *l* of a filament, to which a homogeneous ball of diameter $D = 4\, cm$ is suspended, so that when determining the period of small oscillations T of a ball, consider it as a mathematical pendulum? The error δ with this assumption should not exceed 1%. ($L = 0.069\, m$).

52. A homogeneous ball is suspended on a filament whose length *l* is equal to the radius of the ball R. How many times does the period of small oscillations T_1 of this pendulum exceed the period of small oscillations T_2 of a mathematical pendulum with the same distance from the center of mass to the point of suspension? ($T_1/T_2 = 1.05$).

53. On a drum with a radius $R = 20\, cm$, the moment of inertia of which $J = 0.1\, kg \cdot m^2$, a cord is wound, to the end of which a weight of $m = 0.5\, kg$ is attached. Before the rotation of the drum, the height of the load above the floor $h_0 = 1\, m$. What time *t* does the load fall to the floor? Find the kinetic energy W_k of the load at the moment of impact about the floor and the

tension force of the thread T. The friction must be neglected. ($t = 1.1\, s; W_k = 0.81\, J; T = 4.1\, N$).

54. Two weights with different weights are connected by a thread thrown over a block whose moment of inertia is $J = 50\, kg \cdot m^2$ and the radius is $R = 20\, cm$. The moment of frictional force of the rotating block is $M_r = 98.1\, N \cdot m$. Find the difference in tension forces of the thread $T_1 - T_2$ on both sides of the block, if it is known that the block rotates with an angular acceleration $\varepsilon = 2.36\, rad/s^2$. The block is considered a uniform disk. ($T_1 - T_2 = 1.08\, kH$).

55. A block of mass $m = 1\, kg$ is fixed at the end of the table. Weights 1 and 2 of the same mass $m_1 = m_2 = 1\, kg$ are connected by a thread, thrown over the block. The coefficient of friction of the weight is about the table $k = 0.1$. Find the acceleration a, with which the weights move, and the tension forces T_1 and T_2 of the filaments. The block is considered a uniform disk. Friction in the block is neglected. ($a = 3.53\, m/s^2$; $T_1 = 6.3\, N; T_2 = 4.5\, N$).

56. A disc with a mass $m = 2\, kg$ rolls without sliding along the horizontal plane with a velocity $\upsilon = 4\, m/s$. Find the kinetic energy W_c of the disk. ($W_k = 24\, J$).

57. A ball with a mass $m = 1\, kg$ rolls without sliding, strikes against the wall and rolls away from it. The velocity of the ball before the impact against the wall is $\upsilon = 10\, cm/s$, after the impact $u = 8\, cm/s$. Find the amount of heat Q that is released when the ball hits the wall. ($Q = 2.51\, mJ$).

7.4 CONTINUUM MECHANICS

58. Find the velocity of carbon dioxide flow through the pipe if it is known that in a time $t = 30$ min, a gas mass $m = 0.51\, kg$ flows through the cross section of the pipe. The gas density is $\rho = 7.5\, kg/m^3$. The pipe diameter is $D = 2\, cm.$ ($\upsilon = 0.12\, m/s$).

59. In the bottom of a cylindrical vessel with a diameter $D = 0.5\, m$ there is a circular aperture with a diameter $d = 1\, cm.$ Find the dependence of the rate of lowering the water level in the vessel from the height h of this level. Find the value of this velocity for the height $h = 0.2\, m$. ($\upsilon_1 = 0.8\, mm/s$).

60. There is a vessel with water on the table, in the lateral surface of which there is a small hole located at a distance h_1 from the bottom of the vessel and at a distance h_2 from the water level. The water level in the

vessel is kept constant. What distance l from the vessel (horizontally) the water jet falls on the table in the case; If: a) $h_1 = 25\,cm, h_2 = 16\,cm$; b) $h_1 = 16\,cm, h_2 = 25\,cm$? ($l = 0.4\,m$).

61. The vessel filled with water conects with the atmosphere through a glass tube fixed in the neck of the vessel (Figure 7.1). The crane K is at a distance $h_2 = 2\,cm$ from the bottom of the vessel. Find the velocity υ of water leakage from the tap if the distance between the lower end of the tube and the bottom of the vessel: a) $h_1 = 2\,cm$; b) $h_1 = 7.5\,cm$; c) $h_1 = 10\,cm$. ($a)\ \upsilon = 0\ m/s; b)\ \upsilon = 1.04\ m/s; c)\upsilon = 1.25\ m/s$).

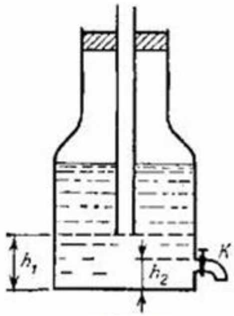


Figure 7.1

62. A cylindrical tank with height $h = 1\,m$ is filled to the brim with water. What time t will all water flow through the hole located at the bottom of the tank if the cross-sectional area s_2 of the hole is 400 times smaller than the cross-sectional area s_1 of the tank? Compare this time with the one that would be needed to drain the same volume of water if the water level in the tank was kept constant at a height of $h = 1\,m$ from the hole. ($t = 3\,s$).

63. Water is poured into the vessel, and a volume of water is poured in time per unit time $V_t = 0.2\,l/s$. What should be the diameter *d* of the hole in the bottom of the vessel, so that the water in it kept at a constant level $h = 8.3\,cm$? ($d = 1.4\,cm$).

64. What pressure P creates a compressor in a paint station, if a jet of liquid paint flows out of it at a speed of $\upsilon = 25\,m/s$? The density of the paint is $\rho = 0.8 \cdot 10^3\ kg/m^3$.

65. Liquid flows through the horizontal pipe *AB* (Figure 7.2). The difference in the levels of this liquid in tubes *a* and *b* is $\Delta h = 10\,cm$. The diameters of tubes *a* and *b* are the same. Find the velocity υ of the fluid flow in the tube *AB*. ($\upsilon = 1.4\,m/s$).

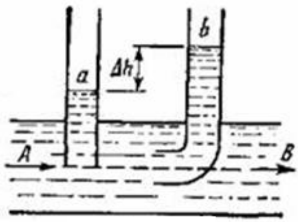


Figure 7.2

66. Air is blown through the *AB* tube (Figure 7.3). For a unit of time through the tube *AB* flows a volume of air $V_t = 5\,l/\min$. The cross-sectional area of the wide part of the tube *AB* is $S_1 = 2\,cm^2$, and

its narrow part and tube abc is equal to $S_1 = 0.5\,cm^2$. Find the difference in the Δh levels of the water poured into the tube abc. Air density $\rho = 1.32\,kg/m^3$. ($\Delta h = 1.6\,mm$).

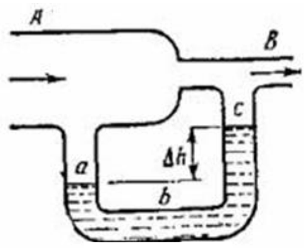


Figure 7.3

67. The ball emerges at a constant velocity υ in a liquid whose density ρ_1 is 4 times greater than the ρ_2 density of the ball material. How many times the frictional force F_{tp}, acting on a floating ball, is greater than the gravitational force mg acting on this ball? ($F/mg = 3$).

68. What is the maximum speed υ that a rain drop with a diameter $d = 0.33\,mm$ can achieve if the dynamic viscosity of air is $\eta = 1.2 \cdot 10^{-5}\ Pa \cdot s$? ($\upsilon = 4.1\,m/s$).

69. A steel ball of diameter $d = 1\,mm$ falls with a constant speed $\upsilon = 0.185\,cm/s$ in a large vessel filled with castor oil. Find the dynamic viscosity η of castor oil. ($\eta = 2\ Pa \cdot s$).

70. A mixture of lead pellets with diameters $d_1 = 3\,mm$ and $d_2 = 1\,mm$ was lowered into a tank with glycerin of height $h = 1\,m$. How much later will smaller pellets fall into the bottom as compared to larger diameter pellets? The dynamic viscosity of glycerol is $\eta = 1.47\ Pa \cdot s$. ($\Delta t = 4\,\text{min}$).

71. A cork ball of radius $r = 5\,mm$ emerges in a vessel filled with castor oil. Find the dynamic and kinematic viscosity of castor oil if the ball emerges at a constant speed $\upsilon = 3.5\,cm/s$. ($\eta = 1.09\ Pa \cdot s, \nu = 12.1\,cm^2/s$).

72. A horizontal capillary is inserted into the lateral surface of a cylindrical vessel of radius $R = 2\ cm$, the inner radius of which is $r = 1\,mm$ and the length $l = 2\ cm$. Castor oil is poured into the vessel, the dynamic viscosity of which is $\eta = 1.2\ Pa \cdot s$. Find the dependence of the rate of decrease in the level of castor oil in the vessel from the height h of this level above the capillary. Find the value of this velocity at $h = 26\ cm$. ($\upsilon = 3 \cdot 10^{-5}\ m/s$).

73. A horizontal capillary is inserted into the lateral surface of the vessel, the inner radius of which is $r = 1\,mm$ and the length $l = 2\,cm$. Glycerin is poured into the vessel, the dynamic viscosity of which is $\eta = 1.0\ Pa \cdot s$. The glycerol level in the vessel is kept constant at a height of $h = 0.18\ m$ above the capillary. How long will it take to get a volume of glycerin from the capillary $V = 5\,cm^3$? ($t = 1.5\,\text{min}$).

74. There is a vessel with a horizontal capillary at the height $h_1 = 5\ cm$ from the bottom of the vessel on the table. The internal radius of the capillary is $r = 1\ mm$ and the length is $l = 1\ cm$. The engine oil is filled into the vessel, the density is $\rho = 0.9 \cdot 10^3\ kg \cdot 10^3\ kg/m^3$ and the dynamic viscosity $\eta = 0.5\ Pa \cdot s$. The level of oil in the vessel is kept constant at a height of $h_2 = 50\ cm$ above the capillary. What distance *l* from the end of the capillary (horizontally) does the jet of oil fall on the table? ($l = 1.1\ cm$).

75. The steel ball falls in a wide vessel filled with transformer oil, the density of which is $\rho = 0.9 \cdot 10^3\ kg \cdot 10^3\ kg/m^3$ and the dynamic viscosity $\eta = 0.8\ Pa \cdot s$. Assuming that the Stokes law takes place at a Reynolds number $\mathrm{Re} \le 0.5$ (if we take the diameter of a ball as the value of D), find the limiting value of the diameter of the ball. ($D = 4.6\ mm$).

76. Assuming that the laminarity of the motion of a liquid (or gas) in a cylindrical tube is preserved at Reynolds number $\mathrm{Re} \le 3000$ (if we take the pipe diameter as the value of D), show that the conditions of problem 76 correspond to laminar motion. The kinematic viscosity of the gas is $\nu = 1.33 \cdot 10^{-6}\ m^2/s$. ($R_e = 1800$).

7.5 MOLECULAR PHYSICS AND THERMODYNAMICS

77. What temperature t has a mass $m = 2\ g$ of nitrogen occupying a volume $V = 820\ cm^3$ at a pressure of $p = 0.2\ MPa$? ($t = 7\ {}^0C$).

78. What volume V occupies the mass $m = 10\ g$ of oxygen at a pressure of $p = 100\ kPa$ and a temperature of $t = 20\ {}^0C$? ($V = 7.6\ l$).

79. The air pressure inside the tightly sealed bottle at a temperature of $t_1 = 7^0C$ was $p_1 = 100\ kPa$. When the bottle was heated, the cork flew out. U_p to what temperature t_2 was the bottle heated, if it is known that the stopper flew out at the air pressure in the bottle $p = 130\ kPa$? ($t_2 = 91\ {}^0C$).

80. How many times does the air density ρ_1 filling the room in winter ($t_1 = 7^0C$) exceed its density ρ_2 in summer ($t_2 = 37^0C$)? The gas pressure is assumed constant. (1.1).

81. Draw mass isotherms $m = 15.5\ g$ oxygen for temperatures: a) $t_1 = 39^0C$; b) $t_2 = 180^0C$.

82. In the middle of the capillary pumped out and sealed from both ends horizontally, there is a column of mercury of length $l = 20\ cm$. If the capillary is placed vertically, then the mercury column will move to

$\Delta l = 10\ cm$. What pressure p_0 was the capillary pumped out? The length of the capillary is $L = 1\ m$. ($p_0 = 375\ mm.\ Hg$).

83. What should be the weight P of the object balloon filled with hydrogen, so that the resultant lifting force of the ball is $F = 0$, i.e. That the ball was in a suspended state? Air and hydrogen are under normal conditions. The pressure inside the ball is equal to the external pressure. The radius of the ball is $r = 12.5\ cm$. ($P = 96\ mH$).

84. At a temperature of $t = 50^0 C$, the saturated water vapor pressure $p = 12.3\ kPa$. Find the density ρ of water vapor. ($\rho = 0.083\ kg/m^3$).

85. Find the density ρ of hydrogen at a temperature $t = 10^0 C$ and a pressure $p = 97.3\ kPa$. ($\rho = 0.081\ kg/m^3$).

86. Some gas at a temperature of $t = 10^0 C$ and a pressure of $p = 200\ kPa$ has a density $\rho = 0.34\ kg/m^3$. Find the molar mass μ of the gas. ($\mu = 0.004\ kg/mol$).

87. In the vessel 1, the volume $V_1 = 3$ liters is gas at pressure $p_1 = 0.2\ MPa$. In vessel 2 the volume $V_2 = 4$ liters is the same gas at the pressure $p_2 = 0.1\ MPa$. The gas temperatures in both vessels are the same. What pressure p will the gas be if we connect the vessels 1 and 2 with a tube? ($p = 140\ kPa$).

88. In the vessel there as $m_1 = 14\ g$ of nitrogen and the mass $m_2 = 9\ g$ of hydrogen at temperature $t = 10\ ^0C$ and at pressure $p = 1\ MPa$. Find the molar mass μ of the mixture and the volume V of the vessel. ($\mu = 0.0046\ kg/mol;\ V = 11.7\ l$).

89. Air contains 23.6% oxygen and 76.4% nitrogen (by weight) at a pressure of $p = 100\ kPa$ and a temperature of $t = 13\ ^0C$. Find the air density ρ and the partial pressures p_1 and p_2 of oxygen and nitrogen. ($\rho = 1.2\ kg/m^3;\ p_1 = 21\ kPa;\ p_2 = 79\ kPa$).

90. A nitrogen molecule flying at a velocity $\upsilon = 600\ m/s$, elastically hits the vessel wall along its normal. Find the momentum of the force $F\Delta t$ obtained by the vessel wall during the impact time. ($F\Delta t = 5.6 \cdot 10^{-23}\ N \cdot s$).

91. The argon molecule, flying at a velocity $\upsilon = 500\ m/s$, elastically hits the vessel wall. The direction of the velocity of the molecule and the normal to the vessel wall make an angle $\alpha = 60^0$. Find the momentum of the force $F\Delta t$ obtained by the vessel wall during the impact time. ($F\Delta t = 3.3 \cdot 10^{-23}\ N \cdot s$).

92. In a vessel with a volume $V = 4$ liters, there is a mass $m = 1\,g$ of hydrogen. How many molecules n contains a unit of volume of a vessel? ($n = 7.5 \cdot 10^{25}\ m^{-3}$).

93. How many N molecules are in a room of volume $V = 80\,m^3$ at a temperature of $t = 17^0C$ and at pressure of $p = 100\ kPa$? ($N = 2 \cdot 10^{27}$).

94. What number of molecules n contains the volume unit of the vessel at a temperature of $t = 10^0C$ and at pressure $p = 1.33 \cdot 10^{-9}\ Pa$? ($n = 3.4 \cdot 10^{11}\ m^{-3}$).

95. In order to obtain a good vacuum in a glass vessel, it is necessary to heat the walls of the vessel when pumping out to remove the adsorbed gas. How high can the pressure in a spherical vessel of radius $r = 10\ cm$ increase if the adsorbed molecules pass from the walls into the vessel? The cross-sectional area of the molecules is $S_0 = 10^{-19}\ m^2$. The temperature of the gas in the vessel is $t = 300\ ^0C$. The molecular layer on the walls is considered to be monomolecular. ($p = 2.4\ Pa$).

96. Find the mean square velocity of air molecules at a temperature $t = 17\ ^0C$. The molar mass of air is $\mu = 0.029\ kg/mol$. ($\sqrt{\overline{\upsilon^2}} = 500\ m/s$).

97. Find the ratio of the mean-square velocities of helium and nitrogen molecules at the same temperatures. ($\sqrt{\overline{\upsilon_1^2}} / \sqrt{\overline{\upsilon_2^2}} = 2.65$).

98. Find the number of molecules n of hydrogen per unit volume of the vessel at a pressure $p = 266.6\ Pa$, if the mean square velocity of its molecules is $2.4\ km/s$. ($n = 4.2 \cdot 10^{24}\ m^{-3}$).

99. The density of a certain gas is $\rho = 0.06\ kg/m^3$, the mean square velocity of its molecules is $500\ m/s$. Find the pressure p that the gas exerts on the walls of the vessel. ($p = 5\ kPa$).

100. The mean square velocity of the molecules of a certain gas under normal conditions $\upsilon_{sr.} = 461\,m/s$. How many molecules n contains the unit of mass of this gas? ($n = 1.88 \cdot 10^{25}\ kg^{-1}$).

101. Find the internal energy W of mass $m = 20\ g$ of oxygen at a temperature $t = 10\ ^0C$. What part of this energy is accounted for by the fraction of the translational motion of the molecules and which part is part of the rotational motion? ($W = 2.2\ kJ; W_{rot} = 1.5\ kJ$).

102. The energy of translational movement of nitrogen molecules in a cylinder is $V = 20$, $W = 5\ kJ$, and the mean square velocity of its molecules

is $2\cdot10^3\ m/s$. Find the mass *m* of nitrogen in the cylinder and the pressure *p*, under which it is located. ($m = 2.5\ g;\ p = 167\ kPa$).

103. The mass $m = 1\,kg$ of a diatomic gas is under pressure $p = 80\ kPa$ and has a density $\rho = 4\ kg/m^3$. Find the energy of the thermal motion W of gas molecules under these conditions. ($N = 1.3\cdot10^{19};W = 0.133\ J$).

104. What number of molecules N of a diatomic gas contains a volume $V = 10\,cm^3$ at a pressure $p = 5.3\,kPa$ and at temperature of $t = 27\ {}^0C$? What energy of the thermal motion W do these molecules have? ($a)\ c_V = 650\ J/kg\cdot K;\ b)\ c_p = 910\ J/kg\cdot K$).

105. Find the specific heat from oxygen for: a) $V = const$; b) $p = const$. ($a)\ c_V = 650\ J/kg\cdot K;\ b)\ c_p = 910\ J/kg\cdot K$).

106. Find the specific heat c_p: a) hydrogen chloride; b) neon; c) nitric oxide; d) carbon monoxide; e) mercury vapors. ($a)\ c_p = 800\ J/kg\cdot K;\ b)\ c_p = 1025\ J/kg\cdot K;\ c)\ c_p = 1040\ J/kg\cdot K;\ d)\ c_p = 103\ J/kg\cdot K$).

107. Find the specific heat c_p/c_v ratio for oxygen. ($c_p/c_v = 1.4$).

108. How many times is the molar heat capacity $C`$ of the explosive gas greater than the molar heat capacity $C``$ of the water vapor obtained during its combustion? The problem is solved for: a) $V = const$; b) $p = const$. ($a)\ c`_v/c``_v = 1.25;\ b)\ c`_p/c``_p = 1.31$).

109. Specific heat capacity of the gas mixture consisting of the quantity $v_1 = 1$ *kmol* of oxygen and some mass m_2 argon, $c_p = 430\dfrac{J}{kg\cdot K}$. What mass of m_2 argon is in the gas mixture? ($m_2 = 60\,kg$).

110. Mass $m = 10\ g$ of oxygen is at a pressure of $p = 0.3\,MPa$ and at temperature of $t = 10^0C$. After heating at $p = const$, the gas occupied the volume $V_2 = 10$ liters. Find the amount of heat Q obtained by the gas and the energy of the thermal motion of the gas molecules *W* before and after heating. ($Q = 7.9\ kJ;W_1 = 1.8\,kJ;W_2 = 7.6\,kJ$).

111. In a closed vessel there is a mass $m = 14\ g$ of nitrogen at a pressure $p_1 = 0.1\,MPa$ and a temperature of $t_1 = 27^0C$. After heating, the pressure in the vessel increased fivefold. U_p to what temperature $t_2 = 27^0C$ was the gas heated? Find the volume V of the vessel and the amount of heat Q which was given the gas. ($Q = 12.4\,kJ;V = 12.4\,l;T_2 = 1500\ K$).

112. How much heat Q should be given to the mass $m = 12\ g$ of oxygen to heat it at $\Delta t = 50^0 C$ for $p = const$? ($Q = 545\ J$).

113. To heat the mass $m = 40\ g$ oxygen from the temperature $t_1 = 16^0 C$ to $t_2 = 40^0 C$, the amount of heat $Q = 628\ J$ was expended. What conditions was the gas heated (at constant volume or at constant pressure)? ($C_x = 20.8\ J/mol \cdot K$).

114. In a closed vessel of volume $V = 2$ liters, nitrogen is present, which density is $\rho = 1.4\ kg/m^3$. How much heat Q should be reported to nitrogen in order to heat it at $\Delta T = 100\ K$? ($Q = 208\ J$).

115. In order to heat a certain mass of gas at $\Delta t_1 = 50\ ^0C$ at $p = const$, it is necessary to spend the quantity of heat $Q_1 = 670\ J$. If the same mass of gas is cooled by $\Delta t_2 = 100\ ^0C$ at $V = const$, then the quantity of heat $Q_2 = 1005\ J$. How many degrees of freedom do the molecules of this gas have? ($i = 6$).

116. Helium is in a closed vessel of volume $V = 2$ liters at a temperature of $t_1 = 20\ ^0C$ and at pressure $p_1 = 100\ kPa$. How much heat Q should helium be given to increase its temperature by $\Delta t = 100\ ^0C$? What will the mean square velocity of its molecules, pressure p_2, helium density ρ_2 and energy of thermal motion W of its molecules be at a new temperature? ($Q = 102\ J; \sqrt{\overline{\upsilon^2}} = 1.57\ km/s; p_2 = 133\ kPa; \rho_1 = \rho_2 = 0.164\ kg/m^3; W = 400\ J$).

117. Find the arithmetic average, the mean square and the most probable υ in the velocity of gas molecules, which at a pressure of $p = 40\ kPa$ has a density $\rho = 0.3\ kg/m^3$. ($\overline{\upsilon} = 579\ m/s; \sqrt{\overline{\upsilon^2}} = 628\ m/s; \upsilon = 513\ m/s$).

118. What temperature T is the mean square velocity of nitrogen molecules greater than their most probable velocity by $\Delta\upsilon = 50\ m/s$? ($T = 83\ K$).

119. Which part of the oxygen molecules at $t = 0^0 C$ has velocities υ from 100 to 110 m/s? ($\Delta N / N = 0.4\%$).

120. Which part of the nitrogen molecules at $t = 150^0 C$ has velocities υ from 300 to 325 m/s? ($\Delta N / N = 2.8\%$).

121. How many times the number of molecules ΔN_1, whose velocities lie in the range from υ to $\upsilon + \Delta\upsilon$, is greater than the number of molecules ΔN_2, whose velocities lie in the interval from $\overrightarrow{\upsilon_{sq}}$ to $\overrightarrow{\upsilon_{sq}} + \Delta\overrightarrow{\upsilon_{sq}}$? ($\Delta N_1 / \Delta N_2 = 1.1\%$).

122. Which part of nitrogen molecules at temperature T has velocities lying in the range from υ to $\upsilon+\Delta\upsilon$, where $\Delta\upsilon = 20\ m/s$, if: a) $T = 400\ K$; b) $T = 900\ K$? ($a)\ \upsilon = 487\ m/s, \Delta N/N = 3.4\%; b)\ \upsilon = 731\ m/s, \Delta N/N = 2.2\%$).

123. Which part of nitrogen molecules at a temperature of $t = 150^0 C$ has velocities lying in the interval from $\upsilon_1 = 300\ m/s$ to $\upsilon_2 = 800\ m/s$? ($N_x/N = 70\%$).

124. Which fraction of the total number N of molecules has velocities: a) greater than the most probable velocity υ_b, b) less than the most probable velocity υ_b? ($a)\ N_1/N = 57\%; b)\ N_2/N = 43\%$).

125. In the vessel there is a mass $m = 2.5\ g$ of oxygen. Find the number N_x of oxygen molecules whose velocities exceed the mean square velocity. ($N_x = 1.9\cdot 10^{22}$).

126. In the vessel there is a mass $m = 8\ g$ of oxygen at a temperature $T = 1600\ K$. How many N_x oxygen molecules have kinetic energy of translational motion exceeding the energy $W_0 = 6.65\cdot 10^{-20}\ J$? ($N_x = 1.8\cdot 10^{22}$).

127. The energy of charged particles is often expressed in electron volts: 1 eV is the energy that the electron acquires after passing through the electric field the potential difference $U = 1V$, with 1 eV = 1.60219 • 10^{-19} J. What temperature T_0 is the average kinetic energy of the translational motion of the molecules $W_0 = 1\,eV$? At what temperature, 50% of all molecules have kinetic energy of translational motion, exceeding the energy $W_0 = 1\,eV$? ($T_0 = 7730\ K; T = 9600\ K$).

128. The molar energy required for the ionization of potassium atoms, $W_i = 418.68\ kJ/mol$. What temperature T of the gas 10% of all molecules have a kinetic kinetic energy of translational motion exceeding the energy W_i? ($T = 1.57\cdot 10^4\ K$).

129. What height h is the air pressure 75% of the sea level pressure? The air temperature is assumed to be constant and equal to $t = 0^0 C$. ($h = 2.3\ km$).

130. The passenger plane performs flights at a height of $h_1 = 8300\,m$. In order not to provide passengers with oxygen masks, a constant pressure is maintained in the cabin with the help of a compressor, corresponding to a height of $h_2 = 2700\,m$. Find the difference Δp of pressures inside and

outside the cabin. The temperature of the outside air is assumed to be equal to $t_1 = 0^0 C$. ($p_1 = 35.8\ kPa;\ p_2 = 72.5\ kPa; \Delta p = 36.3\ kPa$).

131. Find in the previous task how many times the air density ρ_2 in the cabin is greater than the density ρ_1 of the air outside it, if the outside air temperature is $t_1 = -20^0 C$, and the air temperature in the cabin is $t_2 = +20^0 C$. (1.7 *times*).

132. Find the air density ρ: a) near the surface of the Earth; b) at an altitude $h = 4\ km$ from the Earth's surface. The air temperature is assumed to be constant and equal to $t = 0^0 C$. The air pressure at the Earth's surface is $p_0 = 100\ kPa$. ($a)\ \rho = 1.28\ kg/m^3;\ b)\ \rho = 0.78\ kg/m^3$).

133. What height *h* is the density of gas half the density at sea level? The temperature of the gas is assumed to be constant and equal to $t = 0^0 C$. The problem is solved for air, $\mu = 29 \cdot 10^{-3}\ kg/mol$. ($a)\ h = 5.5\ km;\ b)\ h = 80\ km$).

134. Find the mean free path of air molecules under normal conditions. The diameter of the air molecules is $\sigma = 0.3\ nm$. ($\overline{\lambda} = 93\ nm$).

135. Find the average number of collisions of z per unit time of carbon dioxide molecules at a temperature of $t = 100^0 C$ if the mean free path $\lambda = 870\ \mu m$. ($\overline{z} = 4.9 \cdot 10^5\ s^{-1}$).

136. In a vessel of volume $V = 0.5$ liters, oxygen is present under normal conditions. Find the total number of collisions Z between oxygen molecules in this volume per unit time. ($Z = 3 \cdot 10^{31}$).

137. How many times will the number of collisions z decrease per unit time of the molecules of a diatomic gas if the volume of the gas is doubled adiabatically? (2.3. *times*).

138. Find the mean free path of nitrogen molecules at a pressure $p = 10\ kPa$ and a temperature $t = 17^0 C$. ($\overline{\lambda} = 1\ \mu m$).

139. At a certain pressure and temperature, $t = 0^0 C$, the mean free path of oxygen molecules is $\lambda = 95\ nm$. Find the average number of collisions of z per unit time of oxygen molecules, if at the same temperature the oxygen pressure is reduced by a factor of 100. ($\overline{z} = 4.5 \cdot 10^7\ s^{-1}$).

140. Under certain conditions, the mean free path of gas molecules is $\lambda = 160\ nm$; the average arithmetic velocity of its molecules is $95\ km/s$. Find the average number of collisions per unit time of molecules of this gas, if at the same temperature the gas pressure is reduced by 1.27 times. ($\overline{z} = 9.6 \cdot 10^9\ s^{-1}$).

141. There is carbon dioxide in the vessel, the density of which is $\rho = 1.7\ kg/m^3$. The mean free path of its molecules is $79\ nm$. Find the diameter of the molecules of carbon dioxide. ($\sigma = 0.35\ nm$).

142. Find the average time between two consecutive collisions of nitrogen molecules at a pressure of $p = 133\ Pa$ and a temperature of $t = 10^0 C$. ($\overline{\tau} = 1.6 \cdot 10^{-7}\ s$).

143. What pressure p should be created inside the spherical vessel, so that the molecules do not collide with each other if the diameter of the vessel: a) $D = 1\ cm$; b) $D = 10\ cm$; c) $D = 100\ cm$? The diameter of the gas molecules is $\sigma = 0.3\ nm$. ($a)\ p = 931\ mPa;\ b)\ 93.1\ mPa;\ c)\ 9.31\ mPa$).

144. Find the average number of collisions per unit time of molecules of a certain gas, if the average mean free path is $5\ \mu m$, and the mean square velocity of its molecules is $500\ m/s$. ($\overline{z} = 9.2 \cdot 10^7\ s^{-1}$).

145. Find the diffusion coefficient D of hydrogen under normal conditions, if the average mean free path is $0.16\ \mu m$. ($D = 9.1 \cdot 10^{-5}\ m^2/s$).

146. Find the mass m of nitrogen passing through the site $S = 0.01\ m^2$ over time $t = 10\ s$, if the density gradient in the direction perpendicular to the site, $\Delta\rho/\Delta x = 1.26\ kg/m^4$. The temperature of nitrogen is $t = 27^0 C$. The mean free path of nitrogen molecules is $10\ \mu m$. ($m = 2\ mg$).

147. Find the viscosity η of nitrogen under normal conditions, if the diffusion coefficient for it is $D = 1.42 \cdot 10^{-5}\ m^2/s$. ($\eta = 17.8 \cdot \mu Pa \cdot s$).

148. Find the diffusion coefficient D and the viscosity η of air at a pressure $p = 101.3\ kPa$ and a temperature $t = 10^0 C$. The diameter of the air molecules is $\sigma = 0.3\ nm$. ($D = 1.48 \cdot 10^{-5}\ m^2/s;\ \eta = 18.5\ \mu Pa \cdot s$).

149. What is the maximum speed υ that a rain drop with a diameter $D = 0.3\ mm$ can achieve? The diameter of the air molecules is $\sigma = 0.3\ nm$. Air temperature $t = 20^0 C$. To consider that the Stokes law holds for each raindrop ($\upsilon = 2.72 \cdot m/s$).

150. The aircraft flies at a speed of $\upsilon = 360\ km/h$. Assuming that the air layer at the wing of the aircraft, dragged by viscosity, $d = 4\ cm$, find the tangential force F_S acting per unit surface of the wing. The diameter of the air molecules is $\sigma = 0.3\ nm$. Air temperature $t = 0^0 C$. ($F_s = 45\ mN/m^2$).

151. Find the thermal conductivity k of hydrogen, whose viscosity $\eta = 8.6\ \mu Pa \cdot s$. ($K = 90 \cdot mW/m \cdot K$).

152. How much heat Q loses the room in time $t = 1\,h$ through the window due to the thermal conductivity of the air enclosed between the frames? The area of each frame is $S = 4\,m^2$, the distance between them is $d = 30\,cm$. The room temperature is $t_1 = 18^0C$, the outside temperature is $t_2 = -20^0C$. The diameter of the air molecules is $\sigma = 0.3\,nm$. The air temperature between the frames should be considered equal to the average arithmetic temperatures of the room and the outside air. The pressure $p = 10\,kPa$ $(Q = 23.9 \cdot kJ)$.

153. Mass $m = 10\,g$ of oxygen is at a pressure of $p = 300\,kPa$ and a temperature of $t = 10^0C$. After heating at $p = const$, the gas occupied a volume $V = 10$ liters. Find the quantity of heat Q obtained by the gas, the change ΔW of the internal energy of the gas, and work A, which is accomplished by the gas upon expansion ($Q = 7.92\,kJ$; $\Delta W = 5.66\,kJ$; $A = 2.26\,kJ$).

154. The amount of heat $Q = 2.093\,kJ$ is transfered to the diatomic gas. The gas expands at $p = const$. Find a job A gas expansion. ($A = 600 \cdot J$).

155. In a vessel of volume $V = 5$ liters there is a gas at a pressure $p = 200\,kPa$ and a temperature $t = 17^0C$. In the case of isobaric expansion of gas, work was performed $A = 196\,J$. How much was the gas heated? ($\Delta T = 57 \cdot K$).

156. With the isothermal expansion of the mass $m = 10\,g$ of nitrogen at a temperature of $t = 17^0C$, the work $A = 860\,J$ was performed. How many times did the pressure of nitrogen under the expansion change? (2.72 *times*).

157. The work of the isothermal expansion of the mass $m = 10\,g$ of a certain gas from the volume V_1 to $V_2 = 2V_1$ turned out to be equal to $A = 575\,J$. Find the mean square velocity of the gas molecules at this temperature. ($\sqrt{\upsilon^2} = 500\,m/s$).

158. The volume $V_1 = 7.5$ liters of oxygen is adiabatically compressed to a volume $V_2 = 11$, with the pressure $p_2 = 1.6\,MPa$ being established at the end of compression. Under what pressure p_1 was the gas before compression? ($p_1 = 95 \cdot kPa$).

159. The gas widens adiabatically, with its volume doubling, and the thermodynamic temperature drops 1.32 times. How many degrees of freedom i have the molecules of this gas? ($i = 5$).

160. The diatomic gas, which is at a pressure $p_1 = 2\ MPa$ and a temperature of $t = 27^0C$, is compressed adiabatically from the volume V_1 to $V_2 = 0.5 \cdot V_1$. Find the temperature t_2 and the gas pressure p_2 after compression. ($t_2 = 123^0C;\ p_2 = 5.28\ MPa$).

161. The gas under the normal conditions is located in the vessel under the piston. The distance between the bottom of the vessel and the bottom of the piston is $h = 25\ cm$. When a weight of $m = 20\ kg$ was placed on the piston, the piston dropped to $\Delta h = 13.4\ cm$. Assuming the compression is adiabatic, find the ratio c_p / c_v for a given gas. The cross-sectional area of the piston is $S = 10\ cm^2$. Mass of the piston is neglected. ($c_p / c_v = 1.4$).

162. The gas expands adiabatically so that its pressure drops from $p_1 = 200\ kPa$ to $p_2 = 100\ kPa$. Then it is heated at a constant volume to the original temperature, and its pressure becomes $p = 122\ kPa$. Find the ratio c_p / c_v for this gas. Draw a graph of this process. ($c_p / c_v = 1.4$).

163. With adiabatic compression of the quantity $\nu = 1\ kmol$ of diatomic gas, work $A = 146\ kJ$ was performed. How much did the gas temperature increase during compression? ($\Delta T = 7\ K$).

164. How many times will the average quadratic velocity of the molecules of a diatomic gas decrease when the gas volume doubles adiabatically? (1.15 *times*).

165. Mass $m = 10\ g$ of oxygen, which is under normal conditions, is compressed to the volume $V_2 = 1.4$ liters. Find the pressure p_2 and the oxygen temperature t_2 after compression, if oxygen is compressed: a) isothermally; b) adiabatically. Find work A compression in each of these cases. ($a)\ p_2 = 510\ kPa, T_2 = 273\ K,\ A = -1140\ J;$
$b)\ p_2 = 960\ kPa, T_2 = 520\ K,\ A = -1590\ J$).

166. The mass $m = 10\ g$ of air at a pressure $p_1 = 150\ kPa$ and temperature $t_1 = 30^0C$, widens adiabatically and the pressure drops to $p_2 = 100\ kPa$. How many times has the air volume increased? Find the final temperature t_2 and the work A, completed by the gas upon expansion. ($V_2 / V = 1.33_1, T_2 = 270\ K,\ A = -23\ kJ$).

167. The ideal thermal machine operating in the Carnot cycle, receives from the heater the amount of heat $Q_1 = 2.512\ kJ$ per cycle. The temperature of the heater $T_1 = 400\ K$, the temperature of the refrigerator $T_2 = 300\ K$.

Find the work A done by the machine in one cycle and the amount of heat Q_2 given to the refrigerator in one cycle. ($Q_2 = 1.88\ kJ$, $A = 630\ J$).

168. The ideal thermal machine, operating in the Carnot cycle, performs in one cycle the work $A = 2.94\ kJ$ and delivers in one cycle to the refrigerator the amount of heat $Q_2 = 13.4\ kJ$. Find the efficiency of the cycle. ($\eta = 18\%$).

169. The ideal thermal machine; working on the Carnot cycle, performs in one cycle the work $A = 73.5\ kJ$. The temperature of the heater $t_1 = 100^0 C$, the temperature of the refrigerator $t_2 = 0^0 C$. Find the efficiency of the cycle, the amount of heat Q_1 received by the machine in one cycle from the heater, and the amount of heat Q_2 given per cycle to the refrigerator. ($\eta = 26.8\%, Q_1 = 274\ kJ, Q_2 = 200\ kJ$).

170. The ideal thermal machine operates on the Carnot cycle. Air at a pressure $p_1 = 708\ kPa$ and a temperature of $t_1 = 127^0 C$ occupies a volume of $V_1 = 2$ liters. After the isothermal expansion, the air occupied the volume $V_2 = 5$ liters; After adiabatic expansion, the volume became equal to $V_3 = 8$ liters. Find: a) the coordinates of the intersection of isotherms and adiabats; b) work A, performed at each section of the cycle; c) the complete work A, performed for the entire cycle; d) the efficiency of the cycle; e) the amount of heat Q_1 received from the heater in one cycle; f) the amount of heat Q_2 given to the refrigerator in one cycle.

($a)\ V_1 = 2\ l,\ p_1 = 708\ kPa; V_2 = 5\ l,\ p_1 = 284\ kPa; V_3 = 8\ l,$

$p_3 = 146\ kPa; V_4 = 3.22\ l,\ p_4 = 365\ kPa.$

$b)\ A_1 = 1300\ J;\ A_2 = 620\ J;\ A_3 = -1070\ J;\ A_4 = -620\ J.$

$c)\ A = \sum A_i = 230\ J.\ d)\ \eta = 0.175.\ e)\ Q_1 = 1300\ J.\ f)\ Q_2 = 1070\ J.$).

171. The ideal refrigerating machine, operating in the reverse Carnot cycle, performs in one cycle the work $A = 37\ kJ$. In this case, it takes heat from the object with a temperature of $t_2 = -10^0 C$ and transfers heat to the object with temperature $t_1 = 17^0 C$. Find the efficiency of the cycle, the amount of heat Q_2 taken from the cold object in one cycle, and the amount of heat Q_1 transferred to the hotter object in one cycle. ($\eta = 0.093;. Q_1 = 397\ kJ; Q_2 = 360\ kJ$).

172. The room is heated by a refrigeration machine operating in the reverse Carnot cycle. How many times the amount of heat Q obtained by the room from the combustion of wood in the stove is less than the amount

of heat $Q`$ transferred to the room by the refrigeration machine, which is driven by a heat engine consuming the same mass of wood? The heat engine operates between temperatures $t_1 = 100^0C$ and $t_2 = 0^0C$. The room should be maintained at a temperature of $t`_1 = 16^0C$. The ambient temperature is $t`_2 = -10^0C$. $(Q`/Q = 3)$.

173. A steam engine with a power $P = 14.7\ kW$ consumes a mass $m = 8.1\ kg$ of coal with the specific heat of combustion $q = 33\ MJ/kg$ for a time $t = 1\ h$ of operation. The boiler temperature $t_1 = 100^0C$, the temperature of the refrigerator $t_2 = 58^0C$. Find the actual efficiency of the machine and compare it with the efficiency of an ideal thermal machine operating along the Carnot cycle between the same temperatures. $(\eta = 0.2, \eta` = 0.3)$.

174. A steam engine with a power $P = 14.7\ kW$ has a piston area $S = 0.02\ m^2$; Stroke of the piston $h = 45\ cm$. The isobaric process of the aircraft (Figure 7.4) occurs when the piston moves one third of its stroke. The volume V_0 is neglected in comparison with the volumes V_1 and V_2. The steam pressure in the boiler is $p_1 = 1.6\ MPa$, the vapor pressure in the refrigerator is $p_2 = 0.1\ MPa$. How many cycles does the machine do in a time $t = 1\min$ if the adiabatic index $\chi = 1.3$? $(104\ cicles)$.

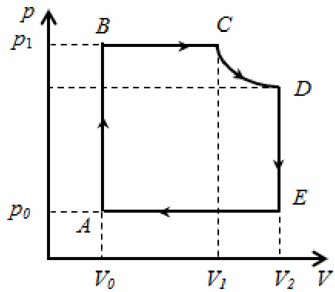


Figure 7.4

175. In the cylinders of the carburetor internal combustion engine, the gas is compressed polytropically to $V_2 = V_1/6$. The initial pressure $p_1 = 90\ kPa$, the initial temperature $t_1 = 127^0C$. Find the pressure p_2 and the temperature t_2 of gas in the cylinders after compression. The polytropic index is $n = 1.3$. ($p_2 = 930\ kPa; T_2 = 686\ K$).

176. In cylinders of a carburetor internal combustion engine, the gas is compressed polytropically so that after compression, the gas temperature becomes $t_2 = 427^0C$. The initial gas temperature is $t_1 = 140^0C$. The compression ratio is $V_2/V_1 = 5.8$. Find the polytropic exponent n. $(n = 1.3)$.

177. The carburetor engine with a power $P = 735.5\ W$ consumes for a time $t = 1\ h$ the minimum weight $m = 265\ g$ of gasoline. Find the loss of gasoline for friction, thermal conductivity, etc. The compression ratio is

$V_1/V_2 = 6.2$. Specific heat of combustion of gas oline $q = 46\ MJ/kg$. The polytropic index $n = 1.2$. (8%).

178. The internal combustion engine of a diesel engine has a degree of adiabatic compression $\varepsilon = 16$ and a degree of adiabatic expansion $\delta = 6.4$. What is the minimum mass m of oil consumed by an engine with a power $P = 36.8\ kW$ for a time $t = 1\ h$? The adiabatic exponent is $\gamma = 1.3$. Specific heat of combustion of oil $q = 46\ MJ/kg$. ($m = 5.9\ kg$).

179. Find the variation ΔS of the entropy in the transformation of the mass $m = 10\ g$ of ice ($t_1 = -20^0 C$) into steam ($t_2 = 100^0 C$). ($\Delta S = 88\ J/K$).

180. Find the change ΔS of entropy when the mass $m = 1\ g$ of water ($t_1 = 0^0 C$) is converted into steam ($t_2 = 100^0 C$). ($\Delta S = 7.4\ J/K$).

181. Find the variation ΔS of entropy for melting the mass $m = 1\ g$ of ice ($t = 0^0 C$). ($\Delta S = 1230\ J/K$).

182. Find the variation ΔS of entropy in the transition of the mass $m = 8\ g$ of oxygen from the volume $V_1 = 10$ liters at a temperature of $t_1 = 80^0 C$ to a volume of $V_2 = 40$ liters at a temperature of $t_2 = 300^0 C$. ($\Delta S = 5.4\ J/K$).

183. Find the change ΔS of entropy when the mass $m = 8\ g$ of hydrogen is transferred from the volume $V_1 = 20\ l$ at a pressure of $p_1 = 150\ kPa$ to a volume of $V_2 = 60\ l$ at a pressure of $p_2 = 100\ kPa$. ($\Delta S = 71\ J/K$).

184. Mass $m = 6.6\ g$ of hydrogen expands isobarically from volume V_1 to volume $V_2 = 2V_1$. Find the variation ΔS of the entropy under this expansion. ($\Delta S = 66.3\ J/K$).

185. Find the variation ΔS of the entropy in the isobaric expansion of the mass $m = 8\ g$ of helium from the volume $V_1 = 10$ liters to the volume $V_2 = 25$ liters. ($\Delta S = 38.1\ J/K$).

186. Find the variation ΔS of entropy in the isothermal expansion of the mass $m = 6\ g$ of hydrogen from the pressure $p_1 = 100\ kPa$ to the pressure $p_2 = 50\ kPa$. ($\Delta S = 17.3\ J/K$).

187. Mass $m = 10.5\ g$ of nitrogen isothermally expanding from volume $V_1 = 2$ liters to volume $V_2 = 5$ liters. Find the variation ΔS of entropy in this process. ($\Delta S = 2.9\ J/K$).

188. Mass $m = 10.5\ g$ of oxygen is heated from the temperature $t_1 = 50^0 C$ to the temperature $t_2 = 150^0 C$. Find the variation ΔS of entropy if

heating occurs: a) isochorically; b) isobaric. ($a)\ \Delta S = 1.76\ J/K;\ b)\ \Delta S = 2.46\ J/K$).

189. When the quantity $\nu = 1\,kmol$ of a diatomic gas is heated, its thermodynamic temperature increases from T_1 to $T_2 = 1.5T_1$. Find the variation ΔS of entropy if heating occurs: a) isochorically; b) isobaric. ($a)\ S_{1\to 2} = 8423.5\ J/K;\ b)\ S_{1\to 2} = 11793\ J/K$).

190. As a result of heating the mass $m = 22\ g$ of nitrogen, its thermodynamic temperature increased from T_1 to $T_2 = 1.2T_1$, and entropy increased by $\Delta S = 4.19\ J/K$. Under what conditions was the heating of nitrogen (at constant volume or at constant pressure)? (*heating was carried out at p = const*).

191. The volume $V_1 = 1\,m^3$ of air at a temperature of $t_1 = 0^0C$ and a pressure of $p_1 = 98\,kPa$ is isothermally expanded from the volume V_1 to the volume $V_2 = 2V_1$. Find the variation ΔS of entropy in this process. ($\Delta S = 500\ J/K$).

192. The change in entropy between the two adiabats in the Carnot cycle $\Delta S = 4.19\ kJ/K$. The temperature difference between two isotherms is $\Delta T = 100\ K$. How much heat Q is converted to work in this cycle? ($\Delta Q = 420\,kJ$).

7.6 ELECTRICITY

193. Find the force F of attraction between the nucleus of the hydrogen atom and the electron. The radius of the hydrogen atom $r = 0.5\cdot 10^{-10}\ m$; the charge of the nucleus is equal in absolute value and opposite in sign to the charge of the electron. ($F = 92.3nN$).

194. Two point charges, being in the air ($\varepsilon = 1$) at a distance $r_1 = 20\,cm$ from each other, interact with some force. What distance r_2 do you need to place these charges in the oil to get the same interaction force? ($r_2 = 8.94\,cm$).

195. How many times the force of gravitational attraction between two protons is less than the force of their electrostatic repulsion? The proton charge is equal in absolute value and opposite in sign to the charge of the electron. ($1.25\cdot 10^{36}$ *times*).

196. Find the force F of the electrostatic repulsion between the nucleus of the sodium atom and the proton bombarding it, assuming that the proton has approached the nucleus, the sodium atom, at a distance $r = 6\cdot 10^{-11}\ m$.

The charge of the sodium nucleus is 11 times that of the proton charge. The influence of the electron shell of the sodium atom is neglected. ($F = 0.7\,N$).

197. How many times the energy W_{el} of the electrostatic interaction of two particles with charge q and mass m is greater than the energy W_{gr} of their gravitational interaction? The problem is solved for: a) electrons; b) protons. ($a) W_e / W_g = 4.17 \cdot 10^{42}; b) W_e / W_g = 1.24 \cdot 10^{36}$).

198. Find the strength E of the electric field at a point in the middle between point charges $q_1 = 8\,nC$ and $q_2 = -6\,nC$. The distance between charges $r = 10\,cm$; $E = 1$. ($E = 50.4\,kV/m$).

199. The negative charge q_0 is placed in the center of the square, at each vertex of which there is a charge $q = 2.33\,nC$. Find this charge if the resultant force $F = 0.1\,N$ acts on each charge q. ($q_0 = -2.23\,nC$).

200. Two point charges $q_1 = 7.5\,nC$ and $q_2 = -14.7.\,nC$ are located at a distance $r = 5\,cm$. Find the intensity E of the electric field at a point $a = 3\,cm$ from the positive charge and $b = 4\,cm$ from negative charge. ($E = 112\,kV/m$).

201. Two balls of the same radius and mass are suspended on filaments of the same length, so that their surfaces come into contact. What charge q should be reported to the balls, so that the tension of the threads becomes $T = 98\,mN$? The distance from the center of the ball to the point of suspension is $l = 10\,cm$; the mass of each ball is $m = 5\,g$ ($q = 1.1\,\mu C$).

202. Two charged balls of the same radius and mass are suspended on filaments of the same length and dropped into a liquid dielectric whose density is equal to ρ and the dielectric constant is equal to ε. What should be the density ρ_0 of the material of the balls, so that the angles of the divergence of the filaments in the air and in the dielectric were the same? ($\rho_0 = \varepsilon\rho/(\varepsilon - 1)$).

203. Find the force F acting on the charge $q = 2\,C$, if the charge is placed: a) at a distance $r = 2\,cm$ from a charged filament with a linear charge density $\tau = 0.2\,\mu C/m$; b) in the field of a charged plane with a surface charge density $\sigma = 20\,\mu C/m^2$; c) at a distance $r = 2\,cm$ from the surface of a charged sphere with a radius $R = 2\,cm$ and a surface charge density $\sigma = 20\,\mu C/m^2$. The dielectric constant of the medium is $\varepsilon = 6$. ($a)\ F = 20\,\mu N; b)\ F = 126\,\mu N; c)\ F = 62.8\,\mu N.$).

204. Find the electric field strength E at a distance $r = 0.2\ nm$ from the monovalent ion. The charge of the ion is considered to be a point charge. ($E = 36\ GV/m$).

205. With what force F_e the electric field of a charged infinite plane acts per unit length of a charged infinitely long thread placed in this field? The linear charge density on the filament is $\tau = 3\ \mu C/m$ and the surface charge density in the plane is $\sigma = 20\ \mu C/m^2$. ($F_l = 3.4\ N/m$).

206. With what force F per unit length, two equally charged infinitely long threads with the same linear charge density $\tau = 3\ \mu C/m$, located at a distance $r_1 = 2\ cm$ from each other, repel? What kind of work A_e per unit length should be done to move these threads to a distance $r_2 = 1\ cm$? ($F_l = 8.1\ N/m;\ A_l = 0.112\ J/m$).

207. A copper sphere of radius $R = 0.5\ cm$ is placed in the oil. The oil density $\rho_\mu = 0.8 \cdot 10^3\ kg/m^3$. Find the charge q of the ball if in a uniform electric field the ball is suspended in the oil. The electric field is directed vertically upwards and its intensity is $E = 3.6\ MV/m$. ($q = 11\ nK$).

208. In a flat horizontally arranged condenser, a charged droplet of mercury is in equilibrium at an electric field strength $E = 60\ kV/m$. The charge of the drop is $q = 2.4 \cdot 10^{-9}\ CGSq$. Find the radius R of the drop. ($R = 0.44\ \mu m$).

209. At a point *A* located at a distance $a = 5\ cm$ from an infinitely long charged filament, the electric field strength $E = 150\ kV/m$. What limit length l of the filament does the found value of the tension will be correct to within 2% if the point *A* is on the normal to the middle of the filament? What is the strength *E* of the electric field at point *A*, if the length of the filament is $l = 20\ cm$? The linear density of a charge on a filament of finite length is assumed equal to the linear density of a charge on an infinitely long thread. Find the linear density of the charge τ on the filament. ($l = 0.49\ m;\ E = 135\ kV/m;\ \tau = 0.41\ \mu K/m$).

210. The electric field strength on the axis of the charged ring has a maximum value at a distance L from the center of the ring. How many times the intensity of the electric field at a point located at a distance of $0.5L$ from the center of the ring will be less than the maximum value of the tension? ($1.3\ times$).

211. The diameter of a charged disk $D = 25\ cm$. What limit distance from a disk along the normal to its center, the electric field can be regarded as a field of an infinitely extended plane? An error with this assumption

should not exceed $\delta = 0.05$. Note. The permissible error is $\delta = (E_2 - E_1)/E_2$, where E_2 is the field strength of the infinitely extended plane, E_1 is the field strength of the disk. ($a = 1.2\ cm$).

212. A ball of mass $m = 40\ mg$, having a positive charge $q = 1\ nC$, moves with a velocity $\upsilon = 10\ cm/s$. What distance r can the ball approach to a positive dot charge $q_0 = 1.33\ nC$? ($r = 6\ cm$).

213. What distance r can two electrons come together to, if they move towards each other at a relative velocity $\upsilon_0 = 10^6\ m/s$? ($r = 5.1 \cdot 10^{-10}\ m.$).

214. Two balls with charges $q_1 = 6.66\ nC$ and $q_2 = 13.33\ nC$ are at a distance $r_1 = 40\ cm$. What kind of work A should be done to bring them closer to a distance $r_2 = 25\ cm$? ($A = 1.2\ \mu J$).

215. Find the potential φ of the point of the field located at a distance $r = 10\ cm$ from the center of the charged sphere of radius $R = 1\ cm$. The problem should be solved if: a) the surface charge density on the ball is $\sigma = 0.1\ \mu C/m^2$; b) the potential of the ball is $\varphi_0 = 300\ V$. ($a)\ \varphi = 11.3\ V; b)\ \varphi = 30\ V$).

216. What work A occurs when a point charge $q = 20\ nC$ is transferred from infinity to a point at a distance $r = 1\ cm$ from the surface of a sphere of radius $R = 1\ cm$ with a surface charge density $\sigma = 10\ \mu C/m^2$. ($A = 113\ \mu J$).

217. A ball with mass $m = 1\ g$ and charge $q = 210\ nC$ is moved from point 1, the potential of which is $\varphi_1 = 600\ V$, to the point 2 whose potential is $\varphi_2 = 0$. Find its velocity at point 1 if at point 2 it becomes equal to $\upsilon_2 = 20\ cm/s$. ($\upsilon_1 = 16.7\ cm/s$).

218. At a distance $r_1 = 4\ cm$ from an infinitely long charged filament there is a point charge $q = 0.66\ nC$. Under the influence of the field, the charge approaches the filament to a distance $r_2 = 2\ cm$, while the work $A = 50\ erg$ is performed. Find the linear charge density τ? ($\tau = 0.6\ \mu K/m$).

219. A point charge $q = 0.66\ nC$ is located near the charged infinitely extended plane. The charge travels along the line of field strength to a distance $\Delta r = 2\ cm$; while the work $A = 50\ erg$ is performed. Find the surface charge density σ in the plane. ($\sigma = 6.6\ \mu K/m^2$).

220. The potential difference between the plates of a flat capacitor $U = 90\ V$. The area of each plate is $S = 60\ cm^2$, its charge is $q = 1\ nC$. What distance d are the plates from each other? ($d = 4.8\ mm$).

221. A flat capacitor can be used as sensitive microbalances. In a flat horizontally located capacitor, the distance between the plates is $d = 3.84\ mm$, there is a charged particle with a charge $q = 1.44 \cdot 10^{-9}\ CGASq$. In order for the particle to be in equilibrium, a potential difference $U = 40\ V$ must be applied between the plates of the capacitor. Find the mass m of the particle. ($m = 5.1 \cdot 10^{-16}\ kg$).

222. In a flat horizontally arranged condenser, the distance between the plates is $d = 1\ cm$, there is a charged droplet of mass $m = 5 \cdot 10^{-11}\ g$. In the absence of an electric field, a drop due to air resistance drops at a certain constant speed. If a potential difference $U = 600\ V$ is applied to the plates of the capacitor, then the droplet drops twice as slowly. Find the charge q of the droplet. ($q = 4.1 \cdot 10^{-18}\ K$).

223. A speck of dust falls between the two vertical plates at the same distance from them. Due to air resistance, the dust particle falls at a constant speed $\upsilon_1 = 2\ cm/s$. What time *t*, after a voltage difference of $U = 3\ kV$ is applied to the plates, a speck of dust reaches one of the plates? What distance *l* along the vertical does the dust particle fly before it hits the plate? The distance between the plates is $d = 2\ cm$, the dust mass $m = 2 \cdot 10^{-9}\ g$, its charge is $q = 6.5 \cdot 10^{-17}\ C$. ($t = 1\ s$).

224. Between two vertical plates, located at a distance $d = 1\ cm$ from each other, a charged bubble ball weighing $m = 0.1\ g$ hangs on the filament. After feeding a potential difference $U = 1\ kV$ to the plates, the filament with the ball is deflected by an angle $\alpha = 10^0$. Find the charge q of the ball. ($q = 1.73\ nK$).

225. An electron, having traveled in a flat capacitor from one plate to the other, acquires a velocity $\upsilon = 10^6\ m/s$. The distance between the plates is $d = 5.3\ mm$. Find the potential difference U between the plates, the electric field strength E in the capacitor and the surface charge density σ on the plates. ($U = 2.8\ V;\ E = 530\ V/n;\ \sigma = 4.7\ nK/m^2$).

226. The electric field is formed by two parallel plates, located at a distance $d = 2\ cm$ from each other. The potential difference $U = 120\ V$ is applied to the plates. What velocity υ will an electron receive under the action of a field, passing a distance $r = 3\ mm$ along the line of tension? ($\upsilon = 2.53 \cdot 10^6\ m/s$).

227. The electron enters a planar horizontally located capacitor at a certain speed parallel to the plates at an equal distance from them. The field

strength in the capacitor is $E = 100\,V/m$; the distance between the plates $d = 4\,cm$. What time *t* after the electron has flown into the capacitor, will it fall on one of the plates? What distance *S* from the beginning of the capacitor does an electron get to the plate if it is accelerated by a potential difference $U = 60\,V$? ($t = 480\,ns$; $S = 22\,cm$).

228. The electron enters a flat horizontally located capacitor parallel to the plates at a velocity $\upsilon_0 = 9\cdot 10^6\ m/s$. The potential difference between the plates is $U = 100\,V$; the distance between the plates is $d = 1\,cm$. Find the total a, normal an, and tangential acceleration of the electron through the time $t = 10\,s$ after the start of its motion in the condenser. ($a_\tau = 15.7\cdot 10^{14}\ m/s^2; a_n = 8\cdot 10^{14}\ m/s^2; a = 17.6\ m/s^2$).

229. The proton and the α particle, moving at the same velocities, enter the flat capacitor parallel to the plates. How many times will the deviation of the proton by the capacitor field be greater than the deflection of the α particle? (2 *times*).

230. A proton and an α particle accelerated by the same potential difference fly into a flat capacitor parallel to the plates. How many times will the deviation of the proton by the capacitor field be greater than the deviation of the α particle? (*The deviation of the proton and α particle will be the same*).

231. A beam of electrons accelerated by a potential difference $U_0 = 300\,V$ passes through an uncharged flat horizontally located condenser parallel to its plates, giving a luminous spot on a fluorescent screen located at a distance $x = 12\,cm$ from the end of the condenser. When charging the capacitor, the spot on the screen is shifted by a distance $Y = 3\,cm$. The distance between the plates is $d = 1.4\,cm$; the length of the capacitor is $l = 6\,cm$. Find the potential difference U applied to the plates of the capacitor. ($U = 28\,V$).

232. Find the capacity from the globe. Read the radius of the globe $R = 6400\ km$. How much will the potential φ of the globe change if it is told to charge $q = 1\,C$? ($C = 710\ \mu F$, $\Delta\varphi = 1400\,V$).

233. The ball, charged to the potential $\varphi = 792\,V$, has a surface charge density $\sigma = 333\,nC/m^2$. Find the radius r of the ball. ($r = 2.1\,cm$).

234. Two balls of the same radius $r = 1\,cm$ and masses $m = 0.15\,kg$ are charged to the same potential $\varphi = 3\,kV$ and are at some distance r_1 from each other. At the same time, their energy of gravitational interaction is

$W_{gr} = 10^{-11}\ J$. The balls approach each other to the distance r_2. The work needed to bring the balls closer, $A = 2 \cdot 10^{-6}\ J$. Find the energy W_{el} of the electrostatic interaction of the balls after they approach each other. ($W_{el} = 2.66 \cdot 10^{-6}\ J$).

235. The area of plates of a flat air condenser $S = 1\,m^2$, the distance between them is $d = 1.5\ mm$. Find the capacitance C of this capacitor. ($C = 5.9\ nF$).

236. The area of plates of a flat air-cooled condenser $S = 0.01\,m^2$, the distance between them is $d = 5\ mm$. The potential difference $U_1 = 300\,V$ is applied to the plates. After the capacitor is disconnected from the voltage source, the space between the plates is filled with ebonite. What will be the potential difference U_2 between the plates after filling? Find capacitances C_1 and C_2 and surface charge densities σ_1 and σ_2 on the plates before and after filling. ($C_1 = 17.7\ pF, C_2 = 46\ pF, \sigma_1 = \sigma_2 = 531\,nK/m^2$).

237. Plate area of a flat capacitor $S = 0.01\,m^2$, the distance between them is $d = 1\,cm$. A potential difference $U = 300\,V$ is applied to the plates. In the space between the plates there is a plane-parallel plate of glass with a thickness $d_1 = 0.5\,cm$ and a parabolic parallel plate with a thickness $d_2 = 0.5\,cm$. Find the electric field strengths E_1 and E_2 and the potential drop U_1 and U_2 in each layer. What will be the capacitance C of the capacitor and the surface charge density σ on the plates? ($U_1 = 75\,V, U_2 = 225\,V, C = 26.6\ pF, \sigma = 0.8\ \mu K/m^2$).

238. A potential difference $U = 100\,V$ is applied between the plates of a flat capacitor, located at a distance $d = 1\,cm$ from each other. A plane-parallel plate of crystalline thallium bromide ($\varepsilon = 173$) with thickness $d_0 = 9.5\,mm$ adjoins one of the plates. After the capacitor is disconnected from the voltage source, the crystal plate is removed. What will be the potential difference U between the plates of the capacitor after this? ($U = 1.8\,kV$).

239. The vacuum cylindrical capacitor has a radius of the inner cylinder $r = 1.5\,cm$ and the radius of the outer cylinder is $R = 1.5\,cm$. A potential difference $U = 2.3\,kV$ is applied between the cylinders. What is the velocity υ of the electron under the action of the field of this condenser, moving from a distance $l_1 = 2.5\,cm$ to a distance $l_2 = 2\,cm$ from the axis of the cylinder? ($\upsilon = 1.46 \cdot 10^7\ m/s$).

240. The cylindrical capacitor consists of an inner cylinder of radius $r = 3\ mm$, two layers of dielectric and an outer cylinder of radius $R = 1\ cm$. The first layer of dielectric thickness $d_1 = 3\ mm$ adjoins the inner cylinder. Find the ratio of the potential drop U_1/U_2 in these layers. ($U_1/U_2 = 1.35$).

241. Find the capacitance C of a spherical capacitor consisting of two concentric spheres with radii $r = 10\ mm$ and $R = 10.5\ cm$. The space between the spheres is filled with oil. Which radius R_0 should have a ball placed in the oil in order to have the same capacity? ($C = 1.17\ nF,\ R_0 = 2.1\ m$).

242. Radius of the inner sphere of the vacuum spherical capacitor $r = 1\ cm$, radius of the outer sphere $R = 4\ cm$. A potential difference $U = 3\ kV$ is applied between the balls. What speed does the electron get, approaching the center of the balls from a distance $x_1 = 3\ cm$ to a distance of $x_2 = 2\ cm$? ($\upsilon = 1.54 \cdot 10^7\ m/s$).

243. By means of an electrometer, the capacitances of the two capacitors were compared. To do this, they were charged to the potential differences $U_1 = 300\,V$ and $U_2 = 100\,V$ and connected the two capacitors in parallel. The potential difference between the capacitor plates measured by the electrometer turned out to be equal to $U = 250\,V$. Find the capacitance ratio C_1/C_2. ($C_1/C_2 = 3$).

244. What extent can the capacitance C of a system consisting of two capacitors of variable capacitance change, if the capacitance C_i of each of them varies from 10 to 450 *pF*? (*The capacitance C of the capacitor system varies from 20 to 900 pF in parallel connection and from 5 to 225 pF in the series*).

245. A capacitor with a capacitance $C = 20\ \mu F$ is charged to a potential difference $U = 100\,V$. Find the energy W of this capacitor. ($W = 0.1\ J$).

246. A sphere of radius $R = 1\ m$ is charged to a potential of $\varphi = 30\ kV$. Find the energy W of the charged sphere. ($W = 0.05\ J$).

247. A ball immersed in kerosene has a potential of $\varphi = 4.5\ kV$ and a surface charge density $\sigma = 11.3\ \mu C/m^2$. Find the radius R, the charge q, the capacitance C, and the energy W of the ball. ($R = 7\ mm;\ q = 7\ nC;\ C = 1.55\ pF;\ W = 15.8\ \mu J$).

248. A ball 1 of radius $R_1 = 10\ cm$, charged to the potential $\varphi_1 = 3\ kV$, after disconnection from the voltage source is connected with a wire (whose capacity can be neglected) first with the remote. Uncharged ball 2, and then

after detachment from the ball 2 with the removed uncharged ball 3. Balls 2 and 3 have radii $R_1 = R_2 = 10\,cm$. Find: a) the initial energy W_1 of the ball 1; b) the energies W_1` and $W_2$` of the balls 1 and 2 after the connection and the operation A of the discharge when connected; c) the energies W_1` and $W_3$` of the balls 1 and 3 after the connection and the A discharge operation when connected. ($a)\,W_1 = 50\,\mu J;\ b)\,W`_1 = W`_2 = 12.5\,\mu J,\ A = 25\,\mu J;$ $c)\,W`_1 = W`_3 = 3.125\,\mu J,\ A = 6.25\,\mu J$).

249. Two metal balls, the first with a charge of $q_1 = 10\,nC$ and a radius of $R_1 = 3\,cm$ and a second with a potential of $\varphi_2 = 9\,kV$ and a radius of $R_2 = 2\,cm$, are connected by a wire whose capacitance can be neglected. Find: a) the potential φ_1 of the first ball before discharge;b) the charge q_2 of the second ball before discharge; c) the energies W_1 and W_2 of each ball before discharge; d) the charge q_1` and the potential $\varphi_1$` of the first ball after the discharge; e) the charge q_2` and the second-ball potential $\varphi_2$` after the discharge; f) the energy W of balls connected by a conductor; g) work A discharge. ($a)\ \varphi_1 = 3\,kV;\ b)\ q_2 = 20\,nC;\ c)\ W_1 = 15\,\mu J,\ W_2 = 90\,\mu J;$ $d)\ q_1^` = 18\,nC,\ \varphi_1^` = 5.4\,kV;$ $e)\ q_2^` = 18\,nC,\ \varphi_2^` = 5.4\,kV;\ f)\ W = 81\,\mu J;\ g)\ A = 24\,\mu J$).

250. A charged ball 1 of radius $R_1 = 2\,cm$ is brought into contact with an uncharged ball 2 whose radius is $R_2 = 3\,cm$. After the balls are disconnected, the energy of the ball 2 is $W_2 = 0.4\,J$. What charge q_1 was on ball 1 to contact with the ball 2? ($q_1 = 2.7\,\mu C$).

251. A thin mica plate is embedded between the plates of a flat capacitor. What pressure p is tested by this plate at an electric field strength $E = 1\,MV/m$? ($p = 26.5\,Pa$).

252. Absolute electrometer is a flat capacitor, the lower plate of which is fixed, and the upper one is suspended to the beam of the scales. With an uncharged capacitor, the distance between the plates is $d = 1\,cm$. What is the difference in the potentials U applied between the plates, if for the same distance $d = 1\,cm$ the other weight cup had to be loaded with a mass $m = 5.1\,g$? The area of the capacitor plates is $S = 50\,cm^2$. ($U = 15\,kV$).

253. A flat capacitor is filled with a dielectric and a potential difference is applied to its plates. Its energy is $W = 20\,\mu J$. After the capacitor was disconnected from the voltage source, the dielectric was taken out of the

condenser. The work that had to be done against the forces of the electric field in order to remove the dielectric, $A = 70\ mJ$. Find the permittivity ε of the dielectric. ($\varepsilon = 4.5$).

254. Find the energy density W_0 of the electric field at a point a) at a distance $x = 2\ cm$ from the surface of a charged sphere of radius $R = 1\ cm$, b) near an infinitely extended charged plane, c) at a distance $x = 2\ cm$ from an infinitely long charged thread. The surface charge density on the ball and plane is $\sigma = 16.7\ \mu C/m^2$, the linear charge density on the filament is $\tau = 167\ nC/m$. The dielectric constant of the medium is $\varepsilon = 2$. ($a)\ W_0 = 97\ mJ/m^3;\ b)\ W_0 = 1.97\ J/m^3;\ c)\ W_0 = 50\ mJ/m^3$).

255. A potential difference $U = 1\ kV$ is applied to plates of a flat capacitor, the distance between which is $d = 3\ cm$. The space between the plates is filled with a dielectric ($\varepsilon = 7$). Find the surface density of bound (polarization) charges σ_{sv}. How much does the surface charge density on the plates change when the capacitor is filled with a dielectric? The problem is solved if the capacitor is filled with a dielectric: a) before the capacitor is disconnected from the voltage source; b) after the capacitor is disconnected from the voltage source. ($a)\ \sigma_1 = 17.7\ \mu K/m^2;\ b)\ \sigma_2 = 2.53\ \mu K/m^2$).

256. The space between the plates of a flat capacitor is filled with glass. The distance between the plates is $d = 1\ mm$. A potential difference $U = 1.2\ kV$ is applied to the capacitor plates. Find: a) the field strength E in the glass; b) the surface charge density σ_d on the capacitor plates; c) the surface density of the bound charges σ_{cv} on the glass; d) the dielectric susceptibility x of the glass. ($a)\ E = 300\ kV/m,\ b)\ \sigma_1 = 15.9\ \mu K/m^2,\ c)\ \sigma_2 = 15.9\ \mu K/m^2,$
$d)\ \chi` = 44.4\ pF/m,\ \chi = 0.4$).

7.7 DIRECT CURRENT

257. The current I in the conductor changes with time t according to the equation $I = 4 + 2t$, where I is in amperes and t is in seconds. How much electricity q passes through the cross-section of the conductor in a time from $t_1 = 2\ s$ to $t_2 = 6\ s$? What constant current I_0 does the same amount of electricity pass through the conductor cross-section during the same time? ($q = 48\ C;\ I_0 = 12\ A$).

258. How many turns of nichrome wire with a diameter $d = 1\,mm$ should be applied t_0 a porcelain cylinder of radius $a = 2.5\,cm$ in order to obtain a furnace with resistance $R = 40\,\Omega$? ($N = 200$).

259. The copper wire coil has a resistance $R = 10.8\,\Omega$. Weight of copper wire $m = 3.41\,kg$. How long is l and what diameter d is the wire wound on the coil? ($l = 500\,m$; $d = 1\,mm$).

260. Find the resistance R of an iron rod of diameter $d = 1\,cm$ if the mass of the rod is $m = 1\,kg$. ($R = 1.8\,m\Omega$).

261. The tungsten filament of an electric bulb at $t_1 = 20^0C$ has a resistance $R = 35.8\,\Omega$. What will be the temperature t_2 of the lamp string, if a current $I = 0.33\,A$ is applied to the network when the voltage $U = 120\,V$ is applied to the network? The temperature coefficient of tungsten resistance is $\alpha = 4.6 \cdot 10^{-5}\,K^{-1}$. ($t_2 = 2200\,^0C$).

262. Coil winding from a copper wire at $t_1 = 14^0C$ has a resistance $R_1 = 10\,\Omega$. After passing the current, the winding resistance became $R_2 = 12.2\,\Omega$. Up to what temperature t_2 did the winding heat up? The temperature coefficient of copper resistance is $\alpha = 4.15 \cdot 10^{-3}\,K^{-1}$. ($t_2 = 70\,^0C$).

263. Find the potential drop U on a copper wire of length $l = 500\,m$ and diameter $d = 2\,mm$, if the current in it is $I = 2\,A$. ($U = 5.4\,V$).

264. Find the potential drop U in the resistances $R_1 = 4\,\Omega$, $R_2 = 2\,\Omega$ and $R_3 = 4\,\Omega$ (Figure 7.5) if the ammeter shows the current $I_1 = 3\,A$. Find the currents I_2 and I_3 in the resistances R_2 and R_3.($U_1 = 12\,V, U_2 = U_3 = 4\,V$; $I_2 = 2\,A, I_3 = 1\,A$).

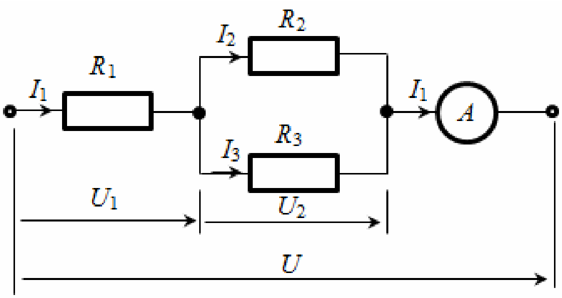


Figure 7.5

265. The element having the EMF $E = 1.1\,V$ and internal resistance $r = 1\,\Omega$, closed to external resistance $R = 9\,\Omega$. Find the current I in the circuit, the potential drop U in the external circuit and the drop in the U_r potential inside the element. What is the efficiency of the element? $(I = 0.11\,A, U = 0.99\,V, U_r = 0.11\,V, \eta = 0.9)$.

266. There are two identical elements with EMF $E = 2\,V$ and internal resistance $r = 0.3\,\Omega$. How to connect these elements (in series or in parallel) in order to obtain a larger current, if the external resistance: a) $R = 0.2\,\Omega$; b) $R = 16\,\Omega$? Find the current I in each of these cases. $(a)\ I_1 = 5\,A, I_2 = 5.7\,A;\ b)\ I_1 = 0.24\,A, I_2 = 0.124\,A)$.

267. Assuming the resistance of the ammeter R_A is infinitesimal, determine the resistance R from the indications of the ammeter and voltmeter (Figure 7.6). Find the relative error $\Delta R / R$ of the resistance found, if in reality the resistance of the ammeter is R_A. Solve the problem for $R_A = 0.2\,\Omega$ and resistance: a) $R = 1\,\Omega$; b) $R = 10\,\Omega$; c) $R = 100\,\Omega$. $(a)\ \Delta R / R = 1\%;\ b)\ \Delta R / R = 10\%;\ c) \Delta R / R = 100\%)$.

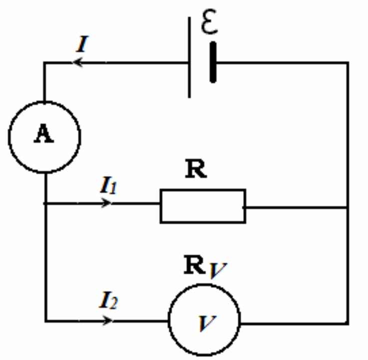


Figure 7.6

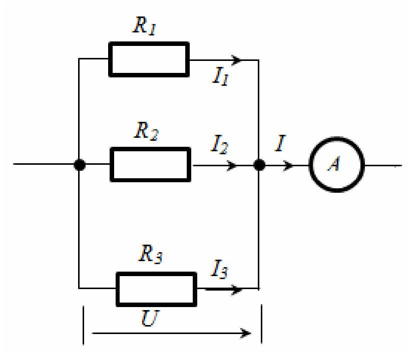


Figure 7.7

268. Resistance $R_2 = 20\,\Omega$ and $R_3 = 15\,\Omega$ (Figure 7.7). The current $I_2 = 0.8\,A$ flows through the resistance R_2. The ammeter shows the current $I = 0.8\,A$. Find the resistance R_1. $(R_1 = 60\,\Omega.)$.

269. Batteries with EMF $E = 100\,V$, resistance $R_1 = R_3 = 40\,\Omega$, $R_2 = 86\,\Omega$ and $R_2 = 34\,\Omega$ (Figure 7.8). Find the current I_2, flowing through the resistance R_2, and the drop of the potential U_2 on it. $(I_2 = 0.4\,A, U_2 = 32\,V)$.

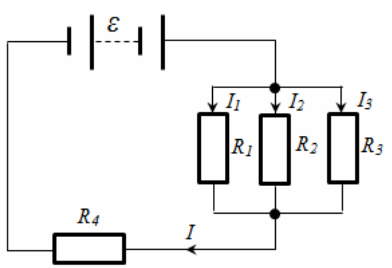


Figure 7.8

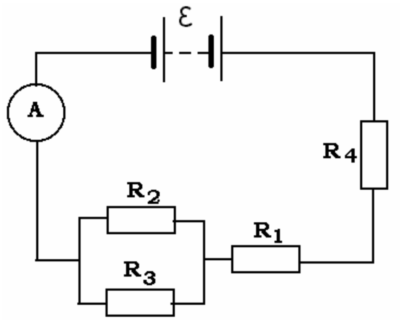


Figure 7.9

270. Battery with EMF $E = 10\,V$ and an internal resistance $r = 1\,\Omega$ has an efficiency of $\eta = 0.8$ (Figure 7.9). The drops in the potential at R_1 and R_4 are $U_1 = 4\,V$ and $U_4 = 2\,V$. Which current I shows the ammeter? Find the drop in the potential U_2 on the resistance R_2. ($I = 2\,A, U_2 = 2\,V$).

271. Resistance $R_1 = R_2 = R_3 = 200\,\Omega$, resistance of the voltmeter $R_V = 1\,k\Omega$ (Figure 7.10). The voltmeter shows the potential difference $U = 100\,V$. Find EMF E batteries. ($\varepsilon = 170\,V$).

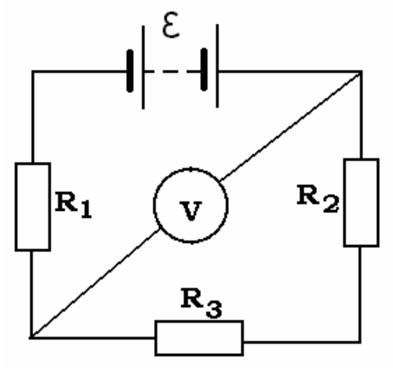


Figure 7.10

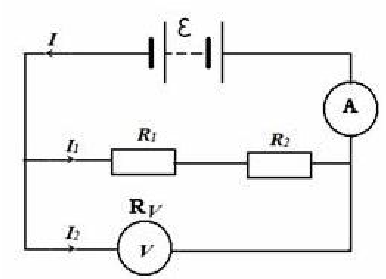


Figure 7.11

272. Find the readings of the ammeter and voltmeter in the diagrams shown in Figure`s 7.11-7.14. Batteries with EMF $E = 110\,V$, resistance $R_1 = 400\,\Omega$ and $R_2 = 600\,\Omega$, resistance of the voltmeter $R_V = 1\,k\Omega$. ($a)\ I = 0.22\,A, U = 110\,V; b)\ I = 0.142\,A, U = 53.2\,V; c)\ I = 0.57\,A, U = 110\,V;$ $d)\ I = 0.089\,A, U = 35.6\,V$).

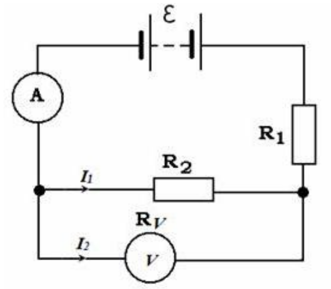


Figure 7.12

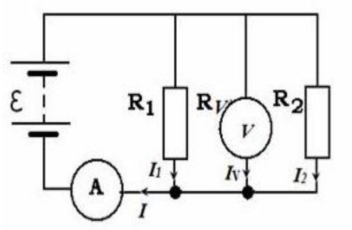


Figure 7.13

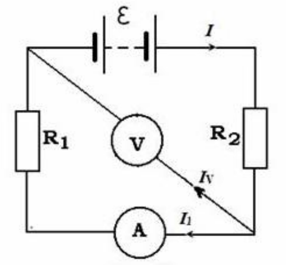


Figure 7.14

273. There is an ammeter with resistance $R_A = 0.18\,\Omega$, whose scale is divided into 100 divisions, designed for measuring currents up to $I = 10\,A$. What resistance R should be taken and how to turn it on, so that this ammeter could measure the current to $I_0 = 100\,A$? How will the cost of dividing the ammeter change? ($R = 0.02\,\Omega$).

274. On the battery with EMF $E = 500\,V$, it is required to transfer energy over a distance $l = 2.5\,km$. Power consumption $P = 10\,kW$. Find the minimum power loss ΔP in the network if the diameter of the copper lead wires is $d = 1.5\,cm$. ($\Delta P = 212\,W$).

275. From the generator with EMF $E = 110\,V$, it is required to transfer energy over a distance $l = 250\,m$. Power consumption $P = 1\,kW$. Find the minimum cross-section S of copper lead wires, if the power losses in the network should not exceed 1%. ($S = 78\,mm^2$).

276. The element is first closed to external resistance $R_1 = 2\,\Omega$, and then to external resistance $R_2 = 0.5\,\Omega$. Find EMF E of the element and its internal resistance r, if it is known that in each of these cases the power released in the external circuit is the same and equal to $P = 2.54\,W$. ($\varepsilon = 4\,V$, $r = 1\,\Omega$).

277. Element with EMF E and the internal resistance r is closed to the external resistance R. The maximum power released in the external circuit, $P = 9\,W$. In this case, the current flows in the circuit $I = 3\,A$. Find the EMF E and the internal resistance r of the element. ($\varepsilon = 6\,V, r = 1\,\Omega$).

278. Batteries with EMF $E = 120\,V$, resistance $R_3 = 30\,\Omega$, $R_2 = 60\,\Omega$ (Figure 7.15). The ammeter shows the current $I = 2\,A$. Find the power P, which is released in the resistance R_1. ($P = 60\,W$).

279. Battery with EMF $E = 100\,V$, its internal resistance $r = 2\,\Omega$, resistance $R_1 = 25\,\Omega$ and $R_2 = 78\,\Omega$ (Figure 7.15). The resistance R_1 is allocated power $P_1 = 16\,W$. What current I shows the ammeter? ($I = 1\,A$).

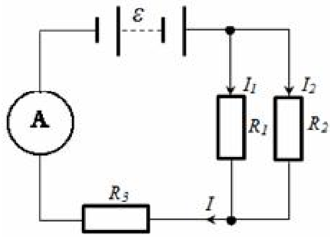


Figure 7.15

280. The potential difference between points A and B is $U = 9\,V$. There are two conductors with resistances $R_1 = 5\,\Omega$ and $R_2 = 3\,\Omega$. Find the amount of heat Q_τ that is released in each conductor per unit time if the conductors between points A and B are connected: a) sequentially; b) in parallel. ($a)\ Q_{\tau 1} = 6.37\ J/s, Q_{\tau 2} = 3.82\ J/s; b)\ Q_{\tau 1} = 16.2\ J/s, Q_{\tau 2} = 27.2\ J/s$).

281. Two light bulbs with resistances $R_1 = 360\,\Omega$ and $R_2 = 240\,\Omega$ are connected in parallel. Which bulb consumes more power? How many times? (1.5 *times*).

282. What volume V of water can be boiled, having spent electric energy $W = 3\,gW \cdot h$? The initial water temperature is $t_0 = 10^0 C$. ($V = 2.9\,l$).

283. What power P consumes the heater of an electric kettle if the volume $V = 1$ liter of water boils through a time $\tau = 5$ minutes? What is the resistance R of the heater, if the voltage in the network is $U = 120\,V$? The initial water temperature is $t_0 = 13.5^0 C$. ($P = 1.2\,kW, R = 12\,\Omega$).

284. The temperature of the water thermostat of volume $V = 1\,l$ is kept constant with a heater of power $P = 26\,W$ 80% of this capacity is spent on heating water. How much will the water temperature in the thermostat go down during the time $\tau = 10$ min, if the heater is switched off? ($\Delta t = 3\ ^0C$).

285. Find the amount of heat Q_τ, released per unit time per unit volume of a copper wire at a current density $j = 300\,kA/m^2$. ($Q_\tau = 1.55\,kJ/s \cdot m^3$).

286. Find the currents I_i in the individual branches of the Wheatstone bridge (Figure 7.16), provided that the current $I_r = 0$ passes through the galvanometer. Element $E = 2V$, resistance $R_1 = 30\,\Omega$, $R_2 = 45\,\Omega$ and $R_3 = 200\,\Omega$.

($I_1 = I_2 = 26.7\,mA$; $I_3 = I_4 = 4\,mA$).

287. Which potential difference U is obtained at the terminals of two elements connected in parallel, if their EMF`s $E_1 = 1.4\,V$ and $E_2 = 1.2\,V$ and internal resistance $r_1 = 0.6\,\Omega$ and $r_2 = 0.4\,\Omega$?

($U = 1.28\,V$).

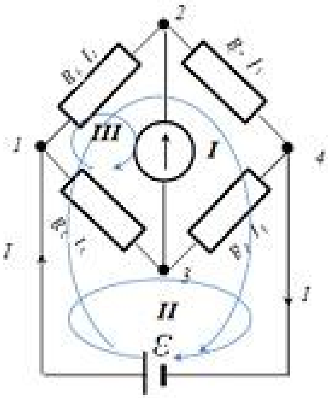


Figure 7.16

288. The batteries have EMF`s $E_1 = 110\,V$ and $E_2 = 220\,V$, resistance $R_1 = R_2 = 100\,\Omega$, $R_3 = 500\,\Omega$ (Figure 7.17). Find the ammeter reading. ($I = 0.4\,A$).

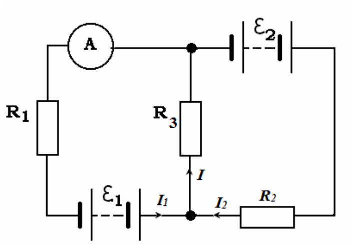


Figure 7.17

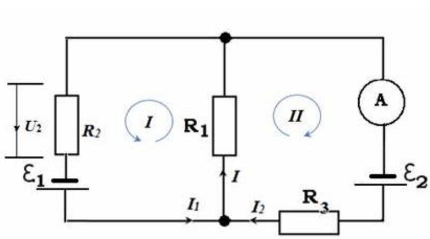


Figure 7.18

289. The batteries have EMF with $E_1 = 2\,V$ and $E_2 = 3\,V$, resistance $R_1 = 1.5\,k\Omega$, resistance of the ammeter $R_A = 0.5\,k\Omega$ (Figure 7.18). The drop in potential at the resistance R_2 is $U_2 = 1V$ U₂ = 1 *V* (current through R_2 is directed from top to bottom). Find the ammeter reading. ($I = 1\,mA$).

290. Two identical elements have EMF $E_1 = E_2 = 2\,V$ and internal resistances $r_1 = r_2 = 0.5\,\Omega$ (Figure 7.19). Find the currents I_1 and I_2, flowing through the resistors $R_1 = 0.5\,\Omega$ and $R_2 = 1.5\,\Omega$, and also the current I through the element with EMF E_1.

($I_1 = 2.28\,A$; $I_2 = 0.56\,A$; $I = 1.72\,A$).

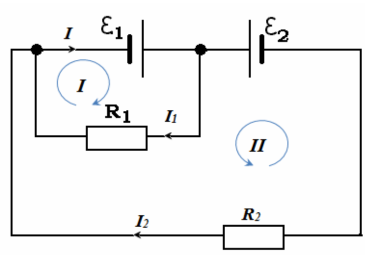


Figure 7.19

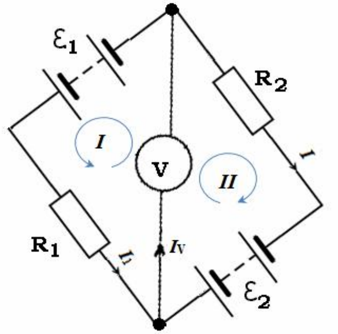


Figure 7.20

291. The batteries have EMF with $E_1 = E_2 = 110\,V$, resistance $R_1 = R_2 = 0.2\,k\Omega$, resistance of the voltmeter $R_V = 1\,k\Omega$ (Figure 7.20). Find the voltmeter reading. ($U = 100\,V$).

292. What time τ in the electrolysis of an aqueous solution of chloride copper ($CuCl_2$), the mass $m = 4.74\,g$ of copper will be released at the cathode, if the current $I = 2\,A$? ($\tau = 2\,h$).

293. What time τ in the electrolysis of copper sulfate, the mass of the copper plate (cathode) will increase by $\Delta m = 99\,mg$? The area of the plate is $S = 25\,cm^2$, the current density is $j = 220\,A/m^2$. Find the thickness d of the copper layer formed on the plate. ($\tau = 10$ min; $d = 4.6\,\mu m$).

294. Find the electrochemical equivalent of K hydrogen. ($K = 1.04 \cdot 10^{-8}\,kg/K$).

295. Ammeter connected in series with the electrolytic bath with $AgNO_3$ solution, shows the current $I = 0.90\,A$. Is the ammeter correct if a mass $m = 316\,mg$ of silver was released during the time $\tau = 5$ min of current passage? (*The ammeter shows less by 0.04 A*).

296. Two electrolytic baths with $AgNO_3$ and $CuSO_4$ solutions are connected in series. What mass of m_2 of copper will be released during the time during which the mass $m_1 = 180\,mg$ of silver was released? ($m_2 = 53\,mg$).

297. In the preparation of aluminum by electrolysis of a solution of Al_2O_3 in a molten cryolite, a current $I = 20\,kA$ was passed with a potential difference on the electrodes $U = 5\,V$. What time τ will the mass $m = 1\,ton$ of aluminum be extracted? Which electrical energy W will be spent in doing this? ($\tau = 149\,h, W = 53.7\,GJ$).

298. What temperature do T mercury atoms have kinetic energy of translational motion sufficient for ionization? The potential for ionization of the mercury atom is $U = 10.4\,V$. ($T = 8 \cdot 10^4\ K$).

299. How many times will the specific thermionic emission of tungsten, which is at $T_1 = 2400\,K$ change, if the tungsten temperature is raised by $\Delta T = 100\ K$? ($j_2 / j_1 = 2.6$).

300. How many times the cathode of thoriated tungsten at a temperature of $T = 1800\ K$ gives a larger specific emission than a pure tungsten cathode at the same temperature? The emission constant for pure tungsten is $B_1 = 0.6 \cdot 10^6\ A/m^2 \cdot K^2$, for thoriated tungsten $B_2 = 0.3 \cdot 10^7\ A/m^2 \cdot K^2$. ($j_2 / j_1 = 1.1 \cdot 10^4$).

7.8 MAGNETISM

301. Find the strength H of the magnetic field at a point a distance of $a = 2\ m$ from an infinitely long conductor, through which the current $I = 5\ A$ flows. ($H = 39.8\ A/m$).

302. Find the intensity H of the magnetic field at the center of a circular wire turn with a radius $R = 1\,cm$ along which the current $I = 1\ A$ flows. ($H = 50\ A/m$).

303. Two rectilinear infinitely long conductors are perpendicular to each other and are in the same plane (Figure 7.21). Find the magnetic field strengths H_1 and H_2 at points M_1 and M_2 if the currents $I_1 = 2\ A$ and $I_2 = 3\ A$. The distances $AM_1 = AM_2 = 1\,cm$ and $BM_1 = CM_2 = 2\ cm$. ($H_1 = 8\ A/m;\ H_2 = 55.8\ A/m$).

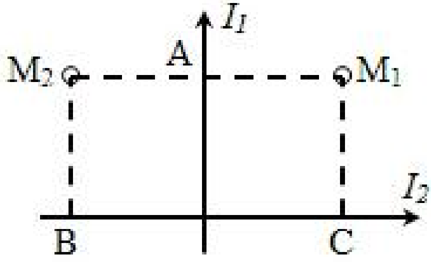


Figure 7.21

304. Two rectilinear infinitely long conductors are arranged in parallel at a distance $d = 10\ cm$ from each other. The currents $I_1 = I_2 = 5\ A$ flow in the opposite directions along the conductors. Find the modulus and

direction of the strength H of the magnetic field at a point a distance of $a = 10\ cm$ from each conductor. ($H = 8\ A/m$).

305. Find the intensity H of the magnetic field produced by a segment AB of a rectilinear conductor with a current at the point C located perpendicular to the middle of this segment at a distance $a = 5\ cm$ from it. The current flows through the conductor $I = 20\ A$. The segment AB of the conductor is visible from point C at an angle of 60^0. ($H = 31.8\ A/m$).

306. The current $I = 20\ A$ runs along a long conductor bent at right angles. Find H the intensity of the magnetic field at the point of this angle lying on the bisector and at a distance $a = 5\ cm$ from the vertex. ($H = 77.3\ A/m$).

307. Current $I = 20\ A$, flowing through a ring of copper wire section $S = 1\ mm^2$, creates in the center of the ring magnetic field strength $H = 178\ A/m$. What is the potential difference U applied to the ends of the wire forming the ring. ($U = 0.12\ V$).

308. Two circular turns with a radius $R = 4\ cm$ each are located in parallel planes at a distance $d = 10\ cm$ from each other. The currents $I_1 = I_2 = 2\ A$ flow through the turns. Find the magnetic field strength H on the axis of turns at a point equally spaced from them. The task is to decide when: a) the currents in the turns flow in one direction; b) currents flow in opposite directions in turns. ($a)\ H = 12.2\ A/m; b)\ H = 0\ A/m$).

309. Two circular turns of radius $R = 4\ cm$ each are located in parallel planes at a distance $d = 5\ cm$ from each other. The currents flow through the turns $I_1 = I_2 = 4\ A$. Find the strength H of the magnetic field at the center of one of the turns. The task is to decide when: a) the currents in the turns flow in one direction; b) currents flow in opposite directions in turns. ($a)\ H = 62.2\ A/m; b)\ H = 38.2\ A/m$).

310. Find the distribution of the magnetic field intensity H along the axis of a circular revolution with a diameter $D = 10\ cm$, along which the current $I = 10\ A$ flows. Write a table of values of H and plot the values for x in the interval $0 \le x \le 10\ cm$ every 2 cm.

311. Two circular turns are located in two mutually perpendicular planes so that the centers of these turns coincide. The radius of each turn is $R = 2\ cm$, the currents in turns $I_1 = I_2 = 5\ A$. Find the strength of the magnetic field at the center of these turns. ($H = 177\ A/m$).

312. A square frame is made of a wire of length $l = 1\ m$. The current flows through the frame $I = 10\ A$. Find the strength H of the magnetic field in the center of the frame. ($H = 35.8\ A/m$).

313. An infinitely long wire forms a circular turn tangent to the wire. The current flows through the wire $I = 5\ A$. Find the radius R of the turn if the magnetic field strength in the center of the turn is $H = 41\ A/m$. ($R = 8\ cm$).

314. A coil of length $l = 30\ cm$ has $N = 1000$ turns. Find the intensity H of the magnetic field inside the coil if the coil passes current $I = 2\ A$. The diameter of the coil is considered small in comparison with its length. ($H = 6.67\ kA/m$).

315. From a wire with a diameter $d = 1\ mm$, a solenoid must be wound, inside of which there must be a magnetic field strength $H = 24\ kA/m$. The limiting current $I = 6\ A$ can be passed through the wire. What number of layers will the coil of the solenoid consist, if the windings are wound tightly to each other? The diameter of the coil is considered small in comparison with its length. ($H = 1.25\ kA/m$).

316. It is required to obtain a magnetic field strength $H = 1\ kA/m$ in a solenoid of length $l = 20\ cm$ and diameter $D = 5\ cm$. Find the number of ampere turns IN required for this solenoid and the potential difference U that must be applied to the ends of the winding from Copper wire diameter $d = 0.5\ mm$. Read the field of the solenoid as homogeneous. ($IN = 200\ A \cdot v; U = 2.7\ V$).

317. Find the distribution of the magnetic field intensity H along the axis of the solenoid, whose length is $l = 3\ cm$ and diameter $D = 2\ cm$. The current $I = 2\ A$ flows through the solenoid. The coil has $N = 100$ turns. Compile a table of values of H and plot a graph for the values of x in the interval $0 \leq x \leq 3\ cm$ every 0.5 *cm*.

318. A capacitor with a capacitance $C = 10\ pF$ is periodically charged from the battery with e. 100 *V* and discharged through a coil in the form of a ring of diameter $D = 20\ cm$, where the plane of the ring coincides with the plane of the magnetic meridian. The coil has $N = 32$ turns. The horizontal magnetic needle placed in the center of the coil is deflected by an angle $\alpha = 45^0$. Switching the capacitor occurs at a frequency of $n = 100\ s^{-1}$. Find from the data of this experiment the horizontal component of H_g of the Earth's magnetic field strength. ($H = 16\ A/m$).

319. A $C = 10\ \mu F$ capacitor is periodically charged from a battery with an EMF of $E = 120\ V$ and discharged through a solenoid of length $l = 10\ cm$. The solenoid has $N = 200$ turns. The average value of the magnetic field strength inside the solenoid is $H = 240\ A/m$. What frequency n does the capacitor switch with? The diameter of the solenoid is considered small in comparison with its length. ($n = 100\ s^{-1}$).

320. In a homogeneous magnetic field of intensity $H = 79.6\ kA/m^2$ there is a square frame whose plane makes an angle $\alpha = 45^0$ with the direction of the magnetic field. The side of the frame is $a = 4\ cm$. Find the magnetic flux Φ penetrating the frame. ($\Phi = 113\ \mu Wb$).

321. In a magnetic field with an induction of $B = 0.05\ T$, a rod of length $l = 1\ m$ rotates. The axis of rotation passing through one of the ends of the rod is parallel to the direction of the magnetic field. Find the magnetic flux Φ intersected by the rod at each revolution. ($\Phi = 157\ \mu Wb$).

322. The frame, the area of which is $S = 16\ cm^2$, rotates in a uniform magnetic field with a frequency of $n = 2\ s^{-1}$. The axis of rotation is in the plane of the frame and is perpendicular to the direction of the magnetic field $H = 79.6\ kA/m$. Find the dependence of the magnetic flux Φ penetrating the frame on the time t and the largest value of $\Phi_{\max}$ of the magnetic flux. ($\Phi = 1.6 \cdot 10^{-4} \cos(4\pi t + \alpha); \Phi_{\max} = 160\ mWb$).

323. The iron sample is placed in a magnetic field of intensity $H = 796\ A/m$. Find the magnetic permeability μ of the iron. ($IN = 500\ A \cdot v$).

324. How many ampere-turns are required to make the volume density of the magnetic field energy inside the solenoid of small diameter and length $l = 30\ cm$ equal $W_0 = 1.75\ J/m^3$? ($IN = 855\ A \cdot v$).

325. The length of the iron core $l_1 = 50\ cm$, the length of the air gap $l_2 = 2\ mm$. Number of ampere-turns in the winding of the toroid $IN = 2000\ A \cdot n$. How many times will the tension decrease, the magnetic field in the air gap, if for the same number of ampere-turns increase the length of the air gap by half? (1.9 *times*).

326. An iron core is placed inside a solenoid of length $l = 25.1\ cm$ and diameter $D = 2\ cm$. The solenoid has $N = 200$ turns. The dependence of the magnetic flux Φ on the current I in the interval $0 \le I \le 5\ A$ in each 1 *A*. Draw the graf for the solenoid on the ordinate axis, plot F (10^{-4} *Wb*).

327. A long rectilinear wire runs through the center of the iron ring perpendicular to its plane, along which the current $I = 25\,A$. The ring has a quadrangular cross-section (Figure 7.22), which dimensions are $l_1 = 18\,mm$, $l_2 = 22\,mm$ and $h = 5\,mm$. Assuming approximately that at any point of the section of the ring the induction is the same and equal to the induction on the midline of the ring, find the magnetic flux Φ penetrating the area of the section of the ring. ($\Phi = 18\,\mu Wb$).

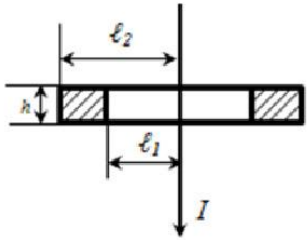


Figure 7.22

328. How many ampere turns are needed to create a magnetic flux of $\Phi = 0.42\,mWb$ in a solenoid with an iron core of length $l = 120\,cm$ and a cross-sectional area $S = 3\,cm^2$? ($\Phi = 18\,\mu Wb$).

329. Find the magnetic flux Φ penetrating the cross-sectional area of the ring of the previous problem, taking into account that the magnetic field at different points of the cross-section of the ring is different. The value of μ is assumed constant and found from the curve of the curve $B = f(H)$ for the value of H on the midline of the ring. ($\Phi = 18\,\mu Wb$).

330. A closed iron core of length $l = 50\,cm$ has a winding of $N = 1000$ turns. The current flows through the winding $I_1 = 1\,A$. What current I_2 should be run through the winding, so that when the core is removed, the induction remains the same? ($I_2 = 620\,A$).

331. It is required to create a magnetic field with induction $B = 1.4\,T$ between the poles of an electromagnet. The length of the iron core is $l_1 = 40\,cm$, the length of the interpolar space is $l_2 = 1\,cm$, the diameter of the core is $D = 5\,cm$. EMF with E should be taken to supply the winding of the electromagnet in order to obtain the required magnetic field using a copper wire with a cross-sectional area $S = 1\,mm^2$? What will be the smallest thickness b of the winding, if we assume that the maximum allowable current density is $I = 3\,MA/m^2$? ($b = 1.2\,cm$).

332. An electromagnet a homogeneous magnetic field with induction $B = 0.1\,T$ is created between the poles. For a wire of length $l = 70\,cm$, placed perpendicular to the direction of the magnetic field, the current $I = 70\,A$ flows. Find the force F acting on the wire. ($F = 4.9\,N$).

333. Two rectilinear long parallel conductors are spaced from each other. Conductors flow the same currents in one direction. Find the currents I_1 and I_2 flowing through each of the conductors, if it is known that in order to extend these conductors by twice the distance, it was necessary to perform work (per unit length of conductors) $A_i = 55\ \mu J/m$. ($I_1 = I_2 = 20\ A$).

334. The galvanometer coil consisting of $N = 600$ turns of wire is suspended on threads of length $l = 10\ cm$ and diameter $d = 0.1\ mm$ in a magnetic field of intensity $H = 160\ kA/m$ so that its plane is parallel to the direction of the magnetic field. The length of the coil frame is $a = 2.2\ cm$ and the width $b = 1.9\ cm$. What is the current I flowing through the winding of the coil if the coil turns by an angle $\varphi = 0.5^0$? The shear modulus of the yarn material is $G = 5.9\ GPa$. ($I = 0.1\ \mu A$).

335. In a homogeneous magnetic field with induction $B = 0.5\ T$ the conductor is moving uniformly with a length of $l = 10\ cm$. The current flows through the conductor $I = 2\ A$. The velocity of the conductor is $\upsilon = 20\ cm/s$ and is directed perpendicular to the direction of the magnetic field. Find the work A of moving the conductor in a time $t = 10\ s$ and the power P spent on this movement. ($A = 0.2\ J;\ P = 20\ mW$).

336. A homogeneous copper disk A of radius $R = 5\ cm$ is placed in a magnetic field with induction $B = 0.2\ T$ so that the plane of the disk is perpendicular to the direction of the magnetic field (Figure 7.23). The current $I = 5\ A$ runs along the radius of the disk ab (a and b are sliding contacts). The disk rotates at a frequency of $n = 3\ s^{-1}$. Find: a) the power P of such an engine; b) direction of rotation of the disk provided that the magnetic field is directed from the drawing to us; c) the torque M acting on the disk. ($a)\ P = 23.6\ mW;\ b)$ *the disc rotates counter-clockwise*; $c)\ M = 12.5 \cdot 10^{-4}\ N \cdot m$).

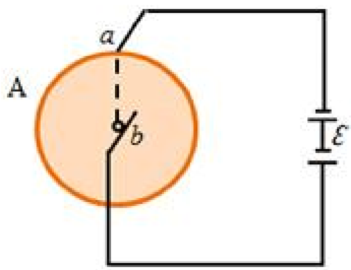


Figure 7.23

337. Find the magnetic flux Φ intersected by the radius ab of the disk A (Figure 7.23) during the time $t = 1\min$ of rotation. The disk radius is $H = 10\ cm$. The magnetic field induction is $B = 0.1\ T$. The disk rotates at a frequency of $n = 5.3\ s^{-1}$. ($\Phi = 1\ Wb$).

338. An electron accelerated by a potential $U = 1\,kV$, difference flies into a homogeneous magnetic field which direction is perpendicular to the direction of its motion. Induction of the magnetic field $B = 1.19\,mT$. Find R the radius of the circle along which the electron moves, T the period of revolution of the electron, and the angular momentum M of the electron. ($R = 9\,cm; T = 30\,ns, M = 1.5 \cdot 10^{-24}\ kg \cdot m^2/s$).

339. The electron flies into a uniform magnetic field, the direction of which is perpendicular to the direction of its motion. The velocity of the electron is $\upsilon = 4 \cdot 10^7\ m/s$. Induction of the magnetic field $B = 1\,mT$. Find the tangential a_τ and the normal a_n acceleration of an electron in a magnetic field. ($a_\tau = 0, a_\tau = const = 7 \cdot 10^{15}\ m/s^2$).

340. Find the kinetic energy of a proton moving along an arc of a circle of radius 60 *cm* in a magnetic field, the induction of which is 1 *T*. ($W = 17.3\,MeV$).

341. A proton and an electron accelerated by the same potential difference fly into a homogeneous magnetic field. How many times the radius of curvature R_1 of the trajectory of the proton is greater than the radius of curvature R_2 of the electron trajectory? ($R_1 / R_2 = m_1 / m_2 = 1840$).

342. A charged particle moves in a magnetic field along the circumference with a velocity $\upsilon = 10^6\ m/s$. Induction, magnetic field $B = 0.3\,T$. The radius of the circle is $R = 4\,cm$. Find the particle charge q if it is known that its energy is $W = 12\ keV$. ($q = 3.2 \cdot 10^{-19}\ K$).

343. An α-particle whose angular momentum $M = 1.33 \cdot 10^{-22}\ kg \cdot m^2/s$, flies into a homogeneous magnetic field perpendicular to the direction of its motion. Induction of the magnetic field $B = 25\ mT$. Find the kinetic energy of the $W\ \ \alpha -$ particle. ($W = 500\,eV$).

344. Single-charged ions of potassium isotopes with relative atomic masses 39 and 41 are accelerated by a potential difference $U = 300\,V$; then they fall into a homogeneous magnetic field perpendicular to the direction of their motion. Induction of the magnetic field $B = 0.08\,T$. Find the radii of curvature R_1 and R_2 of the trajectories of these ions. ($R_1 = 19.5\,cm; R_2 = 20\,cm$).

345. Find the ratio q/m for a charged particle if it moves along a circular arc with a radius $R = 8.3\,cm$, flying at a velocity $\upsilon = 10^6\ m/s$ into a homogeneous magnetic field of intensity $H = 200\ kA/m$. The direction of the velocity of the particle Perpendicular to the direction of the magnetic field. Compare the value found with q/m for an electron, a proton, and an

α particle. ($q/m = 4.8 \cdot 10^7\ C/kg$;for an electron $q/m = 1.76 \cdot 10^{11}\ C/kg$; for the proton $q/m = 9.6 \cdot 10^7\ C/kg$; for the α -particle $q/m = 4.8 \cdot 10^7\ C/kg$).

346. A magnetic field of intensity $H = 8\ kA/m$ and an electric field of intensity $E = 1\ kV/m$ are directed equally. The electron enters the electromagnetic field with a velocity $\upsilon = 10^5\ m/s$. Find the normal an, the tangential a_τ, and the total acceleration of the electron. The problem is solved if the electron velocity is directed: a) parallel to the direction of the electric field; b) perpendicular to the direction of the electric field. ($a)\ a_n = 0,\ a = a_\tau = 1.76 \cdot 10^{14}\ m/s^2;\ b)\ a_\tau = 0,\ a = a_n = 2.5 \cdot 10^{14}\ m/s^2$).

347. An electron accelerated by a potential difference $U = 6\ kV$, flies into a uniform magnetic field at an angle $\alpha = 30^0$ to the direction of the field and moves along a screw trajectory. Induction of the magnetic field $B = 13\ mT$. Find the radius R and the step h of the helical trajectory. ($R = 1\ cm,\ h = 11\ cm$).

348. The electron flies into a flat horizontal capacitor parallel to its plates at a speed of $\upsilon = 10^7\ m/s$. The length of the capacitor is $l = 5\ cm$. The electric field of the capacitor is $E = 10\ kV/m$. When leading condenser, the electron enters a magnetic field perpendicular to the electric field. Induction of the magnetic field $B = 10\ mT$. Find the radius R and the step h of the helical trajectory of an electron in a magnetic field. ($R = 5\ mm,\ h = 3.6\ cm$).

349. In a homogeneous magnetic field with induction $B = 0.1\ T$, a conductor with length $l = 10\ cm$ moves. The velocity of the conductor is $\upsilon = 15\ m/s$ and is directed perpendicular to the magnetic field. Find the EMF induced in the conductor E. ($\varepsilon = -0.15\ V$).

350. A coil with a diameter $D = 10\ cm$, consisting of $N = 500$ turns of wire, is in a magnetic field. Find the average EMF of the induction that appears in this coil if the induction of the magnetic field B increases for a time $\tau = 0.1\ s$ from 0 to 2 *T*. ($\varepsilon = 78.5\ V$).

351. The speed of the aircraft with a jet engine is $\upsilon = 950\ km/h$. Find the EMF of induction E_i appearing at the ends of the aircraft wings, if the vertical component of the Earth's magnetism intensity is $H_B = 39.8\ A/m$, the wingspan of the aircraft is $l = 12.5\ m$. ($\varepsilon = 165\ mV$).

352. In a uniform magnetic field, the induction of which $B = 0.8\ T$, the frame with an angular velocity $\omega = 15\ rad/s$ uniformly rotates. The area of

the frame is $S = 150\,cm^2$. The axis of rotation is in the plane of the frame and makes an angle $\alpha = 30^0$ with the direction of the magnetic field. Find the maximum EMF induction of $E_{\max}$ in a rotating frame. ($\varepsilon_{\max} = 0.09\,V$).

353. A homogeneous copper disk A of radius $R = 5\,cm$ is placed in a magnetic field with induction $B = 0.2\,T$ so that the plane of the disk is perpendicular to the direction of the magnetic field (Figure 7.24). A current can flow along the circuit aba (a and b are sliding contacts). The disk rotates at a frequency of $n = 3\,s^{-1}$. Find the EMF E of such a generator. Indicate the direction of the electric current if the magnetic field is directed from us to the drawing, and the disk rotates counter-clockwise. ($\varepsilon = 4.7\,mV$).

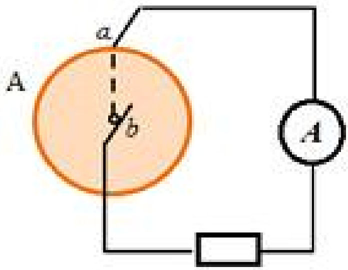


Figure 7.24

354. What is the average the EMF E sup is induced in the coil if the solenoid considered in the previous problem has an iron core? ($\varepsilon_m = 5.1\,V$).

355. In a homogeneous magnetic field, the induction of which is $B = 0.1\,T$, a coil consisting of $N = 200$ turns rotates. The axis of rotation of the coil is perpendicular to its axis and to the direction of the magnetic field. The period of revolution of the coil is $T = 0.2\,s$; cross-sectional area $S = 4\,cm^2$. The axis of rotation is perpendicular to the axis of the coil and to the direction of the magnetic field. Find the maximum the EMF induction in a rotating coil. ($\varepsilon_{\max} = 250\,mV$).

356. How many turns does the coil have, the inductance of which is $L = 1\,mH$, if at a current of $I = 1\,A$ the magnetic flux through the coil is $\Phi = 2\,\mu Wb$? ($N = 500$).

357. In a solenoid with length $l = 50\,cm$, a core is inserted from such an iron species, for which the dependence $B = f(H)$ is unknown. The number of turns per unit length of the solenoid $N_l = 400\,cm^{-1}$; cross-sectional area $S = 10\,cm^2$. Find the magnetic permeability μ of the core material at a current through the winding $I = 5\,A$, if it is known that the magnetic flux penetrating the cross section of the solenoid with the core, $\Phi = 1.6\,mWb$. What is the inductance of the L solenoid under these conditions? ($\mu = 640$; $L = 64\,mH$).

358. In a magnetic field, the induction of which is $B = 0.1\,T$, a square frame of copper wire is placed. The cross-sectional area ofthe wire is

$s = 1\ mm^2$, the area of the frame is $S = 25\ cm^2$. The normal to the plane of the frame is parallel to the magnetic field. How much electricity will q pass through the contour of the frame when the magnetic field disappears? ($q = 74\ mC$).

359. To measure the induction of the magnetic field between the poles of the electromagnet, a coil consisting of $N = 50$ turns of wire is placed and connected to a ballistic galvanometer. The axis of the coil is parallel to the direction of the magnetic field. The cross-sectional area of the coil is $S = 2\ cm^2$. Resistance of the galvanometer $R = 2\ k\Omega$; its ballistic constant is $C = 2 \cdot 10^{-8}\ cells/div$. When the coil is rapidly pulled out of the magnetic field, the galvanometer gives a rejection equal to 50 scale divisions. Find the induction B of the magnetic field. Resistance of the coil compared with the resistance of the ballistic galvanometer is neglected. ($B = 0.2\ T$).

360. Cross-sectional area of a solenoid with an iron core $S = 10\ cm^2$; Length of the solenoid $l = 1\ m$. Find the magnetic permeability μ of the core material if the magnetic flux penetrating the cross section of the solenoid is $\Phi = 1.4\ mWb$. Which current I flowing through the solenoid corresponds to this magnetic flux if it is known that the inductance of the solenoid under these conditions is $L = 0.44\ H$? ($\mu = 1400$; $I = 1.6\ A$).

361. To measure the magnetic permeability of iron, a toroid of length $l = 50\ cm$ and a cross-sectional area $S = 4\ cm^2$ was made from it. One of the windings of the toroid had $N_1 = 500$ turns and was connected to a current source, the other had $N_2 = 1000$ turns and was connected to a galvanometer. Switching the direction of the current in the primary winding to the opposite, we cause in the secondary winding the induction current. Find the magnetic permeability μ of the iron if it is known that when the current direction $I = 1\ A$ is switched in the primary winding, the amount of electricity passed through the galvanometer was $q = 0.06\ C$. Resistance of secondary winding $R = 20\ \Omega$. ($\mu = 1200$).

362. The coil has an inductance of $L = 0.144\ GH$ and a resistance R of $10\ \Omega$. After what time t after the inclusion in the coil current flows, equal to half the steady? ($t = 10\ ms$).

363. A square frame of copper wire of section $s = 1\ mm^2$ is placed in a magnetic field whose induction varies according to the law $B = B_0 \sin \omega t$, where $B_0 = 0.01 T$, $\omega = 2\pi / T$ and T = 0.02 s. The area of the frame is

$S = 25\,cm^2$. The plane of the frame is perpendicular to the direction of the magnetic field. Find the time dependence of t and the largest value of: a) the magnetic flux Φ penetrating the frame; b) the EMF induction E arising in the frame; c) current I flowing along the frame.

($a)\ \Phi = 2.5\cdot10^{-5}\sin100\pi t\ Wb,\ \Phi_{max} = 25\ \mu Wb;$
$b)\ \varepsilon = -7.85\cdot10^{-3}\cos100\pi t\ V,\ \varepsilon_{max} = 7.85\ mV;$
$c)\ I = -2.3\cos100\pi t\ A,\ I_{max} = 2.3\ A$).

364. A current flowing through the coil whose inductance $L = 21\,mH$, changing with time according to the law $I = I_0\sin\omega t$, where $I_0 = 5\,A$, $\omega = 2\pi/T$ and T = 0.02 *s*. Find the dependence on the time t: a) the EMF self-induction E arising in the coil; b) the energy W of the magnetic field of the coil. ($a)\ \varepsilon = -33\cos100\pi t\ V;\ b)\ W = 0.263\sin^2 100\pi t\ J$).

365. Two coils have a mutual inductance $L_{12} = 5\,mH$. In the first coil, the current varies according to the law $I = I_0\sin\omega t$, where $I_0 = 10\,A$, $\omega = 2\pi/T$ and T = 0.02 *s*. Find the dependence on the time *t* the EMF E_2, induced in the second coil, and the largest value of E_{2max} of this EMF. ($\varepsilon_2 = -15.7\cdot10^{-3}\cos100\pi t\ V,\ \varepsilon_{2max} = 15.7\ V$).

7.9 HARMONIC OSCILLATORY MOTION AND WAVES

366. Write the equation of harmonic oscillatory motion with amplitude $A = 50\ mm$, period T = 4 *s* and initial phase $\varphi = \pi/4$. Find the displacement x of the oscillating point from the equilibrium position at $t = 0$ and $t = 1.5\ s$. Draw a graph of this movement. ($x = 50\sin\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)mm,\ x_1 = 35.2\ mm,\ x_2 = 0$).

367. Write the equation of harmonic oscillatory motion with amplitude $A = 5\ cm$ and period T = 8 *s*, if the initial phase φ of oscillations is: a) 0; b) π / 2; c) π; d) 3π / 2; e) 2π. Draw a graph of this movement in all cases.

($a)\ x = 50\sin\frac{\pi}{4}t\ cm,\ b)\ x = 5\sin\left(\frac{\pi}{4}t + \frac{\pi}{2}\right)cm,$
$c)\ x = 5\sin\left(\frac{\pi}{4}t + \pi\right)cm,\ d)\ x = 5\sin\left(\frac{\pi}{4}t + \frac{3\pi}{2}\right)cm,$
$e)\ x = 5\sin\frac{\pi}{4}t\ cm$).

368. What time after the beginning of the movement, the point that performs harmonic oscillations will shift from the equilibrium position to half the amplitude? The period of oscillations is T = 24 s, the initial phase is $\varphi_0 = 0$. ($t = 2\ s$).

369. The initial phase of the harmonic oscillation is $\varphi = 0$. What fraction of the period will the velocity of the point be equal to half its maximum speed? ($t = \frac{T}{6}$).

370. What time after the beginning of the oscillation, the point that performs the oscillatory motion along the equation $x = 7\sin\frac{\pi}{2}t$, runs from the equilibrium position to the maximum displacement? ($t = 1\ s$).

371. Amplitude of harmonic oscillation $A = 5\ cm$, period T = 4 s. Find the maximum velocity $\upsilon_{\max}$ of the oscillating point and its maximum acceleration $a_{\max}$. ($\upsilon_{\max} = 7.85\ cm/s;\ a_{\max} = 12.3\ cm/s^2$).

372. The equation of motion of a point is given in the form $x = 2\sin\left(\frac{\pi}{2}t + \frac{\pi}{4}\right)cm$. Find the oscillation period T, the maximum velocity $\upsilon_{\max}$ and the maximum acceleration $a_{\max}$ point. ($T = 4\ s;\ \upsilon_{\max} = 3.14\ cm/s;\ a_{\max} = 4.93\ cm/c^2$).

373. The equation of motion of a point is given in the form $x = \sin\left(\frac{\pi}{6}t\right)$. Find the times t in which the maximum speed and the maximum acceleration are reached. ($t = 3, 9, 15\ s, ...$).

374. The point performs harmonic oscillations. The oscillation period is $T = 2s$, the amplitude is $A = 50cm$, the initial phase is $\varphi = 0$. Find the velocity υ at the time when the displacement of the point from the equilibrium state is $x = 25\ mm$. ($\upsilon = 13.6\ cm/s$).

375. The initial phase of the harmonic oscillation is $\varphi = 0$. When the point is shifted from the equilibrium position $x_1 = 2.4\ cm$, the velocity of the point is $\upsilon_1 = 3\ cm/s$, and at a displacement of $x_2 = 2.8\ cm$ its velocity is $\upsilon_2 = 2\ cm/s$. Find the amplitude A and the period T of this oscillation. ($A = 3.1\ cm;\ T = 4.1\ s$).

376. The equation of oscillation of a material point with mass $m = 16\ g$ has the form. $x = 0.1\sin\left(\frac{\pi}{8}t + \frac{\pi}{4}\right)m$. Draw a graph of the dependence on the

time t (within one period) of the force F acting on the point. Find the maximum force $F_{\max}$. ($F_{\max} = 246\ \mu N$).

377. The equation of oscillations of a material point of mass $m = 10\ g$ has the form $x = 5\sin\left(\frac{\pi}{5}t + \frac{\pi}{4}\right)cm$. Find the maximum force $F_{\max}$ acting on the point and the total energy W of the oscillating point. ($F_{\max} = 197\ \mu N; W = 4.93\ \mu J$).

378. The equation for the oscillation of a material point of mass $m = 16\ g$ has the form. $x = 2\sin\left(\frac{\pi}{4}t + \frac{\pi}{4}\right)cm$. Draw a graph of the dependence on time *t* (within one period) of the kinetic W_k, the potential W_p, and the total W energy of the point.

379. Find the ratio of the kinetic energy W_k of the point performing the harmonic oscillation to its potential energy W_p for the time moments: a) t = T/12; B) t = T/8; C) t = T/6. The initial phase of oscillations is $\varphi_0 = 0$. ($a) W_k / W_p = 3; b) W_k / W_p = 1; c) W_k / W_p = 1/3$).

380. Find the ratio of the kinetic energy W_k of the point performing the harmonic oscillation to its potential energy W_p for the moments when the displacement of the point from the equilibrium position is: a) $x = A/4$; b) $x = A/2$; c) $x = A$, where A is the amplitude of the oscillations. ($a) W_k / W_p = 15; b) W_k / W_p = 3; c) W_k / W_p = 0$).

381. The amplitude of the harmonic oscillations of the material point $x = 2\ cm$, the total energy $W = 0.3\ \mu J$. What displacement of X from the equilibrium position, does the force $F = 22.5\ \mu N$ act on the oscillating point? ($x = 1.5\ cm$).

382. A load was hung on the spring. The maximum kinetic energy of the load oscillations is $W_{k\max} = 1\ J$. The amplitude of the oscillations is $A = 5\ cm$. Find the rigidity k of the spring. ($k = 805\ N/m$).

383. How will the period of vertical oscillations of the load hanging on two identical springs change, if from the series connection of the springs go to their parallel connection? (*decrease by a factor of 2*).

384. A hydrometer weighing $m = 0.2\ kg$ is floating in a liquid. If you load it a little into the liquid and release it, it will start to oscillate with a period of *T* = 3.4 *s*. Assuming that the oscillations are undamped, find the

density of the liquid ρ in which the hydrometer floats. Diameter of a vertical cylindrical tube of a hydrometer $d = 1\,cm$. ($\rho = 0.89 \cdot 10^3\ kg/m^3$).

385. Find the amplitude A and the initial phase φ_0 of the harmonic oscillation obtained from the addition of equally directed oscillations given by the equations: $x = 0.2\sin\left(5t + \frac{\pi}{2}\right)$ and $x = 0.03\sin\left(5t + \frac{\pi}{4}\right)$. ($A = 4.6\ cm, \varphi_0 = 62^0 46`$).

386. As a result of the addition of two equally directed harmonic oscillations with identical amplitudes and identical periods, the resultant oscillation with the same period and the same amplitude is obtained. Find the phase difference $\varphi_2 - \varphi_1$ of the added oscillations. ($\varphi_2 - \varphi_1 = 2\pi/3$).

387. Find the amplitude A and the initial phase φ of the harmonic oscillations given by the equations: $x = 4\sin t\ cm$ and $x = 3\sin\left(t + \frac{\pi}{2}\right)$. Write the equations of the resulting oscillation. ($A = 5\ cm; \varphi = 36^0 52`, x = 5\sin(\pi t + \pi/5)\,cm$).

388. The equations of two harmonic oscillations have the form $x_1 = 3\sin 4\pi t\ cm$ and $x_2 = 6\sin 10\pi t\ cm$. Draw a graph of these oscillations. Adding these oscillations graphically, plot the resulting oscillation. Draw a spectrum of the resulting oscillation.

389. The point participates in two oscillations of the same period with the same initial phases. The amplitudes of the oscillations are $A_1 = 3\,cm$ and $A_2 = 4\,cm$. Find the amplitude A of the resulting oscillation if the oscillations are made; a) in one direction; b) in two mutually perpendicular directions. ($a)\ A = 7\ cm; b)\ A = 5\ cm$).

390. The point participates in two mutually perpendicular vibrations $x = \sin \pi t$ and. $y = 2\sin\left(\pi t + \frac{\pi}{2}\right)$. Find the trajectory of the resulting motion of the point and draw it with scale. ($x^2/1 + y^2/4 = 1$).

391. The period of damped oscillations T = 4 s, logarithmic decrement χ = 1.6, the initial phase is $\varphi = 0$. For $t = T/4$, the displacement of the point is $x = 4.5\ cm$. Write the equation of motion for this oscillation. Construct a graph of this fluctuation within two periods. ($x = 6.7e^{-0.4t}\sin\frac{\pi}{2}t\ cm$).

392. Logarithmic decrement of damping of the mathematical pendulum $x = 0.2$. How many times will the amplitude of oscillations decrease at the same time as the complete swing of the pendulum? ($A_1/A_2 = 1.22$).

393. Find the logarithmic damping decrement χ of a mathematical pendulum, if the amplitude of the oscillations decreases by a factor of 2 during a time $t = 1\,\min$. The length of the pendulum is $l = 1\,m$. ($\chi = 0.023$).

394. A mathematical pendulum performs damped oscillations with a logarithmic decrement of attenuation $\sigma = 0.2$. How many times does the total acceleration of the pendulum decrease in its extreme position by one oscillation? (at 1.22 *times*).

395. Amplitude of damped oscillations of a mathematical pendulum over time $t = 1\,\min$ decreased by half. How many times will the amplitude decrease over time $t = 3\,\min$? (at 8 *times*).

396. A mathematical pendulum of length $l = 0.5\,m$, derived from the equilibrium position, deviated by the first vibration at $x = 5\,cm$, and the second (in the same direction) by $x = 4\,cm$. Find the time during which the amplitude decreases e times, where e is the base of the natural logarithm $\chi = 6$? ($t = 6.4\,s$).

397. Find the wavelength λ of the oscillation, whose period $T = 10^{-14}\,s$. The propagation velocity of the oscillations is $c = 3 \cdot 10^8\ m/s$. ($\lambda = 3\,\mu m$).

398. Sound vibrations having a frequency $\nu = 500\,Hz$ and amplitude $A = 0.25\,mm$ propagate in the air. Wavelength $\lambda = 700\,nm$. Find the speed with the propagation of the oscillations and the maximum velocity $\upsilon_{\max}$ of the air particles. ($c = 350\,m/s, \upsilon_{\max} = 0.785\,m/s$).

399. The equation of undamped oscillations has the form $x = 10\sin\frac{\pi}{2}t$. Find the wave equation if the propagation velocity of the oscillations is $c = 300\,m/s$. Write and graphically represent the oscillation equation for a point spaced at a distance $l = 600\,m$ from the source of oscillations. Write and graphically represent the oscillation equation for wave points at time $t = 4\,s$ after the oscillations begin. ($x = 10\sin\left(\frac{\pi}{2}t - \frac{\pi l}{6 \cdot 10^4}\right)cm, t = 4\,s\ x = 10\sin\left(2\pi - \frac{\pi l}{6 \cdot 10^4}\right)cm$).

400. The equation for undamped oscillations is as follows $x = \sin 2.5\pi t$. Find the displacement x from the equilibrium position, the velocity υ and the acceleration a of the point located at a distance $l = 20\,m$ from the oscillation source, for the instant $t = 1\,s$ after the oscillations begin. The propagation velocity of the oscillations is $c = 100\,m/s$. ($x = 0, \upsilon = 7.85\,cm/s, a = 0$).

401. Find the phase difference of two points that are from the source at distances $l_1 = 10\,m$ and $l_2 = 16\,m$. The period of oscillations is $T = 0.4\,s$, the wave propagation velocity is $\upsilon = 300\,m/s$. ($\Delta\varphi = \pi$).

402. Find the offset from the equilibrium position of a point separated from the oscillation source at a distance $l = \lambda/12$, for the time instant $t = T/6$. The amplitude of the oscillations is $A = 0.05\,m$. ($x = 2.5\,cm$).

403. The offset from the equilibrium position of the point, which is from a source of oscillation at a distance of $l = 4\,cm$, at the time $t = T/6$ is equal to half the amplitude. Find the length of the traveling wave. ($\lambda = 0.48\,m$).

404. Find the wavelength λ of the oscillations if the distance between the first and fourth antinodes of the standing wave is $l = 15\,cm$. ($\lambda = 0.1\,m$).

7.10 ELECTROMAGNETIC OSCILLATIONS AND WAVES

405. The oscillatory circuit consists of a capacitor with a capacitance $C = 888\ pF$ and a coil with an inductance of $L = 2\ mH$. What wavelength λ is the contour tuned to? ($\lambda = 2500\,m$).

406. Which wavelength range can the oscillatory circuit be tuned on if its inductance is $L = 2\ mH$, and the capacitance can vary from $C = 69\ pF$ to $C = 533\ pF$? ($\lambda_1 = 700\,m\ to\ \lambda_2 = 1950\,m$).

407. What kind of inductance L should be included in the oscillatory circuit, so that at a capacitance $C = 2\ \mu F$ obtain a frequency $\nu = 1000\ Hz$? ($L = 12.7\ mH$).

408. A coil with an inductance of $L = 30\ \mu H$ is connected to a flat capacitor with a plate area of $S = 0.01\,m^2$ and a distance $d = 0.1\,mm$ between them. Find the permittivity ε of the medium, which fills the space between the plates, if the contour is tuned to a wavelength $\lambda = 750\ m$. ($\varepsilon = 6$).

409. The equation of variation with time of the potential difference on the capacitor plates in the oscillatory circuit has the form $U = 50\cos 10^4 \pi t\ V$. Capacitor capacitance $C = 0.1\,\mu F$. Find the period T of oscillations, the loop inductance L, the law of variation with time t of the current I in the circuit and the wavelength λ corresponding to this circuit. ($T = 0.2\ ms,\ L = 10.15\ mH,\ I = -157\sin 10^4 \pi t\ mA,\ \lambda = 60\ km$).

410. The equation of variation with time of current in the oscillatory circuit has the form $I = -0.02\sin 400\pi t\ A$. Inductance of the circuit $L = 1\,H$. Find the period T of oscillations, the capacitance C of the contour, the maximum energy of the W_m field and the maximum energy W_{el} of the electric field. ($T = 5\ ms,\ C = 0.63\ \mu F,\ U = 25.2\,V,\ W_m = 0.2\ mJ,\ W_{el} = 0.2\ mJ$).

411. Find the ratio of the energy W_m / W_{el} of the magnetic field of the vibrational circuit to the energy of its electric field for the time instant $T/8$. ($W_m / W_{el} = 1$).

412. The oscillatory circuit consists of a capacitor with a capacitance of $C = 7\ \mu F$ and a coil with an inductance of $L = 0.23\ H$ and an impedance of $R = 40\ \Omega$. The capacitor plates have a charge of $q = 0.56\ mC$. Find the period T of the oscillations of the circuit and the logarithmic damping decrement χ of the oscillations. Write the equation of variation with time t of the potential difference U on the capacitor plates. Find the potential difference at instants of time equal to: $T/2$, T, $3T/2$ and $2T$. Draw a graph $U = f(t)$ within two periods. ($T = 8\ ms,\ \chi = 0.7, U = 80e^{-87t} \cos 250\pi t\ V, U_1 = -56.5\ V,$ $U_2 = 40\ V, U_3 = -28\ V, U_4 = 20\ V$).

413. The oscillation circuit consists of a capacitor with a capacitance $C = 0.2\ \mu F$ and a coil with an inductance of $L = 5.07\ mH$. What logarithmic decrement χ the potential difference on the capacitor plates in a time $t = 1\ ms$ will decrease threefold? What is the resistance R of the circuit? ($\chi = 0.22, R = 11.1\ \Omega$).

414. The oscillation circuit consists of a capacitor with a capacitance $C = 405\ nF$, a coil with an inductance of $L = 10\ mH$ and a resistance $R = 2\ \Omega$. How many times will the potential difference on capacitor plates change over a single oscillation period? (*at* 1.04 *times*).

415. The oscillatory circuit consists of a capacitor with a capacitance $C = 2.22\ nF$ and a coil of length $l = 20\ cm$ from a copper wire of diameter $d = 0.5\ mm$. Find the logarithmic decrement of the damping of the oscillations. ($\chi = 0.018$).

416. The oscillating circuit has a capacitance $C = 1.1\ nF$ and an inductance of $L = 5\ mH$. The logarithmic damping decrement is $x = 0.005$. What time, due to attenuation, will 99% of the circuit energy be lost? ($t = 6.8\ ms$).

417. A coil of length $l = 50\ cm$ and a cross-sectional area $S = 10\ cm^2$ is included in the alternating current circuit with a frequency $\nu = 50\ Hz$. The number of turns of the coil is $N = 3000$. Find the resistance R of the coil if the phase shift between voltage and current is $\varphi = 60^0$. ($R = 4.1\ \Omega$).

418. The coil winding consists of $N = 500$ turns of copper wire, the cross-sectional area of which is $s = 1\ mm^2$. The length of the coil is $l = 50\ cm$, its

diameter is $D = 5\ cm$. What frequency ν of alternating current, the impedance Z of the coil is twice as large as its active resistance R? ($\nu = 300\ Hz$).

419. Two capacitors with capacitances $C_1 = 0.2\ \mu F$ and $C_2 = 0.1\ \mu F$ are connected in series to an alternating current circuit with a voltage $U = 220\ V$ and a frequency $\nu = 50\ Hz$. Find the current I in the circuit and the potential drop U_{C1} and U_{C2} on the first and second capacitors. ($I = 4.6\ mA, U_{C1} = 73.4\ V, U_{C2} = 146.6\ V$).

420. A coil of length $l = 25\ cm$ and radius $r = 2\ cm$ has a winding of $N = 1000$ turns of copper wire, the cross-sectional area of which is $s = 1\ mm^2$. The coil is included in the AC circuit of frequency $\nu = 50\ Hz$. What part of the impedance Z of the coil is the active resistance R and the inductive resistance X_L? ($74\%; 68\%$).

421. A coil with an active resistance $R = 10\ \Omega$ and an inductance L is included in the AC circuit with a voltage $U = 127\ V$ and a frequency $\nu = 50\ Hz$. Find the coil inductance L if it is known that the coil absorbs power $P = 400\ W$ and phase shift between voltage and current $\varphi = 60^0$. ($L = 55\ mH$).

422. Find the formulas for the impedance of the circuit Z and the phase shift φ between voltage and current for different methods of switching on the resistance R, capacitance C and inductance L. Consider the cases: a) R and C are connected in series; b) R and C are included in parallel; c) R and L are connected in series; d) R and L are included in parallel; e) R, L and C are connected in series.

($a)\ Z = \sqrt{R^2 + 1/(\omega C)^2},\ tg\varphi = 1/R\omega C;\ b)\ z = \dfrac{R}{\sqrt{R^2\omega^2C^2+1}},\ tg\varphi = -R\omega C;$

$c)\ Z = \sqrt{R^2 + (\omega L)^2},\ tg\varphi = \omega L/R;$

$d)\ z = \dfrac{R\omega L}{\sqrt{R^2+(\omega L)^2}},\ tg\varphi = R/\omega L;\ e)\ Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2},\ tg\varphi = \dfrac{\omega L - 1/\omega C}{R}$).

423. A capacitor with a capacitance of $C = 1\ \mu F$ and a resistor with an impedance $R = 3\ k\Omega$ are included in the alternating current circuit with a frequency $\nu = 50\ Hz$. Find the impedance Z of the circuit, if the capacitor and the resistor are switched on: a) in series; b) in parallel. ($a)\ Z = 4.38\ k\Omega,\ b)\ Z = 2.18\ k\Omega$).

424. In a circuit of an alternating current with voltage $U = 220\ V$ and frequency $\nu = 50\ Hz$ capacitance $C = 35.4\ \mu F$, resistance $R = 100\ \Omega$ and

inductance $L = 0.7\ H$ are sequentially connected. Find the current I in the circuit and the voltage drops U_C, U_R and U_L on the capacitance, resistance and inductance. ($I = 1.34\ A, U_C = 121 V, U_R = 134 V, U_L = 295 V$).

7.11 WAVE OPTICS

425. What is the potential difference U applied between the electrodes of the helium discharge tube, if the maximum Doppler shift of the helium line ($\lambda = 492.2\ nm$) is observed to be equal to $\Delta\lambda = 0.8\ nm$ when observed along the α-particle beam? ($U = 2.5\ kV$).

426. When photographing the spectrum of the Andromeda star, it was found that the titanium line ($\lambda = 495.4\ nm$) was shifted to the violet end of the spectrum by $\Delta\lambda = 0.17\ nm$. How does the star move with respect to the Earth? ($\upsilon = 103\ km/s$).

427. How many times will the distance between adjacent interference fringes increase on the screen in Young's experiment if the green filter ($\lambda = 500\ nm$) is replaced by red, ($\lambda = 650\ nm$)? (*at* 1.3 *times*).

428. In Young's experiment, the apertures were illuminated with monochromatic light ($\lambda = 600\ nm$). The distance between the holes is $d = 1\ mm$, the distance from the holes to the screen is $L = 3\ m$. Find the position of the first three light strips. ($y_1 = 1.8\ mm; y_2 = 3.6\ mm; y_3 = 5.4\ mm$).

429. In Jung's experiment, a glass plate of thickness $h = 12\ cm$ is placed in the path of one of the interfering beams perpendicular to the ray. How much can the refractive indices differ in different places of the plate, so that the change in the path difference from this inhomogeneity does not exceed $\Delta = 1\ \mu m$? ($\Delta n \le= 5 \cdot 10^{-5}$).

430. A soap film, arranged vertically, forms a wedge due to the flow of liquid. When observing interference fringes in the reflected light of a mercury arc ($\lambda = 546.1\ nm$), it turned out that the distance between the five bands is $l = 2\ cm$. Find the angle γ of the wedge. Light falls perpendicular to the surface of the film. The refractive index of soapy water is $n = 1.33$. ($\gamma = 11^0$).

431. The installation for obtaining Newton's rings is illuminated by monochromatic light incident on the normal to the surface of the plate. Observation is conducted in reflected light. The radii of two adjacent dark rings are equal to $r_k = 4\ mm$ and $r_{k+1} = 4.38\ mm$. The radius of curvature of the lens is $R = 6.4\ m$. Find the ordinal numbers of the rings and the wavelength λ of the incident light. ($k = 5; k + 1 = 6; \lambda = 0.5\ \mu m$).

432. The installation for obtaining Newton's rings is illuminated by a monochromatic light incident on the normal to the surface of the plate. The radius of curvature of the lens is $R = 8.6\ m$. The observation is performed in reflected light. By measurements it is established that the radius of the fourth dark ring (assuming the central dark spot as zero) $r_4 = 4.5\ mm$. Find the wavelength λ of the incident light. ($\lambda = 589\ nm$).

433. The installation for obtaining the rings of Newton is illuminated by white light falling along the normal to the surface of the plate. The radius of curvature of the lens is $R = 5\ m$. Observation is conducted in transmitted light. Find the radii r_b and r_r of the fourth blue ring ($\lambda_b = 400\ nm$) and the third red ring ($\lambda_r = 630\ nm$). ($r_b = 2.8\ mm; r_r = 3.1\ \mu m$).

434. The installation for obtaining the rings of Newton is illuminated by a monochromatic light incident on the normal to the surface of the plate. Observation is conducted in reflected light. The distance between the second and the twentieth dark rings is $l_1 = 4.8\ mm$. Find the distance l_2 between the third and sixteenth dark rings of Newton. ($l_2 = 3.66\ mm$).

435. The installation for obtaining Newton's rings is illuminated by light with a wavelength $\lambda = 589\ nm$ incident along the normal to the surface of the plate. The radius of curvature of the lens is $R = 10\ m$. The space between the lens and the glass plate is filled with liquid. Find the refractive index n of the liquid if the radius of the third light ring in the transmitted light is $r_3 = 3.65\ mm$. ($n = 1.33$).

436. The installation for obtaining Newton's rings is illuminated by monochromatic light with a wavelength $\lambda = 600\ nm$ incident along the normal to the surface of the plate. Find the thickness h of the air layer between the lens and the glass plate at the point where the fourth dark ring is observed in reflected light. ($h = 1.2\ \mu m$).

437. The installation for obtaining Newton's rings is illuminated by monochromatic light with a wavelength $\lambda = 500\ nm$ incident along the normal to the surface of the plate. The space between the lens and the glass plate is filled with water. Find the thickness h of the water layer between the lens and the plate at the point where the third light ring is observed in reflected light. ($h = 470\ nm$).

438. In the Michelson interferometer experiment, to shift the interference pattern by $k = 500$ bands, it was necessary to move the mirror to a distance $L = 0.161\ mm$. Find the wavelength λ of the incident light. ($\lambda = 644\ nm$).

439. To measure the refractive index of ammonia, a pumped tube of length $l = 14\ cm$ was placed in one of the arms of the Michelson interferometer. The ends of the tube were covered with plane-parallel glasses. When the tube was filled with ammonia, the interference pattern shifted to $k = 180$ bands for a wavelength $\lambda = 590\ nm$. Find the refractive index n of ammonia. ($n = 1.00038$).

440. A white light beam falls normal to the surface of a glass plate with a thickness $d = 0.4\ \mu m$. The refractive index of glass is $n = 1.5$. Which wavelengths λ lying within the visible spectrum (from 400 to 700 *nm*) are amplified in reflected light? ($\lambda = 480\ nm$).

441. Light from a monochromatic source ($\lambda = 600\ nm$) falls normally on the diaphragm with a hole diameter $d = 6\ mm$. Behind the diaphragm at a distance $l = 3\ m$ from it there is a screen. What number of k Fresnel zones fits in the aperture hole? What will be the center of the diffraction pattern on the screen: dark or light? ($k = 5$).

442. Find the radii r_k of the first five Fresnel zones, if the distance from the light source to the wave surface is $a = 1\ m$, the distance from the wave surface to the observation point is $b = 1\ m$. The wavelength of light is $\lambda = 500\ nm$.
($r_1 = 0.50\ mm,\ r_2 = 0.71\ mm,\ r_3 = 0.86\ mm,\ r_4 = 1\ mm,\ r_5 = 1.12\ mm$).

443. Find the radii r_k of the first five Fresnel zones for a plane wave, if the distance from the wave surface to the observation point is $b = 1\ m$. The wavelength of light is $\lambda = 500\ nm$.
($r_1 = 0.71\ mm,\ r_2 = 1\ mm,\ r_3 = 1.22\ mm,\ r_4 = 1.41\ mm,\ r_5 = 1.58\ mm$).

444. A diffraction pattern is observed at a distance from a point source of monochromatic light ($\lambda = 600\ nm$). At a distance $a = 0.5l$, a circular opaque barrier with a diameter $D = 1\ cm$ is placed from the source. Find the distance l if the barrier covers only the central Fresnel zone. ($l = 167\ m$).

445. A normally parallel beam of monochromatic light ($\lambda = 589\ nm$) falls on a gap of width $a = 2\ \mu m$. What angles of φ will diffraction minimum of light be observed at?
($r_1 = 0.50\ mm,\ r_2 = 0.71\ mm,\ r_3 = 0.86\ mm,\ r_4 = 1\ mm,\ r_5 = 1.12\ mm$).

446. A normally parallel beam of monochromatic light ($\lambda = 500\ nm$) falls on a slot of width $a = 20\ \mu m$. Find the width *A* of the image of the gap on the screen, remote from the gap by a distance $l = 1\ m$. The width of the

image is the distance between the first diffraction minima located on both sides of the main maximum of the illumination. ($A = 5\,cm$).

447. A normally parallel beam of monochromatic light with a wavelength λ falls on a gap of width $a = 6\lambda$. What angle φ will the third diffraction minimum of light be observed at? ($\varphi = 30^0$).

448. How many strokes of N_0 per unit length does the diffraction grating have if the green mercury line ($\lambda = 546.1\,nm$) in the first-order spectrum is observed at an angle $\varphi = 19^0 8`$? ($N_0 = 600\,mm^{-1}$).

449. The beam of light normally falls on the diffraction grating. The sodium line ($\lambda = 589\,nm$) gives the diffraction angle $\varphi_1 = 17^0 8`$in the first-order spectrum. Some lines give the diffraction angle $\varphi_2 = 24^0 12`$ in the second-order spectrum. Find the wavelength λ_2 of this line and the number of strokes N_0 per unit length of the lattice. ($\lambda_2 = 409.9\,nm$, $N_0 = 500\,mm^{-1}$).

450. A beam of monochromatic light normally falls on the diffraction grating. The third-order maximum is observed at an angle $\varphi = 36^0 48`$ to the normal. Find the lattice constant d, expressed in the wavelengths of the incident light. ($d = 5\lambda$).

451. What should be the constant d of the diffraction grating in order that in the first order the lines of the potassium spectrum be allowed $\lambda_1 = 404.4\,nm$ and $\lambda_2 = 404.7\,nm$? The width of the lattice is $a = 3\,cm$. ($d = 22\,\mu m$).

452. The constant of the diffraction grating $d = 2\,\mu m$. What difference in wavelengths $\Delta\lambda$ can this lattice solve in the yellow-ray region ($\lambda = 600\,nm$) in the second-order spectrum? The width of the lattice is $a = 2.5\,cm$. ($\Delta\lambda = 24\,pm$).

453. The constant of the diffraction grating $d = 2.5\,\mu m$. Find the angular dispersion $d\varphi / d\lambda$ of the lattice for $\lambda = 589\,nm$ in the first-order spectrum. ($d\varphi / d\lambda = 4.1 \cdot 10^5\,rad/m$).

454. Angular dispersion of the diffraction grating for $\lambda = 668\,nm$ in the first-order spectrum $d\varphi / d\lambda = 2.02 \cdot 10^5\,rad/m$. Find the period d of the diffraction grating. ($d = 5\,\mu m$).

455. Find the linear dispersion D of the diffraction grating under the conditions of the previous problem, if the focal length of the lens projecting the spectrum onto the screen is $F = 40\,cm$. ($D = 81\,\mu m / nm$).

456. For which wavelength λ, does the diffraction grating have an angular dispersion $d\varphi / d\lambda = 6.3 \cdot 10^5 \; rad/m$ in the third-order spectrum? The lattice constant is $d = 5 \; \mu m$. ($\lambda = 510 \; nm$).

457. Which focal length F should a lens have projecting on the screen the spectrum obtained by means of a diffraction grating that the distance between two lines of potassium $\lambda_1 = 404.4 \; nm$ and $\lambda_2 = 404.7 \; nm$ in the first-order spectrum be equal to $l = 0.1 \, Mm$? The lattice constant is $d = 2 \; \mu m$. ($F = 0.65 \, m$).

458. Find the full polarization angle when the light is reflected from the glass, the refractive index of which is $n = 1.57$. ($i_B = 57^0 30`$).

459. The limiting angle of total internal reflection for a certain substance is $i = 45^0$. Find for this substance an angle i_B of complete polarization. ($i_B = 54^0 44`$).

460. Find the refractive index n of the glass if, when reflected from the light, the reflected beam is completely polarized at a refraction angle $\beta = 30^0$. ($n = 1.73$).

461. A beam of light passes through a liquid poured into a glass ($n = 1.5$) vessel, and is reflected from the bottom. The reflected beam is completely polarized when it falls, to the bottom of the vessel at an angle $i_B = 42^0 37`$. Find the refractive index n of the liquid. At what angle i should the light beam coming into the bottom of the vessel go in this liquid, so that complete internal reflection comes? ($n = 1.63, i = 66^0 56`$).

462. Find the angle φ between the main planes of the polarizer and the analyzer if the intensity of natural light passing through the polarizer and the analyzer decreases by 4 times. ($\varphi = 45^0$).

463. Find the reflection coefficient ρ of the natural light incident on the glass ($n = 1.54$) at the full-polarization angle i_B. Find the degree of polarization P of the rays passed into the glass. ($\rho = 0.083; P = 9.1\%$).

464. Find the reflection coefficient ρ and the degree of polarization P_1 of the reflected rays when the natural light falls on the glass $n = 1.5$) at an angle $i = 45^0$. What is the degree of polarization of the P_2 refracted beams? ($\rho = 5.06\%; P_1 = 83\%; P_2 = 4.42\%$).

7.12 QUANTUM NATURE OF LIGHT AND WAVE PROPERTIES OF PARTICLES

465. Find the mass m of the photon: a) red light rays ($\lambda = 700 \; nm$); b) *X*-rays ($\lambda = 25 \; nm$); c) gamma rays ($\lambda = 1.24 \; pm$). ($a) \; m = 3.2 \cdot 10^{-36} \; kg, b) \; m = 8.8 \cdot 10^{-32} \, kg, c) \; m = 1.8 \cdot 10^{-30} \; kg$).

466. Find the energy ε, the mass m and the photon momentum p if the corresponding wave length $\lambda = 1.6\ pm$? ($m = 1.38 \cdot 10^{-30}\ kg, \varepsilon = 1.15 \cdot 10^{-13} J,\ p = 4.1 \cdot 10^{-22}\ kg \cdot m/s$).

467. What speed υ should an electron move, so that its kinetic energy is equal to the energy of a photon with a wavelength of $\lambda = 521\ nm$? ($\upsilon = 9.2 \cdot 10^5\ m/s$).

468. What energy should a photon have to make its mass equal to the rest mass of an electron? ($\varepsilon = 0.51\ MeV$).

469. The impulse carried by a monochromatic photon beam through the area $S = 2\ cm^2$ for a time $t = 0.5\ \min$ is equal to $p = 3 \cdot 10^{-9}\ \dfrac{kg \cdot m}{s}$. Find for this beam energy E, falling per unit area per unit time. ($E = 150\ J/s \cdot m^2$).

470. What temperature T the kinetic energy of a diatomic gas molecule will be equal to the energy of a photon with a wave length $\lambda = 589\ nm$? ($T = 9800\ K$).

471. Find the mass m of a photon whose momentum is equal to the momentum of the hydrogen molecule at a temperature of $t = 21^0 C$. The velocity of the molecule is assumed equal to the mean square velocity. ($m = 2.1 \cdot 10^{-32}\ kg$).

472. The wavelength of light corresponding to the red boundary of the photoelectric effect, for a certain metal $\lambda_0 = 275\ nm$. Find the minimum energy ε of the photon causing the photoelectric effect. ($\varepsilon = 4.5\ eV$)

473. The wavelength of light corresponding to the red boundary of the photoelectric effect, for a certain metal $\lambda_0 = 275\ nm$. Find the work function of an electron from a metal, the maximum velocity of electrons emitted from a metal by light with a wavelength $\lambda = 180\ nm$, and the maximum kinetic energy of $W_{\max}$ electrons. ($A = 4.5\ eV, \upsilon_{\max} = 9.1\ m/s, W_{\max} = 3.8 \cdot 10^{-19}\ J$).

474. Find the frequency ν of the light that tears out electrons from the metal, which are completely delayed by the potential difference $U = 3\ V$. The photoelectric effect begins at a light frequency $\nu_0 = 6 \cdot 10^{14}\ Hz$. Find the work function of the electron from metal. ($A = 2.48\ eV, \nu = 13.2 \cdot 10^{14}\ H$).

475. Photons with an energy of $e = 4.9\ eV$ excite electrons from the metal with an output function $A = 4.5\ eV$. Find the maximum momentum

p_{max} transmitted to the surface of the metal upon the ejection of each electron. ($p_{\max} = 3.45 \cdot 10^{-25}\ kg \cdot m/s$).

476. Find the Planck constant h if it is known that the electrons extracted from the metal with light with a frequency $\nu_1 = 2.2 \cdot 10^{15}\ s^{-1}$ are completely delayed by the potential difference $U_1 = 6.6\,V$, and those expelled by light with the frequency $\nu_2 = 4.6 \cdot 10^{15}\ s^{-1}$ - the potential difference U_2 = 16.5 V? ($h = 6.6 \cdot 10^{-34}\ J \cdot s$).

477. Find the light pressure P on the walls of an electric 100-Watt lamp. The bulb of the lamp is a spherical vessel of radius $r = 5\,cm$. The walls of the lamp reflect 4% and pass 6% of the incident light. Consider that all the power consumed goes to the radiation. ($P = 10.4\ \mu Pa$).

478. A monochromatic light beam ($\lambda = 490\,nm$), falling along the normal to the surface, produces a pressure $p = 4.9\ \mu Pa$. What is the number of photons I per unit time per unit area of this surface? The coefficient of light reflection $\rho = 0.25$. ($I = 2.9 \cdot 10^{21}\ c^{-1} \cdot m^{-2}$).

479. What was the wave length λ_0 of X-ray radiation, if the scattered radiation wavelength was equal to $\lambda = 25.4\ pm$ when Compton scattering of this radiation with graphite at an angle $\varphi = 60^0$? ($\lambda_0 = 24.2\ pm$).

480. X-rays with a wavelength $\lambda_0 = 21\,pm$ experience Compton scattering at an angle $\varphi = 90^0$. Find the variation $\Delta\lambda$ of the X-ray wavelength for scattering, as well as the energy. W_e and the momentum of the recoil electron. ($\Delta\lambda = 2.42\ pm, W_e = 6.6\,keV,\ p_e = 4.4 \cdot 10^{-23}\ kg \cdot m/s$).

481. Under Compton scattering, the energy of the incident photon is distributed equally between the scattered photon and the recoil electron. The scattering angle is $\varphi = \pi/2$. Find the energy W and the momentum p of the scattered photon. ($W = 0.26\,MeV,\ p_e = 9.3 \cdot 10^{-12}\ kg \cdot m/s$).

482. The energy of X-rays is $\varepsilon = 0.6\,MeV$. Find the recoil energy we if the X-ray wavelength after Compton scattering has changed by 21%. ($W_e = 0.1\,MeV$).

483. Find the de Broglie wavelength λ for electrons that have passed the potential difference $U_1 = 1V$ and $U_2 = 100\,V$. ($\lambda_1 = 1.23\,nm, \lambda_2 = 0.123\,nm$).

484. Find the de Broglie wavelength λ for: a) an electron moving with a velocity $\upsilon = 10^6\ m/s$; b) hydrogen atom moving with an average square

velocity at temperature $T=300\ K$; c) a ball of mass $m=1\ g$ moving with velocity $\upsilon=1\ cm/s$. ($a)\ \lambda=730\ pm, b)\ \lambda=144\ pm, c)\ \lambda=6.6\cdot10^{-29}\ m$).

485. Find the de Broglie wavelength λ for an electron with kinetic energy: a) $W_1=10\ keV$; b) $W_2=1\ MeV$. ($a)\ \lambda=12.2\ pm, b)\ \lambda=0.87\ pm$).

486. A charged particle accelerated by a potential difference $U=210\ V$ has a de Broglie wavelength $\lambda=2.02\ pm$. Find the mass m of the particle if its charge is numerically equal to the charge of the electron. ($m=1.67\cdot10^{-27}\ kg$).

7.13 ATOM OF BORA. X-RAYS

487. Find the radii r_k of the three first Bohr electron orbits in the hydrogen atom and the velocity υ_c of the electron on them. ($r_1=53\ pm, r_2=212\ pm, r_3=477\ pm; \upsilon_1=2.19\cdot10^6\ m/s,$ $\upsilon_2=1.1\cdot10^6\ m/s, \upsilon_3=7.3\cdot10^5\ m/s$).

488. Find the kinetic W_k, the potential W_p, and the total energy of the electron in the first Bohr orbit. ($W_k=13.6\ eV, W_p=-27.2\ eV, W=-13.6\ eV$).

489. Find the kinetic energy W_k of the electron located on the *n*-th orbit of the hydrogen atom, for *n* = 1, 2, 3, and ∞. ($W_{k1}=13.6\ eV, W_{k2}=3.40\ eV, W_{k3}=1.51\ eV, W_{k4}=0$).

490. Find the period T of revolution of an electron on the first Bohr orbit of a hydrogen atom and its angular velocity ω. ($T=1.43\cdot10^{-16}\ s, \omega=4.4\cdot10^{16}\ rad/s$).

491. Find the smallest $\lambda_{\min}$ and the largest $\lambda_{\max}$ of the wavelengths of the spectral lines of hydrogen in the visible region of the spectrum. ($\lambda_{\min}=365\ nm, \lambda_{\max}=656\ nm$).

492. Find the largest wavelength $\lambda_{\max}$ in the ultra-violet region of the hydrogen spectrum. What is the minimum speed $\upsilon_{\min}$ should electrons have, so that when the hydrogen atoms are excited by electron beams, this line appears? ($\upsilon_{\min}=1.90\cdot10^6\ m/s, \lambda_{\max}=121\ nm$).

493. Find the ionization potential U_i of the hydrogen atom. ($U_i=13.6\ V$).

494. Find the first excitation potential U_1 of the hydrogen atom. ($U_1=10.2\ V$).

495. Which minimum energy $W_{\min}$ (in electron-volts) should have electrons so that when the hydrogen atoms are excited by the impacts of these electrons, all the lines of all series of the hydrogen spectrum appear? What is the minimum speed $\upsilon_{\min}$ should these electrons have? ($W_{\min} = 13.6\, eV, \upsilon_{\min} = 2.2 \cdot 10^6\ m/s$).

496. Which minimum energy $W_{\min}$ (in electron-volts) should have electrons so that when hydrogen atoms are excited by the impacts of these electrons, the spectrum of hydrogen has three spectral lines? Find the wavelengths λ of these lines. ($W_{\min} = 12.1\, eV, \lambda_1 = 121\, nm, \lambda_2 = 103\, nm, \lambda_3 = 656\, nm$).

497. What limits should the wavelength λ of monochromatic light lie so that when the hydrogen atoms are excited by the quanta of this light, three spectral lines are observed? ($97.3 \le \lambda \le 102.6\, nm$).

498. How much did the kinetic energy of an electron in a hydrogen atom change when a photon is atomized at a wavelength of $\lambda = 486\, nm$? ($\Delta W = 2.56\, eV$).

499. What extent should the wavelengths λ of monochromatic light lie, so that when the hydrogen atoms are excited with quanta of this light, the radius of the orbit r_k of an electron increases by a factor of 9? ($97.3 \le \lambda \le 102.6\, nm$).

500. A light beam from a discharge tube filled with atomic hydrogen normally falls on the diffraction grating. The lattice constant is $d = 5\ \mu m$. Which transition of the electron corresponds the spectral line observed with the aid of this lattice in the spectrum of the fifth order at an angle $\varphi = 41^0$? ($n = 3, k = 2$).

501. Find the wavelength of de Broglie λ for an electron moving along the first Bohr orbit of a hydrogen atom. ($\lambda = 0.33\, nm$).

502. Find the radius r_1 of the first Bohr electron orbit for singly ionized helium and the velocity υ_1 of the electron on it. ($r_1 = 26.6\ pm, \upsilon_1 = 4.37 \cdot 10^6\ m/s$).

503. Find the first excitation potential U_1: a) once ionized helium; b) doubly ionized lithium. ($a) U_1 = 40.8\, V, b) U_1 = 91.8\, V$).

504. Find the wavelength λ of a photon corresponding to the transition of an electron from the second Bohr orbit to the first in a singly ionized helium atom. ($\lambda = 30.4\, nm$).

505. Find the lattice constant d of the rock salt, knowing the molar mass $\mu = 0.058\ kg/mol$ of rock salt and its density $\rho = 2.2 \cdot 10^3\ kg/m^3$. Crystals of rock salt have a simple cubic structure. ($d = 281\ pm$).

506. The potential difference $U = 60\ kV$ is applied to the electrodes of the X-ray tube. The smallest wavelength of X-rays obtained from this tube, $\lambda = 21.6\ pm$. Find the Planck constant h from these data. ($h = 6.6 \cdot 10^{-34}\ J \cdot s$).

507. Find the wavelength that determines the short-wavelength boundary of the continuous X-ray spectrum for cases when the potential difference U is applied to the X-ray tube equal to: 30, 40, 50 kV. ($\lambda_1 = 41.3\ pm, \lambda_2 = 31\ pm, \lambda_3 = 24.8\ pm$).

508. The wavelength of the gamma radiation of radium $\lambda = 1.6\ pm$. What kind of potential difference U should be applied to the X-ray tube in order to obtain X-rays with this wavelength? ($U = 770\ kV$).

509. Find for aluminum the thickness $x_{1/2}$ of the half attenuation layer for X-rays of a certain wavelength. The mass absorption coefficient of aluminum for this wavelength is $\mu_m = 5.3\ m^2/kg$. ($x_{1/2} = 0.5\ mm$).

510. How many times will the intensity of X-rays with wavelength $\lambda = 21\ pm$ decrease when an iron layer with thickness $d = 0.15\ mm$ passes? The mass coefficient of iron absorption for this wavelength is $\mu_m = 1.1\ m^2/kg$. (*at* 3.7 *times*).

511. In the following table, for some materials, the layer thickness $x_{1/2}$ of the half attenuation of X-rays, whose energy is $W = 1\ MeV$, is given for some materials. Find the linear μ and mass μ_m absorption coefficients of these materials for a given energy of X-rays. For what wavelength λ of X-rays is this data obtained? (

$$a)\ \mu_1 = 6.7\ m^{-1},\ \mu_{m1} = 6.7 \cdot 10^{-3}\ m^2/kg;$$
$$b)\ \mu_2 = 16\ m^{-1},\ \mu_{m2} = 6.2 \cdot 10^{-3}\ m^2/kg;$$
$$c)\ \mu_3 = 44\ m^{-1},\ \mu_{m3} = 5.6 \cdot 10^{-3}\ m^2/kg;$$
$$d)\ \mu_4 = 77\ m^{-1},\ \mu_{m4} = 6.8 \cdot 10^{-3}\ m^2/kg$$

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512. How many half-attenuation layers are needed to reduce X-ray intensity by 80 times? ($n = 6.35$).

7.14 RADIOACTIVITY

513. How many polonium atoms decay in a time $\Delta t = 1$ day from $N_0 = 10^6$ atoms? ($\Delta N = 5025\ day^{-1}$).

514. Find the activity a of the mass $m = 1\,g$ of the gradium. ($a = 3.7 \cdot 10^{10}\ Bq$).

515. Find the mass m of radon, the activity of which is $a = 3.7 \cdot 10^{10}\ Bq$. ($m = 6.5 \cdot 10^{-9}\ kg$).

516. Find the mass m of polonium ${}^{210}_{84}Po$, whose activity is $a = 3.7 \cdot 10^{10}\ Bq$. ($m = 0.22\,mg$).

517. Find the decay constant λ of radon, if it is known that the number of radon atoms decreases by 18.2% during the time $t = 1$ day. ($m = 2.1 \cdot 10^{-6}\ s^{-1}$).

518. Find the specific activity of a_m uranium U_{92}^{235}. ($a_m = 7.9 \cdot 10^{7}\ Bq/kg$).

519. The Geiger-Müller ionization counters have a certain «background» in the absence of a radioactive preparation. The presence of background can be caused by cosmic radiation or radioactive contamination. What is the radon mass $m = 1$ corresponding to the background that gives 1 counter rejection in a time $t = 5\,s$? ($m = 3.5 \cdot 10^{-20}\ kg$).

520. The activity of some radioactive isotope is investigated using an ionization counter. At the initial time, the counter gives 75 scraps in a time $t = 10\ s$. How many scraps in a time $t = 10\ s$ give a counter after the time $t = T_{1/2}/2$? Read $T_{1/2} >> 10\,s$. (53 *garbage*).

521. Some radioactive isotope has a decay constant of $10^{-7}\ s^{-1}$. What time will 75% of the original mass of atoms decay? (t = 40 $days$).

522. Natural uranium is a mixture of three isotopes $U_{92}^{234}, U_{92}^{235}, U_{92}^{238}$. The content is negligible (0.006%), it accounts U_{92}^{235} for 0.71%, and the rest mass (99.28%) is U_{92}^{238}. The half-lives of $T_{1/2}$ of these isotopes are $2.5 \cdot 10^{5}$ years, $7.1 \cdot 10^{8}$ years and $4.5 \cdot 10^{9}$ years, respectively. Find the percentage of radioactivity contributed by each isotope to the total radioactivity of natural uranium. (${}^{238}_{92}U,\ {}^{235}_{92}U,\ {}^{234}_{92}U$).

523. Kinetic energy of an α-particle emitted from the nucleus of a radium atom in radioactive decay $W_1 = 4.78\,MeV$. Find the velocity υ of the α particle and the total energy W that is released when the α particle is emitted. ($\upsilon = 1.52 \cdot 10^{7}\ m/s, W = 4.87\ MeV$).

524. How much heat Q is released during the decay of radon by the activity $a = 3.7 \cdot 10^{10}\ Bq$: a) for a time $t = 10\,h$; b) for the average lifetime τ?

The kinetic energy of the α-particle emitted from the radon is $W = 5.5\ MeV$. ($a) Q = 0.12 \cdot kJ, b) Q = 16\ kJ$).

525. The mass $m = 1\ g$ of uranium U_{92}^{238}, in equilibrium with the products of its decay, releases power $P = 1.07 \cdot 10^{-7}\ W$. Find the molar heat Q_μ, released by uranium for the average lifetime τ of uranium atoms. ($Q_\mu = 5.2 \cdot 10^{12} \cdot J / mol$).

526. Find the activity of a radon, formed from the mass $m = 1\ g$ of radium in a time $t = 1\ h$. ($a = 2.8 \cdot 10^8\ Bq$).

527. As a result of the decay of the mass $m = 1\ g$ of radium, during a time $t = 1$ year a certain mass of helium was formed, occupying a volume $V = 43\ mm^3$ under normal conditions. Find from these data the constant Avogadro. ($N_A = 6 \cdot 10^{23}\ mol^{-1}$).

528. A drug containing a mass of $m_0 = 1.5\ g$ of radium is placed in the ampoule. What is the mass m of radon accumulated in this ampule after the time $t = T_{1/2} / 2$, where $T_{1/2}$ is the half-life of radon? ($m = 4.8 \cdot 10^{-9}\ kg$).

529. A number of radium atoms are placed in a closed vessel. What time t the number of radon atoms N in this vessel will differ by 10 % of the number of radon atoms $N`$, which corresponds to the radioactive equilibrium of radium with radon in this vessel? Draw a curve for the dependence of the variation of $N / N`$ in the vessel on the time t in the interval $0 \le t \le 6T_{1/2}$, taking the radon half-life $T_{1/2}$ as a unit of time. ($t = 12.6\ days$).

530. Some number of radon atoms $N`$ is placed in a closed vessel. Draw a curve for the dependence of the change in the number of radon atoms $N / N`$ in the vessel on time in the interval $0 \le t \le 20$ days every 2 days. Radon decay constant $\lambda = 0.181$ days^{-1}. From the curve $N / N` = f(t)$ find the half-life of $T_{1/2}$ radon. ($T_{1/2} = 3.8\ days$).

531. The following table shows the results of measuring the dependence of the activity a of a radioactive element on the time t. Find the half-life of the $T_{1/2}$ element. ($T_{1/2} \approx 4\ h$).

532. Radon is placed in the ampoule, the activity of which is $a = 14.8 \cdot 10^9\ Bq$. What time t after filling the vial the activity of radon will be equal to $a = 2.23 \cdot 10^9\ Bq$? ($t = 10.4\ days$).

533. The lead contained in the uranium ore is the final product of the decay of the uranium series, so the age of the ore can be determined from the ratio of the mass of uranium in the ore to the mass of lead in it. Find the age of the uranium

ore if it is known that the weight $m_w = 1\,kg$ of uranium ${}^{238}_{92}U$ in this ore accounts for the mass $m_{pb} = 320\,g$ of lead ${}^{205}_{82}Pb$. ($t = 3\cdot10^9$ *years*).

534. Knowing the half-lives of $T_{1/2}$ radium and uranium, find the number of uranium atoms per radium atom in natural uranium. Consider that the radioactivity of natural uranium is due mainly to the isotope ${}^{238}_{92}U$. ($N = 2.8\cdot10^6$).

535. What minimum mass of ore containing 42% of pure uranium, can one obtain a mass $m_0 = 1\,g$ of radium? ($m = 7\cdot10^3\ kg$).

536. α-particles from the radium isotope emerge at a velocity $\upsilon = 1.5\cdot10^7\ m/s$ and hit the fluorescent screen. Assuming that the screen consumes $P_I = 0.25\,W/cd$ power per unit of light intensity, find the light intensity I of the screen if all α particles emitted by the mass $m = 1\,\mu g$ of radium. ($I = 1.1\cdot10^{-7}\ cd$).

537. What proportion of the initial mass of the radioactive isotope decays during the lifetime of this isotope? (63.2 %).

538. Find activity a and mass $m = 1\,\mu g$ of polonium Po^{210}_{84}? ($a = 1.67\cdot10^8\ Bq$).

539. Find the specific activity of the artificially obtained radioactive isotope strontium Sr^{90}_{38}? ($a_m = 5.25\cdot10^{15}\ Bq/kg$).

540. To the mass $m_1 = 10\,mg$ of radioactive isotope Ca^{45}_{20}, the mass $m_2 = 30\,mg$ of non-radioactive isotope was added Ca^{40}_{20}. How much decreased the specific activity a of the at radioactive source? ($\Delta a_m = 4.9\cdot10^{17}\ Bq/kg$).

541. What is the mass of m_2 of the radioactive isotope Bi^{210}_{83} that must be added to the mass $m_1 = 5\,mg$ of the non-radioactive isotope Bi^{210}_{83}, so that after a time $t = 10$ days after that the ratio of the number of decayed atoms to the number of non-decaying atoms is 50%? The decay constant of the isotope is $\lambda = 0.14$ day^{-1}. ($m_2 = 11\,mg$).

542. Which isotope is formed from Th^{232}_{90} after 4-α decays and 2-β decays. (${}^{216}_{84}Po$).

543. Which isotope is formed from U^{238}_{92} after three α-decays and two β-decays? (${}^{226}_{88}Ra$).

544. Which isotope is formed from U^{239}_{92} after two β-decays and one α-decay? (${}^{235}_{92}U$).

545. Which isotope is formed from Li_3^8 after one β-decay and one α-decay? ($_2^4He$).

546. Which isotope is formed from after S_{51}^{133} four β-decays? ($_{55}^{133}Cs$).

547. Kinetic energy of an α-particle emitted from the nucleus of a polonium atom Po_{84}^{214} in radioactive decay $W_k = 7.68\,MeV$. Find: a) the velocity υ of the α particle; b) the total energy W, which is released during the emission of the α particle; c) the number of pairs of N ions formed by the α particle, assuming that the energy $W_0 = 34\,eV$ is required to form one pair of ions in air; d) saturation current I_n in the ionization chamber from all α-particles emitted by polonium. The activity of polonium $a = 3.7\cdot 10^4\ Bq$. ($a)\ \upsilon = 1.92\cdot 10^7\ m/s,\ b)\ W = 7.83\,MeV,$ $c)\ N = 2.26\cdot 10^5,\ d)\ I = 1.33\cdot 10^{-9}\ A$).

7.15 NUCLEAR REACTIONS

548. Find the number of protons and neutrons that make up a) Mg_{12}^{24}, b) Mg_{12}^{25}; at) Mg_{12}^{26}. (*a) 12 protons and 12 neutrons, b) 12 protons and 13 neutrons, c) 12 protons and 14 neutrons*).

549. Find the binding energy of the isotope core Li_3^7. ($W = 39.3\,MeV$).

550. Find the binding energy of the isotope nucleus He_2^4. ($W = 28.3\,MeV$).

551. Find the binding energy of the nucleus of an aluminum atom Al_{13}^{27}. ($W = 225\,MeV$).

552. Find the binding energy W of the nuclei: a) H_1^3; b) He_2^3. Which of these cores is more stable? ($a)\ W = 8.5\,MeV,\ b)\ W = 7.7\,MeV$).

553. Find the binding energy W_0 per nucleon in the nucleus of the oxygen atom O_8^{16}. ($W_0 = 7.97\,MeV$).

554. Find the binding energy W of the deuterium nucleus H_1^2. ($W = 2.2\,MeV$).

555. Find the binding energy W_0 per nucleon in the nuclei: a) Li_3^7, b) N_7^{14}, c) Al_{13}^{27}, d) Ca_{20}^{40}, f) Cu_{29}^{63}, e) Cd_{48}^{113}, g) Hg_{80}^{200}, h) U_{92}^{238}. Construct the dependence $W_0 = f(A)$, where A is the mass number.
($a)\ W = 5.6\,MeV,\ b)\ W = 7.5\,MeV,\ c)\ W = 8.35\,MeV,$
$d)\ W = 8.55\,MeV,\ e)\ W = 8.75\,MeV,\ f)\ W = 8.5\,MeV,$
$g)\ W = 7.9\,MeV,\ h)\ W = 7.6\,MeV$).

556. Find the energy released during the reaction $Li_3^7 + H_1^1 \to He_2^4 + He_2^4$. ($Q = 17.3\,MeV$).

557. Find the energy Q absorbed during the reaction $N_7^{14} + He_2^4 \to H_1^1 + O_8^{17}$. ($Q = 1.18\,MeV$).

558. Find the energy ΔW that is released during the reaction: a) $H_1^2 + He_2^3 \to H_1^1 + H_2^4$, b) $H_1^2 + H_1^2 \to He_2^3 + n_0^1$. ($a)\ Q = 4.04\,MeV, b)\ Q = 3.26\,MeV$).

559. Find the energy Q, released during the reactions: a) $H_1^2 + He_2^3 \to H_1^1 + H_2^4$, b) $Li_3^6 + H_1^2 \to He_2^4 + He_2^4$, c) $Li_3^6 + H_1^1 \to He_2^3 + He_2^4$. ($a)\ Q = 18.3\,MeV, b)\ Q = 22.4\,MeV, c)\ Q = 4.02\,MeV$).

560. What mass m of water can be heated from 0°C to boiling if all the heat released during the reaction Li_3^7 $s(p,\alpha)$ is used, with the total mass decomposition of $m = 1\,g$ of lithium? ($m = 570\,t$).

561. Write the missing designations in the reactions :

$Al_{13}^{27}(n,\alpha)x$, b) $F_9^{19}(p,x)$ O_8^{16}, c) $Mn_{25}^{55}(x,n)$ Fe_{26}^{55}, d) $Al_{13}^{27}(\alpha, p)x$, e) $N_7^{14}(n,x)$ O_6^{14}, f) $x(p,\alpha)\ Na_{11}^{22}$.

562. Find the energy Q released during the reaction $Li_3^7 + H_1^2 \to Be_4^8 + n_0^1$. ($Q = 15\,MeV$).

563. Find the energy Q released during the reaction $Be_4^9 + H_1^2 \to B_5^{10} + n_0^1$. ($Q = 4.35\,MeV$).

564. When a nitrogen isotope N_4^7 is bombarded with neutrons, a carbon isotope C_5^{14} is obtained, which turns out to be β-radioactive. Write the equations of both reactions. (${}^{14}_{7}N + {}^{1}_{0}n \to {}^{14}_{6}C + {}^{1}_{1}H$, ${}^{14}_{6}C \to {}^{0}_{-1}e + {}^{14}_{7}N$).

565. When an isotope of aluminum Al_{13}^{27} is bombarded with α particles, a radioactive isotope of phosphorus P_{15}^{30} is obtained, which then decays with the release of a positron. Write the equations of both reactions. Find the specific activity a_μ of the at isotope P_{15}^{30} if its half-life $T_{1/2} = 130\,s$. ($a_m = 1.1 \cdot 10^{23}\ Bq/kg$).

566. When a lithium isotope Li_3^6 is bombarded with deuterons (deuterium nuclei H_1^2), two α-particles are formed. In this case, the energy $Q = 23.3\,MeV$ is released. Knowing the masses of the deuteron d and α particle, find the mass m of the lithium isotope Li_3^6. ($m = 6.015\,a.m.u$).

567. The neutron source is a tube containing beryllium Be_4^9 powder and gaseous radon. During the reaction of α-particles of radon with beryllium, neutrons arise. Write the reaction for obtaining neutrons. Find the mass m of radon introduced into the source during its manufacture, if it is known that this source gives, after a time $t=5$ days after its manufacture, the number of neutrons per unit time $a_2 = 1.2 \cdot 10^6\ s^{-1}$. The reaction yield $k = 1/4000$, i.e, only one α-particle from $n = 4000$ causes the reaction. ($m = 2.1 \cdot 10^{-9}\ kg$).

568. In the reaction $N_7^{14}(\alpha,\ p)$, the kinetic energy of the α particle is $W_1 = 7.7\ MeV$. At what angle φ does the proton fly out to the direction of motion of the α particle if it is known that its kinetic energy is $W_2 = 8.5\ MeV$? ($\varphi = 32^0$).

569. Find the threshold W of the nuclear reaction $N_7^{14}(\alpha,\ p)$. ($W = 1.52\ MeV$).

570. Find the threshold W of the nuclear reaction $Li_3^7(p, n)$. ($W = 1.89\ MeV$).

571. When a lithium isotope Li_3^7 is bombarded with protons, two α particles are formed. The energy of each α-particle at the time of their formation is $W_2 = 9.15\ MeV$. What is the energy of W_1 bombarding protons? ($W_1 = 1\ MeV$).

572. Find the lowest energy of a γ-quantum sufficient to carry out the reaction $Mg_{12}^{24}(\gamma, n)$. ($h\nu = 16.6\ MeV$).

573. What is the mass of uranium U_{92}^{235} consumed for a time $t = 1$ day at a nuclear power plant with a power $P = 5000\ kW$? The acceptance rate is 17%. To consider that for each act of decay energy is released $Q = 200\ MeV$. ($m = 31\ g$).

574. In nuclear physics, the number of charged particles bombarding a target is assumed to be characterized by their total charge expressed in microampere-hours (μAh). What number of charged particles does the total charge $q = 1\ \mu A/h$ correspond? The problem to be solved for: a) electrons; b) α-particles. ($a)\ N = 2.2 \cdot 10^{16}, b)\ N = 1.1 \cdot 10^{16}$).

575. In the case of an elastic central collision of a neutron with a stationary nucleus of a retarding substance, the kinetic energy of a neutron decreased by a factor of 1.4. Find the mass m of the nuclei of the slowing agent. ($m = 12\ a.m.u$).

576. What part of the initial speed will be the neutron velocity after an elastic central collision with the fixed isotope core $^{23}_{11}Na$? (92%).

577. To obtain slow neutrons, they are passed through substances containing hydrogen (for example, paraffin). What is the largest part of its kinetic energy that a neutron of mass m_0 can transmit to a) a proton (mass m_0), (b) the nucleus of a lead atom (mass $207\,m_0$)? The largest part of the transmitted energy corresponds to an elastic central collision. ($a) \approx 100\,\%; b)\, 1.9\,\%$).

578. The stream of charged particles enters a homogeneous magnetic field with induction $B = 3\,T$. The velocity of the particles is $\upsilon = 1.52 \cdot 10^7\ m/s$ and is directed perpendicular to the direction of the field. Find the charge q of each particle if it is known that the force $F = 1.46 \cdot 10^{-11}\ N$ acts on it. ($q = 3.2 \cdot 10^{-19}\ C$).

579. The meson of cosmic rays has an energy $W = 3\,GeV$. The rest energy of the meson is $W_0 = 1000\,MeV$. What distance l in the atmosphere can a meson pass during its lifetime τ by the laboratory clock? The intrinsic lifetime of the meson is $\tau_0 = 2\ \mu s$. ($l \approx 18\,km$).

580. The positron and the electron combine to form two photons. Find the energy $h\nu$ of each of the photons, assuming that the initial energy of the particles is negligible. What is the wavelength λ of these photons? ($W = 0.58\,MeV; \lambda = 2.4\ pm$).

581. The electron and positron are formed by a photon with energy $h\nu = 2.62\,MeV$. What was the total kinetic energy $W_1 + W_2$ of a positron and an electron at the time of origin? ($W_1 + W_2 = 1.60\ MeV$).

582. The neutron and antineutron combine to form two photons. Find the energy $h\nu$ of each of the photons, assuming that the initial energy of the particles is negligible. ($h\nu = 940\ MeV$).

583. The maximum radius of curvature of the trajectory of particles in the cyclotron $R = 35\,cm$; frequency of the potential difference applied to the duants is $\nu = 13.8\,MHz$. Find the magnetic induction B of the field necessary for synchronous operation of the cyclotron, and the maximum energy W of the emitted protons. ($B = 0.9\,T, W = 4.8\ MeV$).

584. Ion current in the cyclotron when working with α-particles $I = 15\ \mu A$. How many times is such a cyclotron more productive than mass $m = 1\ g$ of radium? (*a thousand times*).

585. The cyclotron produces deuterons with an energy $W = 7\ MeV$. The magnetic induction of the cyclotron field is $B = 1.5\ T$. Find the maximum radius of curvature R of the deuteron trajectory. ($R = 36\ cm$).

586. What energy W can the α particles be accelerated in the cyclotron if the relative increase in the particle mass $k = (m - m_0)/m_0$ should not exceed 5%? ($W = 188\ MeV$).

587. Energy of deuterons accelerated by a synchrotron, $W = 200\ MeV$. Find for these deuterons the ratio m/m_0 (where m is the mass of the moving deuteron and m_0 is its rest mass) and the velocity υ. ($m/m_0 = 1.1$; $\beta = 0.44$; $\upsilon = 1.32 \cdot 10^8\ m/s$).

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м. Кропивницький,
вул. Добровольського, 1,
тел.. 39-44-37.